FE Analysis and experimental validation of land subsidence due to ground water level variation

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Abstract

Variation in groundwater with excessive withdrawal is one of the causes of land subsidence. Withdrawal of groundwater level can cause an increase in effective stress at thick clay layers, leading to land subsidence. In Rafsanjan plain in Iran, due to the excessive use of groundwater, land subsidence has caused earth fissures and damages to the buildings and installations. Variation in water table level may happen due to the seasonal extraction of groundwater in spring and summer for farming and replacement due to rainfall in fall and winter. The behavior of soils due to the rise of ground water table is non-linear; therefore, a part of settlements may not be recoverable and therefore, it must be considered in the analysis. Modeling of this phenomenon and formulation was considered by finite element method in this research. Finite Element method was used based on element equilibrium and fluid continuity equations. The nonlinear behavior of soil was applied by the bilinear model. The rate of settlements and variation in pore pressure caused by the oscillation of groundwater table were determined by using this model. A good correlation was observed between the results of finite element model and actual data in Rafsanjan area in Kerman Province.

Keywords: Consolidation; Finite Element Analysis; Ground Water; Kerman Province; Land Subsidence.

1. Introduction

Groundwater, as the world's largest freshwater resource, is critically important for irrigated agriculture and hence, for global food security. Yet, depletion is widespread in large groundwater systems in both semi-arid and humid regions of the world [1], [2]. Groundwater exploitation is a major cause of land subsidence [3]. The excessive use of ground water causes water table level to be drawn down and increases the effective stresses applied on soil layers. Additional effective stress will result in compression and consolidation of under layer soils; also, if the layers consist of compressible soils which are followed by settling and lowering the ground surface, it is called land subsidence [4], [5]. The ground surface subsidence profile is strongly influenced by the presence of the clay zones [6]. It has been proven that the major cause of land subsidence in Kerman Province, Iran, especially in Rafsanjan and Zarand Regions, is due to the extensive withdrawal of ground water from agricultural wells. Land settling may cause earth fissure that can damage structures and pipelines. Recently, land subsidence in the city of Kerman has changed the topography of the city, creating problems for city sewage system. Since the produced effective stresses and consequently, land settlements are due to the amount of water table level decline, it is essential to prevent any future decline of ground water. One way to prevent or reduce it is by seasonal pumping of ground water [4], [5]. Kang-He and Xing-Yu studied one-dimensional nonlinear consolidation of double-layered soil [7] and Baligh and Levadoux introduced a one-dimensional consolidation theory for an inelastic normally consolidated clay layer subjected to cyclic loading [8]. Finite element modeling based on Biot's three-dimensional consolidation theory and formulation was developed in a cylindrical coordinate system using fluid continuity and elements of equilibrium equations [9], [10]. In this paper, land subsidence due to cyclic loading caused by seasonal pumping of ground water level oscillation was studied by the finite element method. Elasto plastic (Bilinear) model was used to model the inelastic behavior of soil under cyclic loading. In this research, it was assumed that ground water level oscillated horizontally, that is, it was
reliable in a broad file except near the pumping wells. A laboratory model developed to simulate land subsidence in laboratory and calibrate and compare it with the finite element model results.

2. Materials and methods

2.1. Non-linear behavior imposed by cyclic loading

When a normally consolidated soil body is affected by cyclic loading, it is at the normally consolidated condition until the end of the first half cycle of loading. It will become over consolidated during the first half cycle of unloading and the first part of the next half cycle of loading until the average effective stress in soil layer is smaller than the maximum mean effective stress at the end of the last half cycle of loading. Then it becomes normally consolidated until the end of the half cycle. This procedure is repeated in all cycles until reaching the steady-state condition. Over consolidation time at the beginning of loading half cycles is increased with the increment of cycle numbers, reaching to all time of the half cycle of loading in the steady-state condition. Because of the large difference in clays compressibility over consolidated and normally consolidated conditions, it is very important to apply the effect of stress-strain history in the analysis. In Fig. 1, the inelastic behavior of soil during a loading cycle has been shown. In the first cycle of loading, stress path is according to [1-2] route and for unloading half cycles, it is [2-3-4] route. But in the second and next loading half cycles, stress-strain route is [4-3-5-6] and the location of points 3 and 5 is near the preconsolidation stress. Preconsolidation stress at each cycle is the maximum effective stress produced at the end of the last half cycles of loading and it is increased with the number of cycles, reaching the constant value in the steady-state condition.

In Fig. 1, α is the ratio of deformation coefficient in the over consolidated state to the normally consolidated state. In other words, at each time, if the average effective stress in each element has the maximum magnitude of precedence stress, that element will be in the normally consolidated condition and in the reverse condition, it will be over consolidated and therefore, must be taken into account in analysis.

2.2. Finite element formulation

The basic formulation presented here is based on Biot’s three-dimensional consolidation theory. In Biot’s theory, the soil skeleton is treated as a porous elastic solid and the laminar pore fluid is coupled by the conditions of compressibility and of continuity.

In the computations, cylindrical coordinates were assumed to be used in axial symmetric conditions. Also, they could be used for the modeling of water pumped out from a single well. In such a coordinate system, both radial and axial flows can take place, which are symmetric. In order to simulate this condition by finite element, the exact behavior should be achieved by actual mathematical equations. For each reason, Biot's governing equation was selected as shown below:

\[
C_r \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + C_z \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} - \frac{\partial p}{\partial t}
\]  

(1)
where \( u_e \) is excess pore water pressure, \( P \) is mean total stress, \( Z \) and \( r \) are axial and radial directions, \( t \) is time, and \( C_r \) and \( C_z \) are coefficients of consolidation in radial and axial directions, respectively.

The equilibrium equation, with the assumption of zero volumetric force, can be written as follows:

\[
\frac{\partial \sigma'_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\partial u_r}{\partial r} = 0
\]
\[
\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma'_z}{\partial z} + \frac{\partial u_z}{\partial z} = 0
\]

(2)

The stress-strain relations for such a condition can be written as follows:

\[
\begin{bmatrix}
\sigma'_r \\
\sigma'_z \\
\tau_{rz}
\end{bmatrix}
= E \begin{bmatrix}
1 & \frac{\nu}{1-\nu} & 0 \\
\frac{\nu}{1-\nu} & 1 & 0 \\
0 & 0 & \frac{1-2\nu}{2(1-\nu)}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_r \\
\varepsilon_z \\
\gamma_{rz}
\end{bmatrix}
\]

(3)

Where \( E \) refers to modules of elasticity, \( \nu \) is Poisson’s ratio, \( \sigma' \) is effective stress, and \( \varepsilon \) is strain; also:

\[
\begin{bmatrix}
q_r \\
q_z
\end{bmatrix}
= \frac{1}{\gamma_c} \begin{bmatrix}
K_r & 0 \\
0 & K_z
\end{bmatrix}
\begin{bmatrix}
\frac{\partial u_r}{\partial r} \\
\frac{\partial u_z}{\partial z}
\end{bmatrix}
\]

(4)

Where \( q_r, q_z \) are volumetric flow rates per unit area into and out of the element, and \( K_r, K_z \) are coefficients of permeability in redial and axial directions, respectively.

For fully saturated soil and incompressible fluid condition, outflow from an element of soil equals the reduction in the volume of element. Hence:

\[
\frac{\partial q_r}{\partial r} + \frac{\partial q_z}{\partial z} = \frac{d}{dt} \left( \frac{\partial u_r}{\partial r} + \frac{\partial v}{\partial z} \right)
\]

(5)

Where \( u \) and \( v \) are displacements in \( r \) and \( z \) directions, respectively. By combining Eqs. (1) and (2), we get:

\[
\frac{K_r}{\gamma_c} \frac{\partial^2 u_r}{\partial r^2} + \frac{K_z}{\gamma_c} \frac{\partial^2 u_z}{\partial z^2} + \frac{d}{dt} \left( \frac{\partial u_r}{\partial r} + \frac{\partial v}{\partial z} \right) = 0
\]

(6)

As usual, in a displacement method, the final coupled variables are \( u, v, \) and \( u_e \). These are now discretized in the normal way:

\[
\begin{align*}
u &= Xu \\
v &= Xv \\
u_e &= Xu_e
\end{align*}
\]

(7)

Where \( X \) is the vector of shape function. When discretization and the Galerkin process are completed, Eqs. (2) and (6) lead to the pair of equilibrium and continuity equations, which are:

\[
KM \frac{\partial u}{\partial r} + Cu \frac{\partial u}{\partial t} = F
\]
\[
CT \frac{\partial v}{\partial t} - KPu_{e} = 0
\]

(8)

Where, for a four-nodded element,
\[ \mathbf{r} = [u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4]^T \]

\[ \mathbf{u}_c = [u_{c1}, u_{c2}, u_{c3}, u_{c4}]^T \]

Where \( \mathbf{D} \) is elasticity matrix and \( \mathbf{K_M} \) is the elastic stiffness matrix:

\[ \mathbf{K_M} = \int \mathbf{B}^T \mathbf{D} \mathbf{B} \, dr \, dz \]  \hspace{1cm} (10)

Where \( \mathbf{B} \) is \( \mathbf{AX} \), and \( \mathbf{X} \) is the vector of shape function; also

\[ \mathbf{A} = \begin{bmatrix}
\frac{\partial}{\partial r} & 0 \\
0 & \frac{\partial}{\partial r} \\
\frac{\partial}{\partial z} & \frac{\partial}{\partial r} \\
1 & 0 \\
\end{bmatrix} \]  \hspace{1cm} (11)

\( \mathbf{K_P} \), the fluid stiffness matrix, is

\[ \mathbf{K_P} = \int \left( C, \frac{\partial \mathbf{X}_i}{\partial r}, \frac{\partial \mathbf{X}_j}{\partial r} + C, \frac{\partial \mathbf{X}_i}{\partial z}, \frac{\partial \mathbf{X}_j}{\partial z} \right) \, dr \, dz \]  \hspace{1cm} (12)

\( \mathbf{C} \) is a rectangular coupling matrix that can be written as follows:

\[ \mathbf{C} = \int \mathbf{X}_i, \frac{\partial \mathbf{X}_j}{\partial r} \, dr \, dz \]  \hspace{1cm} (13)

\( \mathbf{F} \) is the external loading vector. Eq. (8) must be integrated in time. To integrate Eq. (8) with respect to time, there are many methods available, but we consider only the simplest linear interpolation in time using finite difference; thus:

\[ \theta \mathbf{K_M} \mathbf{r}_i + \theta \mathbf{C} \mathbf{u}_i = (\theta - 1) \mathbf{K_M} \mathbf{r}_0 + (\theta - 1) \mathbf{C} \mathbf{u}_0 + \mathbf{F} \]

\[ \theta \mathbf{C}^T \mathbf{r}_0 - \theta \mathbf{C}^T \mathbf{r}_i = \theta (\theta - 1) \Delta \mathbf{K_P} \mathbf{u}_0 \]  \hspace{1cm} (14)

In the above equations, if \( \theta \geq 0.5 \), the system will be stable without any condition; in the Crank-Nicholson type of approximation, \( \theta \) is made equal to 0.5 or in the Galerkin approximation, \( \theta \) is equal to 0.67. By using \( \theta = 0.5 \) in Crank-Nicholson method, Eq. (14) can be written as follows:

\[ \begin{bmatrix}
\mathbf{K_M} & \mathbf{C} \\
\mathbf{C}^T & -\frac{\Delta t}{2} \mathbf{K_P} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{r}_{n+1} \\
\mathbf{u}_{n+1} \\
\end{bmatrix}
= \begin{bmatrix}
-\mathbf{K_M} & -\mathbf{C} \\
\mathbf{C}^T & -\frac{\Delta t}{2} \mathbf{K_P} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{r}_n \\
\mathbf{u}_n \\
\end{bmatrix}
+ \begin{bmatrix}
2 \mathbf{F} \\
0 \\
\end{bmatrix} \]  \hspace{1cm} (15)

Therefore, values that are unknown can be calculated at time \( t=t_{n+1} \) based on the known parameters at time \( t=t_n \). For initial conditions, at time \( t=0 \), all values are known.

After finding governing matrix equations for a single element, the assembled matrices for total elements can be obtained and boundary conditions can be introduced.

As explained in section 2, to apply the effect of inelasticity, stress-strain history is kept separately for each element and the calculated stress is compared with stress history; also, if at any time of solution, the calculated stress had the maximum value of the precedent stresses, that element would be normally consolidated and to form stiffness matrix for the next time step, normally consolidated properties of that element material could be used. On the other hand, in the reverse condition, that element should be over consolidated and the related specifications must be used.

By solving such equations at any time, horizontal and vertical deformations \( (u,v) \) at various nodal points can be found and strain values for each element can be calculated.
3. Results and discussion

3.1. Precision of finite element model

In order to calibrate and confirm the finite element model, a laboratory model was developed and compared with the numerical model results. The laboratory model was prepared using one-dimensional consolidation apparatus. Clay specimens used in odometer test were taken from Nogh field in Rafsanjan. Fig. 2 shows odometer test results along with numerical results for finite element model, with material properties and similar testing boundary conditions. In the above test, the initial thickens of the specimen was 2.84cm; the applied load included 650 kPa precompression pressure and 100 kPa additional cyclic stresses. Estimated compressibility and consolidation coefficients were $7.9 \times 10^{-3} \text{kPa}^{-1}$ and $2.06 \times 10^{-2} \text{cm}^2/\text{min}$, respectively.

![Fig. 2: Comparison of Numerical Analysis and Laboratory Test Results.](image)

As shown in Fig. 2, there was a good correspondence between numerical and laboratory tests results.

3.2. Evaluation of vertical load due to water level decline

The equivalent external load due to water table decline can be computed from Fig. 3 If water table drops to be to equal $h$, then:

$$h = h_1' - h_1 = h_2 - h_2'$$

$$\sigma_{v1}' = \gamma_{sat} h_1' + (\gamma_{sat} - \gamma_n) h_2'$$

$$\sigma_{v1} = \gamma_{sat} h_1 + (\gamma_{sat} - \gamma_n) h_2$$

$$\sigma_{v1} = \gamma_{sat} (h_1 + h_1') + (\gamma_{sat} - \gamma_n) (h_2 - h_1)$$

$$\Delta \sigma' = \sigma_{v1}' - \sigma_{v1} = [\gamma_{sat} - (\gamma_{sat} - \gamma_n)] h$$

Where $\sigma_{v1}'$ is the initial vertical effective stress, $\sigma_{v1}$ is the final vertical effective stress, and $\Delta \sigma'$ is the estimated vertical load at the top layer of clay.
3.3. Numerical results

Formulation of finite element analysis for subsidence problem was discussed in the previous section. A computer program in Matlab was developed to predict and examine various soil behaviors and conditions under cyclic loading. In order to verify the computer model, an analysis for simple behaviors such as one-dimensional consolidation was performed. As an example, for the examination of the model, properties of Rafsanjan aquifer in Kerman Province were considered, as shown in Fig. 3. It should be noted that values of E and other material properties could be varied in depth or other directions.

A section with the height of 200 meters and the width of 1000 meters was discretized to 160 rectangular elements with 189 nodes.

At the first stage of this study, in order to examine the consolidation process for continuous loading with cyclic loading under inelastic condition, it was assumed that water table suddenly was dropped by about one meter and water flowed in axial and radial directions under axi-symmetric conditions.

Seasonal withdrawal of ground water caused water table level to oscillate by one meter at one year period. This simulation was very close to actual filed condition under pumping groundwater through wells. The settlement analysis for these cases is shown in Fig. 4.

Final settlement calculated for one meter water level decline was about 12cm and occurred after 50 years; also, final settlement for cyclic condition was about 7.5 cm.

It can be seen from Fig. 4 that the amount of subsidence was higher for continuous condition, as compared to the cyclic condition. There was about 38 percent decrease in settlement in the cyclic condition. Therefore, cyclic pumping and dewatering can be a useful method to reduce land subsidence.
Excess pore water pressure dissipation during the first cycle of loading has been shown in Fig. 5.

![Fig. 5: Pore Water Pressure Distribution on the Clay Layer at the First Cycle of Loading.](image)

It can be seen in Fig. 5 that there was some negative pore-water pressure in the clay layer during the unloading half cycle. Static pore water pressure was excluded from our computations. In the unloading cycles, the soil body was expanded, so water flowed into the pores of soil skeleton and because of the clay’s low permeability, suction and negative pore-water pressure were introduced.

Fig. 6 shows excess pore water pressure dissipation in the clay layer during a complete cycle of loading at the steady-state condition.

![Fig. 6: Pore Water Pressure Dissipation during a Cycle of Loading in the Clay Layer at the Steady-State Condition.](image)

It is interesting to note that there was no possibility of complete consolidation under cyclic loading at the present condition in Rafsanjan field. Pore water pressure distributions at the end of the half cycle of loadings have been shown in Fig. 7.

![Fig. 7: Pore Water Pressure Distribution in the Clay Layer at the End of Half Cycles until the Steady-State Condition.](image)

In the Fig. 7, it can be seen that after an adequately large number of cycles, there was no change in pore water pressure distribution at the end of half cycles. Also, it can be noticed that excess pore water pressure was not dissipated completely at steady-state condition; this was the main cause of settlement reduction in cyclic condition as compared to
static loading. For a thorough study of loading period time effect on settlements, the above analysis was repeated with different periods and the same loading quantity. The settlement analysis results for different loading period times have been shown in Fig. 8. For better understanding, only the upper bound of settlements has been illustrated in Fig. 8. It is observable in Fig. 8 that the amount of settlements was increased by the increase in the loading period.

To study inelasticity effect on the settlements, the above analysis was repeated with normally consolidated specifications of soils holding the assumption of elastic behavior. In Fig. 9, settlement history has been compared for a loading with a one year period in elastic and inelastic conditions.

It could be seen in Fig. 9 that the amount of calculated settlements would have been underestimated if the effect of inelasticity had been neglected. In Fig. 10, settlement analysis results for elastic condition with different periods of loading have been shown.
It could be observed that there was a relation between loading period and settlements in elastic condition, the same as inelastic state, and maximum settlements were decreased with the decrease in the loading period. Comparing Figs. 10 and 8 indicated that cyclic loading in both elastic and inelastic states caused settlement reduction and calculated settlements in inelastic state were greater than those in the elastic state for any period of loading.

4. Conclusion

In this study was first developed for static loading. Then, the nonlinear behavior was introduced and compared with the static loading. Settlement in cyclic and nonlinear conditions was found to be 62% of the static case. The calculated settlement in the elastic condition was 54% of the static condition. There was 8% difference between elastic and nonlinear conditions. Elastic assumption was tantamount to neglecting over consolidation effect, showing that in the case of cyclic loading on the normally consolidated clays, the effect of preconsolidation had to be taken into the account.

Finally, the effect of load type and its period on settlements was studied. It was found that the type of loading and period influenced the maximum settlement. Settlement was increased with loading period and load-time diagram under area was increased too.

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References