A new approach using hybrid power series – cuckoo search optimization algorithm to solve electrostatic pull-in instability and deflection of nano cantilever switches subject to van der waals attractions

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Abstract

A hybrid Power Series (PS) and Cuckoo Search via L’evy Flights (CS) optimization algorithm (PS-CS) method is utilized to obtain a solution for the deflection and pull-in instability of a nano cantilever switch in the presence of the van der Waals attractions, electrostatic forces and fringing filed effects. In order to obtain a relation for deflection of the beam, a trial solution including adjustable coefficients, satisfying the boundary conditions of the governing, is proposed. The cuckoo search optimization algorithm is executed to find the adjustable parameters of the trial solution satisfying the governing equation of the nanobeam. The results are compared with the available results in the literature as well as numerical solution. The results indicate the remarkable accuracy of the present approach. The minimum initial gap and the critical free standing detachment length of the nano actuator that does not stick to the substrate due to the van der Waals attractions, as an important parameter in pull-in instability of the nano switches, is calculated. Utilizing the results of the PS-CS, the stress distribution inside the nano actuator is determined at the onset of the pull-in instability.

Keywords: Cantilever Nano Actuator; Pull-In Instability; Electromechanical Switches.

1. Introduction

Many of the micro and nano devises including, switches in some microchips, pressure sensors, chemical sensors, values and pumps, are constructed using suspended beams and plates [1-3]. A cantilever nano switch is a beam suspended over a substrate. Applying voltage difference between the beam and the substrate causes the beam to deflect. The defection of the beam is determined at the onset of the pull-in instability. Lin and Zaho [7] utilized a lumped model to analyze the pull-in instability of the micro actuators. They obtained analytical relations for pull-in instability of the actuators. Soroush et al. [3] utilized a distributed parameter model to examine the pull-in instability and deflection of the nano actuators. They [3] utilized the Adomian decomposition method to obtain a solution for deflection and pull-in instability of nano actuators. The results show that the Adomian series are capable to obtain a solution for pull-in instability of the nano actuators; however, the accuracy of the results is not good. Hence, Noghrehabadi et al. [9] tried to increase the accuracy of the Adomian power series solution by using the Pade approximations. The Pade approximants increased the accuracy of the solution. However, Pade approximants simplified the power series solution to a rational form, and in some cases the denominators could take zero digits which in this case the results are not applicable. Ramezani et al. [9, 10] utilized the Green method to obtain a closed-form solution of the pull-in instability in nanocantilevers. They [9, 10] assumed a second order shape function for the shape of nanobeam. Recently, artificial intelligence techniques have been utilized to solve various types of differential equations [11-14]. Meade and Fernandez [11] and Lagaris, Likas, Nakos [12] proposed the neural networks as a trial function with adjustable parameters for solving boundary value differential equations. Malek and Beidokhti [13] proposed a new approach for solving...
differential equations using a power series as a trial function with adjustable parameters and the Nelder-Mead optimization algorithm to adjust the parameters.

Cuckoo Search via L’evy Flights algorithm proposed by Yang and Deb [15] as a population base optimization algorithm. This algorithm mimics the obligate brood parasitism of some cuckoo species which lay their eggs in the nests of other Cuckoo species. The effectiveness of this algorithm has been demonstrated in many recent studies [16-18].

In the present study a combination of power series and Cuckoo search via Levy flights optimization algorithm is proposed as a new approach to obtain a relation for deflection of nano cantilever beams in the presence of van der Waals attractions. The pull-in instability, free standing length of the beam and the Stress resultants are also evaluated.

2. Mathematical model

A schematic view of a nano-cantilever beam suspended over a conductive substrate is depicted in Fig. 1.

![Fig. 1: Schematic View of the Physical Model of the Cantilever Nano Switch.](image)

The cross section of the beam is rectangular with thickness of h and width w. Length of the beam is denoted by L. The initial gap space between the cantilever beam and substrate plane is denoted by g. Considering the van der Waals attractions, electrostatic forces and fringing effects, the governing equation for the distributed model of the beam is written as [3]:

\[ \frac{d^4u}{dx^4} = \frac{\alpha}{(1-u(x))^3} + \frac{\beta}{(1-u(x))^2} + \gamma \frac{\partial^2u}{\partial x^2} \]  

(1-a)

Subject to the following boundary conditions at the basis and the natural end:

\[ u(0)=u'(0)=0, \text{ at } x=0 \quad \text{and} \quad u''(1)=u'''(1)=0, \quad \text{ at } x=1 \]  

(1-b)

Where x is the non-dimensional position along the beam measured from the clamped end, and prime denotes the differentiation with respect to x. α is the non-dimensional van der Waals parameter, β is the non-dimensional electrostatic parameter and γ is the non-dimensional fringing filed parameter. The non-dimensional parameters are defined as follow [3]:

\[ \alpha = \frac{Asw^4}{6E_\text{eff}I}, \quad \beta = \frac{egwV^2}{2E_\text{eff}I}, \quad \gamma = 0.65 \frac{g}{w}, \quad x = \frac{X}{L}, \quad u = \frac{y}{g} \]  

(2)

Where y is the beam deflection, \( \varepsilon_0 = 8.854 \times 10^{-12} \text{C}^2\text{N}^{-1}\text{m}^{-2} \) is the permittivity of the vacuum, \( V \) is the applied external voltage, \( h = 1.055 \times 10^{-34} \text{Js} \) is Planck’s constant divided by \( 2\pi \), and \( c = 2.998 \times 10^8 \text{m/s} \) is the light speed. \( E_\text{eff} \) is the effective Young’s modulus, and I is the moment of inertia of the beam cross section (wh\(^4\)/12) [3].

3. Cuckoo search L’evy flight

Yang and Deb [15] proposed the Cuckoo Search L’evy flight (CS) method as a new meta-heuristic algorithm to deal with unconstrained optimization problems. The Cuckoo search L’evy flight algorithm is inspired by the obligate brood parasitic behavior of some cuckoo species in combination with the L’evy flight behavior of some birds and fruit flies in nature. CS method simply follows three rules as follows [15]:

1) Each cuckoo lays one egg at a time, and dump its egg in a randomly chosen nest; 2) The best nests with high quality of eggs will carry over to the next generations; 3) The number of available host nests is fixed, and the egg laid by a Cuckoo is discovered by the host bird with a probability \( p_c \in [0,1] \).

As the CS method tries to maximize a fitness function, in the case of minimization the fitness function is considered as the minus of the original fitness function. Each egg in a nest symbolizes a solution, and a cuckoo egg symbolizes a new candidate solution. The goal is to use the new and potentially better solutions (cuckoo egg) to replace a not so good solution in the nests. Utilizing the mentioned three basic rules of CS method, the main steps of the CS algorithm are listed as the pseudo code in Fig. 2.

![Fig. 2: Pseudo Code of the Cuckoo Search (CS) [15].](image)

For generating a new solution (a new cuckoo egg) for next generation \( x^{(i+1)} \), a L’evy flight is required to be performed as follow (3) where \( \kappa > 0 \) is the step size of the search, related to the optimization parameter scales of the optimization problem. The default use

\[ x_1^{(i+1)} = x_1^{(i)} + \kappa \times \text{rand}^{-\lambda}, \quad 1 \leq \lambda \leq 3 \]  

Of \( \kappa = 1 \)

Is a good choice in most cases. The product denotes the entry wise multiplication which is similar to those used in Particles Swarm Optimization (PSO) method [19, 20].

More details about the implementation of the cuckoo search L’evy flight method can be found in the work of Yang and Deb [15].

4. Problem formulation

The deflection of the beam subject to the external forces can be obtained from the solution of Eq. (1-a) subject to the boundary conditions of Eq. (1-b). In order to find a solution for Eq. (1), consider a uniform discretization of the governing differential equation of Eq. (1-a) with m arbitrary points [12, 13]. In each discretized point the governing equation can be written as:
Eq. (4) represents Eq. (1-a) is discretized points. Now, let’s assume \( \gamma_T(x, \bar{a}) \) as a function (a trial solution) which approximate the solution to Eq. (1). \( \bar{a} \) denotes a vector containing the adjustable parameters. As the solution of Eq. (1) should satisfy the governing equation, the Eq. (4) should hold true for each point in the domain of the solution. Substituting the assumed trial solution in Eq. (4) should also satisfy the equation in each discrete point of the domain of the solution. The error of the trial solution in the domain of the solution will be computed using the following equation:

\[
E(t_i) = \sum_{i=1}^{m} \left( \frac{d^4u(x_i)}{dx^4} \right) + \left( \frac{\alpha}{(1-u(x_i))^2} \right) + \left( \frac{\beta}{(1-u(x_i))} \right), \quad x_i \in [0, 1]
\]

The value of \( E(t_i) \) in Eq. 5 should be zero if the trial solution satisfies the governing differential equation, Eq. (1). However, as \( \gamma_T(x, \bar{a}) \) is the approximate solution of Eq. (1), the value of \( E(t_i) \) is not zero. The boundary conditions of the governing equation should also be satisfied. Therefore, the trial solution is written as the sum of two parts, the first part will satisfy the boundary conditions exactly. The second part with the aid of the first part constructs a trial solution for Eq. (1) as follow:

\[
\gamma_T(x, \bar{a}) = a_1x^5 + a_2x^4 - (10a_1 - 4a_2)x^3 \\
+ (20a_1 + 6a_2)x^2 + x^2(x-1)^2N(x, \bar{a})
\]

(6)

Where the coefficients of \( a_1 \) and \( a_2 \) as well as the vector of \( \bar{a} \) are the adjustable parameters. The term of \( N(x, \bar{a}) \) is a power series in the form of \( \sum_{i=1}^{m} a_i u_i \) involving the remaining adjustable parameters (i.e. \( a_1 \ldots a_m \)). It should be noticed that the trial function in the form of Eq. (6) exactly satisfies the boundary conditions of Eq. (1-b). Now, the adjustable parameters of \( a_1, a_2 \) and \( \bar{a} \) should be evaluated such a way that \( E(t_i) \) in Eq. (5) be minimum. Here, the Cuckoo search l’evy flight is utilized to find the values of \( a_1 \) and \( a_2 \) and the vector \( \bar{a} \) regarding to minimize \( E(t_i) \) in Eq. (6). The Cuckoo search l’evy flight optimization algorithm is coded using MATLAB 2009 program. The calculations were also executed using the MATLAB Software.

5. Results

21 sample points (\( m=21 \)) with uniform space of 0.5 were chosen in the domain of the solution (\( x_i \in \{0, 0.05, 0.01 \ldots 1\} \)). In the Cuckoo search l’evy flight optimization algorithm the number of the nests (nests) and the fraction of worse nests (\( P_a \)) were fixed at 15 and 0.25 respectively. 300 iterations were also selected for generations number (Number of Iterations).

As a test of the convergence of the solution, the deflection of a typical nano cantilever actuator (\( a=0.3, \beta=0.2, g/w=1.0 \)) is computed. In this case, the results of present study are compared with the numerical results as well as the results of Adomian decomposition method [3]. The numerical results are obtained using the dsolve function of MAPLE 14.0 mathematical software [21], [22]. Table 1 shows the variation of the cantilever tip deflection (\( u_{wp} \)) as a function of the dimensionless length of the beam (\( x \)) for different selected terms of the size of the trial solution. The error in Table 1 denotes the difference between the tip deflection (evaluated using the Adomian method [3] or the present solution) and that of the numerical solution. Table 1 shows that the trial solution with six terms of the power series or more provides excellent results.

<table>
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<tr>
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<tr>
<td>Numeral</td>
<td>0.09181</td>
<td>0.09788</td>
</tr>
<tr>
<td>4 Terms series</td>
<td>( u_{wp} ) = 0.078</td>
<td>Errors = 0.0121</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.005698</td>
</tr>
<tr>
<td>6 Terms series</td>
<td>( u_{wp} ) = 0.0986</td>
<td>Errors = 0.0077</td>
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<tr>
<td></td>
<td></td>
<td>0.090810</td>
</tr>
<tr>
<td>7 Terms series</td>
<td>( u_{wp} ) = 0.0987</td>
<td>Errors = 0.0038</td>
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<tr>
<td></td>
<td></td>
<td>3.07E-05</td>
</tr>
<tr>
<td>8 Terms series</td>
<td>( u_{wp} ) = 0.0940</td>
<td>Errors = 0.0032</td>
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<td></td>
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<td>2.06E-06</td>
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<tr>
<td>9 Terms series</td>
<td>( u_{wp} ) = 0.0886</td>
<td>Errors = 0.0022</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.86E-07</td>
</tr>
<tr>
<td>10 Terms series</td>
<td>( u_{wp} ) = 0.0922</td>
<td>Errors = 0.0014</td>
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<td>1.50E-08</td>
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It is also clear that as the size of the power series in the trial solution rises the accuracy of the solution would also increases. Table 1 also shows that eight terms of the power series in the trial solution provides results with error of the order of \( 10^{-5} \), but the Adomian series solution with eight terms of the series solution provides results with error of the order of \( 10^{-1} \). Therefore, it is clear that the present method with the same size of the series could provide very accurate results. As the eight terms in the trial solution provide accuracy, the results in the rest of the paper are evaluated using eight terms of the power series for convenience.

The corresponding trial power series and the power series obtained by Adomian method [3] for the results of Table 1 (i.e. \( a=0.3, \beta=0.2 \) and \( g/w=1 \)) can be summarized as follows respectively:

\[
u(x) = 0.178337302x^2 - 0.113517738x^3 + 0.02487749x^4 + 0.000028350x^5 + 0.002011236x^6 - 0.001214524x^7 + 0.000285856x^8
\]

(7)

\[
u(x) = 0.183945211x^2 - 0.116761146x^3 + 0.026250000x^4 - 0.000730671253x^5 - 0.000198771951x^7 + 0.000732989145x^8
\]

(8)
Indeed, Eqs. (7) and (8) represent the non-dimensional shape of the cantilever beam after deflection.

5.1. Pull-in instability study

The pull-in stability of the cantilever nano switch can be evaluated using the solution of Eq. (1) by setting $du(1)/dξ→∞$ where $ξ$ is the parameter of study (i.e. $α$, $β$ or $g/w$). When the nano switch reaches to the pull-in instability any increase in the magnitude of $ξ$ would results in large deflection of the beam (i.e. large variation of tip displacement). In the pull-in condition, the values of $α$, $β$ or $g/w$ are denoted by subscript of PI. Any raise of $ξ$ to the values larger than the pull-in value, results in instability of the beam; in this case the governing equation cannot provide any real solution for deflection of the beam. In a case in which there is not any applied voltage between the electrode and the substrate, the suspended beam is solely subject to the intermolecular force. If the van der Waals attraction be large enough to induce pull-in instability the nano actuator will switch without any applied external voltage. In this situation the non-dimensional value of van der Waals parameter can be obtained by setting $β=0$ and $du(1)/dξ→∞$.

5.1.1. Free standing parameters

As mentioned in the previous section, when the separation space ($g$) between the substrate and the suspended cantilever beam is small enough, the movable beam might collapse onto the substrate because of the van der Waals attraction forces without applying any external voltage. Fig. 3 depicts the variation of non-dimensional shape of the cantilever parameter ($u_{PI}$) as a function of the van der Waals parameter ($α$) when the applied external voltage difference is zero ($β=0$).

$$u(x) = \frac{0.607231800x^2 - 0.298579773x^3 + 0.001136843x^4 - 0.001377942x^5 + 0.058577877x^6 - 0.043236974x^7 + 0.011599078x^8}{0.605 E h_{\text{eff}} L w}$$

(9)

A comparison between the results of present study (PS-CS method) and the results of previous studies is performed in Fig. 3. This figure shows that the accuracy of present PS-CS method is excellence compared with the numerical method. For a nano switch with a specified value of the separation space ($g$) and in the absence of any external voltage, the maximum free standing length of the actuator, in which the nano actuator does not go through the pull-in instability, is called the detachment length ($L_{\text{max}}$) [10]. Alternatively, for a specified cantilever nano switch with an specified value of the length ($L$), there is a minimum separation space, ($g_{\text{min}}$), in which the cantilever beam would not stick to the substrate because of the van der Waals attraction forces [10]. The detachment length ($L_{\text{max}}$) and minimum separation space ($g_{\text{min}}$) of the cantilever actuator are the basic parameters for design of nano electro mechanical switches. These parameters can be evaluated by using the critical value of van der Waals parameter, i.e. $αC$. Substituting the critical value of the van der Waals parameter ($αC=1.21$) into the definition of van der Waals parameter (i.e. $α$ in Eq. 2); the relations for evaluation of $L_{\text{max}}$ and $g_{\text{min}}$ are obtained as follow:

$$L_{\text{max}} = \left( \frac{0.605 \pi E_{\text{eff}} h_{\text{eff}}^2}{w} \right) ^{\frac{1}{4}}$$

$$g_{\text{min}} = \left( \frac{L_{\text{w}}^4}{0.605 \pi E_{\text{eff}} h_{\text{eff}}^2} \right) ^{\frac{1}{4}}$$

(10)

5.1.2. Electrostatic force

Figs. (4) and (5) show the tip deflection of a wide ($g/w=0$) and narrow ($g/w=1$) cantilever beam as a function of electrostatic parameter, respectively, when the van der Waals attractions are negligible ($α=0$). Figs. (4) and (5) show that the tip deflection of a narrow beam is larger than the tip deflection of a wide beam. A comparison between the results of PS-CS and previous results is shown in Figs. (4) and (5). The results show that the lamped parameter model cannot properly follow the variation of tip deflection accurately. The lamped parameter model under estimated the tip deflection while the Green method overestimated the tip deflection of the nano beam.

![Fig. 3: Tip Deflection as A Function of Van Der Waals Parameter A.](image1)

![Fig. 4: Tip Deflection as A Function of Electrostatic Force (B) when A=0 and G/W=0.](image2)

![Fig. 5: Tip Deflection as A Function of the Electrostatic Parameter (B) when A and G/W=1.](image3)
Figs (6) and (7) show the effect of fringing filed (for different values of g/w parameter) on the pull-in values of elector static parameter (β_P) and the corresponding tip deflection of the nano beam u_P respectively, when α=0.

Fig. (6) demonstrates that the Lamped parameter method [3] as well as the Adomian method [3] underestimated the pull-in value of the electro static parameter (β_P) while the Greens method [10] overestimated magnitude of β_P. Fig. (7) shows that the Green method [10] and the Adomian method [3] overestimated the tip deflection of the beam at the pull-in instability, and the lamped parameter method [3] under estimated the pull-in tip deflection of the beam. Figs. (6) and (7) show that the evaluated pull-in values of β_P and u_P are in very good agreement with the numerical results.

5.1.3. Electrostatic and intermolecular force at nanoscale separations

Fig. (8) shows the deflection of a cantilever nano-beam subject to simultaneous effects of electrostatic and van der Waals forces. This figure shows that when the applied external voltage is zero (β=0) the shape of the beam is not at rest, and There is an initial deflection is because of the van der Waals forces (α=0.5). A comparison between the evaluated shape of the beam using the numerical method, eight terms of Adomian series [3] and eight terms of PS-CS method is performed in Fig. 8. As seen, the difference between Adomian results and the numerical solution is significant at the onset of pull-in instability. The results of PS-CS method indicate that the pull-in occurs at β_P=0.66 when α=0.5 and g/w=1. The corresponding power series is obtained as follow:

\[
u(x) = 0.742382895x^2 - 0.371064051x^3 + \\
0.006908911x^4 - 0.002064628x^5 + 0.070527153x^6 \\
-0.052763152x^7 + 0.014290035x^8
\]  

(11)

Fig. (9) Shows the pull-in value of the electro static parameter (β_P) as a function of van der Waals parameter (α_P) for the narrow (g/w=1) and wide (g/w=0) beams. This figure depicts that as the intermolecular force (α) increases the required applied voltage (β_P) in which the pull-in instability occurs decreases. The variation of electro static parameter as a function of the van der Waals parameter is almost linear. The pull-in tip deflection of the beam as a function of the van der Waals parameter is plotted in Fig. (10). This figure shows that as the van der Waals forces increases the tip deflection of the beam increases. A comparison between the results of PS-CS, numerical results as well as Adomian [3] and lamped model [3] is performed in Figs. (9) and (10). As seen the results of PS-CS are in very good agreement with the numerical results.

Fig. 8: Deflection of A Typical Nano-Beam Under Both Electrostatic and Intermolecular Loading when B Increases from Zero to Instability Point. A=0.5 and G/W=1, Collapse Occurs when B Reaches Values Greater Than Its Critical Value B=0.66

Fig. 9: Relationship Between A and B Pull-In for Narrow and Wide Beams.
5.1.4. Stress resultants

The stress distribution in the nanobeam is one of the basic requirements in the nano switches designs. The maximum value of the shear stress at the onset of the pull-in instability is very important because of its crucial effect on the failure of the switch. Utilizing the Euler beam theory, the shear stresses can be directly evaluated using the stress resultants [23]. In order to determine the critical values of stress resultants, $F_{\text{Pl, max}}$ and $M_{\text{Pl, max}}$ are introduced as the dimensionless maximum value of the shear force and bending moment at the onset of instability, respectively as follow

$$
F_{\text{Pl, max}} = \frac{F_0 L^3}{E_{\text{eff}} l g}, \quad M_{\text{Pl, max}} = \frac{M_0 L^2}{E_{\text{eff}} l g}
$$

(12)

Where $F_0$ and $M_0$ are the shear force and the bending moment at the cross-section of the beam fixed end ($x=0$). By these definitions, $M_{\text{Pl, max}}$ and $F_{\text{Pl, max}}$ equal to $u''(x=0)$ and $-u''(x=0)$, respectively [23].

$F_{\text{Pl, max}}$ and $M_{\text{Pl, max}}$ as a function of the van der Waals force for wide and narrow types of cantilever nanobeams are depicted in Figs. (11) and (12), respectively. As seen, the increase of the van der Waals parameter ($\alpha$) increases the maximum value of the shear forces. Therefore, the nano switches with smaller separation spaces (large values of $\alpha$) are subject to larger shear stresses. In addition, a comparison between the previous reported results and the results of present study is performed in Figs. (11) and (12). These figures show good agreement between the results of PS-CS and numerical results.

6. Conclusions

A combination of power series as a trial solution and cuckoo search optimization algorithm is successfully utilized to obtain a relation for the shape of cantilever nano switches in the presence of van der Waals attractions and electro static forces. The pull-in instability of the nano switch is also studied. The results of present approach are compared with the numerical results as well as the reported results of Adomian, Green and Lamped methods in the literature. The comparison between numerical solution and the present approach demonstrates that the PS-CS approach overcomes the shortcoming of the accuracy of the previous approaches, and it is capable to deal with the problem robustly. Utilizing the obtained results, a relation for the detachment length and minimum separation space for nano cantilever switches is obtained. Finally, the stresses at the onset of pull-in instability are evaluated.

References


[9] Ramezani A, Alasty A, Akbari J., Closed-form approximation and numerical validation of the influence of van der Waals force on...
electrostatic cantilevers at nano-scale separations. Nanotechnology, 2008; 19 015501 (10pp).


