*International Journal of Engineering & Technology*, 7 (3.36) (2018) 211-217

International Journal of Engineering & Technology

*Website: www.sciencepubco.com/index.php/IJET*

***Research paper***

# Creation the Overlapping Ordinary Sets by Complementary Approach

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**Abstract**

The present paper reports definition of overlapping ordinary sets and represent how to nest only one set, comparable sets, and disjoint sets by using basic concepts of the set theory, in especially absolute complement operations. It found that the most useful entrance to develop a definition of overlapping ordinary sets is reasonable definition of incomparable sets. *Overlap relation*, *Overlap operation* and *Overlap function* was studied in a manner featured. It is shown that the double complement with different types of complement operations (Absolute, Relative), are not necessary equal to the same results. In addition to the paper studied separable overlapping ordinary sets.

***Keywords***: *Overlapping Ordinary Sets, Harmonic and non-harmonic sets, Overlap operation, Overlap relation, Overlap function, Overlap system, Overlap diagram.*

## 1. Introduction

There are at least two differences between the fuzzy set and overlapping ordinary sets: First, the values memberships function of fuzzy set (denoted by $X_f$), are $X_f = [0,1]$ [1],and of overlapping ordinary sets (denoted by $X_O$), are $X_O = \{0,1\}$.Second, the nest set can arise at most by only one fuzzy set, and can arise by at least two overlapping ordinary sets. Though $X_o \subset X_f$, but; not all the facts of fuzzy set are hold in overlapping ordinary sets.

Aristotelian logic interested the overall relations between the universal concepts (sets). The definitions of those relations are not enough clear, for this reason, in this paper we consider the logical definitions of $A \subset B,\ A \supset B,\ A = B,$ and $A \sim B$ ,as the partial,inclusion,equality, and disjoint relationship respectively, as they defined in set theory.

In this paper, the partial relationship can be defined by $A \subset B$, and defined the inclusion relationship by $A \supset B$. In addition, the relation of disjoint can be symbolized by symbol $\sim$, which mean $A \cap B = \varnothing$. The order of the sets have own importance in the relations covered by this paper.

By using, the logical definitions of relationships between the universal concepts could eliminate the gap between Aristotelian logic and Mathematical logic. Overlapping ordinary sets and their nest set are main subject of this paper will be defined later on.

The objective of this paper is setting up overlapping ordinary sets from various sources: from one set, or from sets between them one of the relations mentioned above, in addition to the separation of overlapping ordinary sets. The fundamental operations adopted in this paper are complement, intersection and implication.

## 2. Basic Concepts and Definitions

Overlapping Ordinary Sets, abbreviated by (OOS), are formed by intersection operation, with some specifics conditions. The creation of (OOS) or nesting the sets is closely linked precisely to the concept of incomparable set versus comparable sets. It is known; for any two element *x*, *y* in *X*; if $x \leq y \ or \ x \geq y$ then *x* and *y* are comparable [2], otherwise the elements are incomparable, but; there is need for another definition for incomparable sets, take into regarded the comparison between the sets, not between elements. So the relations ≤ and ≥ do not meet this purpose. May be a set, for instance *A*, denoted to region or cluster ,or objects number, so on, in the same sense; *A* is any non-empty set , so as set *B*. Based on this note, the incomparable sets must define under the strict inclusion relation $A \supset B$ and under strict partial relation $A \subset B$, conjunction with the difference operation.

*Definition* 1 **:**( *Incomparable Sets*)the non-empty sets and *B* are *incomparable sets* if and only if the difference sets of them are non-empty sets.

There is no redundant when we say that the phrase "The difference sets, which are both non-empty", is the restriction for the symmetric difference of the sets, which are (OOS). Thus; if $A - B$ and $B - A$ are non-empty sets, and their intersection set is empty set, then that *A* and *B* are incomparable sets. In another hand, if *A*, and *B* are incomparable sets, then $A - B \ \cup B - A$ is symmetric difference of *A* and *B*.

Note that the incomparability denoted by $||$ ,and the comparability dented by $\perp$[3].

*Definition* 2**:**(*Overlapping Ordinary Sets*)Let *A* and *B* are non–empty sets, and ordinary sets, then they are *Overlapping Ordinary Sets* if and only if:

*i*) They are incomparable sets. And

*ii*) Their intersection set is non-empty set.

The conditions (*i*) and(*ii*)are necessary and sufficient for any (OOS), also for to create any (OOS) from different sources.
There are no symbol indicates to (OOS) in Ordinary Set Theory, owe shall suggest the following symbol:

$$A\S B \qquad (1)$$

Such that; § is read: overlapping; $A\S B$ is read: *A* and *B* is overlapping ordinary sets, or *A* and *B*are overlapped. According to abovementioned definition can be formulated it as implication

$$A\S B \Rightarrow (N \subset A \wedge N \subset B) \qquad (2)$$

One can also establish the conditions mentioned in the Def. 2 as the follows

$$\left.\begin{array}{l} i)\ A \not\subset B\ , B \not\subset A \\ ii)\ A \cap B\ =\ N_{AB} \end{array}\right\} \qquad (3)$$

Such that $N_{AB}$is *nest set*.

Sometimes we abbreviate symbol nest set by *N*. In [4] the authors said that " $C_k \not\subset C_i$ , $C_k \not\subset C_i$ " during discussing overlapping clusters, maybe there is a printing mistake, the correct is" $C_k \not\subset C_i$ , $C_i \not\subset C_k$ .
*Comment* That is necessary to note the variance between the phrase (overlapping ordinary sets) and the phrase (set of overlapping ordinary sets).The meaning of the first phrase is *sets which are compose the overlapping ordinary sets denoted by*$A\S B$, whereas the meaning of the second phrase is *union of the sets which are overlapped with each other, denoted by A*∪*B*, or *A* and *B*.

*Definition*3**:***Nest Set* is all the common elements between the overlapping ordinary sets.
From Def.3, it is clear that elements $x_i$ in *N* such that $x_i \in A \cap B$ , and this means the common element(s) have the same characteristics in *A* and *B*, then can be written as the following:

$$A = \{a_{b1}, a_{b2}, \dots, a_c, a_d, \dots, a_z\},$$
$$B = b_{a1}, b_{a2}, \dots, b_n, b_c, \dots, b_z\}$$

The elements $a_{b1}$ and $b_{a1}$ are identical, in special case they are equal. For example, let *A*is represented the set "Birds" which are contains all the birds, and let *B* is represented the set "white objects" which are contains all white objects. The elements" Birds White" denoted by $a_b$ ,as if the item code shows the statements "$Birds_{white}$", and elements “White Birds" denoted by$b_a$ as if the item code "$White\ _{birds}$".
The different between the two items does not preclude coinciding things that refer to it. So can be written as the following:

$a_{b1} = b_{a1},\ a_{b2} = b_{a2}, etc,$and $N = \{a_{bi}\}\ or\ \{b_{ai}\}$.

(See Figure.1)
Whatever the finite number of the (OOS)has only one nest set for them, and the smallest nest set is singleton set. The following proposition is illustrate this fact.

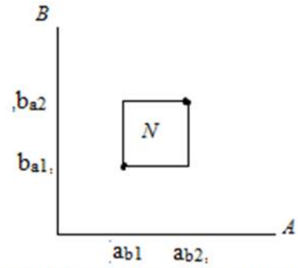

*FIG1. The nest set of non-empty sets A and B*

*Proposition*1: For any finite number of the overlapping ordinary sets $S_i$ , since *i* is the finite natural number (contrary to zero), so $\lim_{i\to a} S_i \succeq \{n\}$ , $n \in N_{si}$ , such that $N_{si}$ is the nest set of $S_i$ , $2 \le i \le a$ , $\{n\}$ singleton set, and *a* is largest number of (OOS).
Suppose the set$A\S B\S C\ \S \dots \S Z = \bigcup_i^a S_i$, it is clear that $\bigcap_i^a S_i = A \cap B \cap \dots \cap Z_a$ Equal to the nest set $N_{si}$of$S_i$, it is containing at least an element. That is interprets that the $\lim_{i\to a} S_i \succeq \{n\}$.

## 2.1. Equation of Overlapping Ordinary Sets

The total elements of the two sets which are (OOS) denoted by equation *T*, such that total of elements of universal set is $T_U = A + B - N + R$ , *R* is the set of all the elements a round *A* and *B* inside the rectangle. See FIG.(2).
Observed that the universal set equal to

$$U = A \cup B - N \cup R \qquad (4)$$

By putting $E = A \cup B$ can rewrite Eq. 4 as

$$U = E - N \cup R$$

It is useful and convenientconsiderEq.4 as instrument to prove some another facts. That would explain the relations between the five sets as seen in the FIG.2, and that *R* is separated from the union sets *A* and *B*, and consequently one can write

$$R \sim (A \cup B)\ and\ \ R \subset N' \qquad (5)$$

$$R \cap (A' \cap B') \subset N' \qquad (6)$$

$$R \subset (A' \cup B') \subset N' \qquad (7)$$

More advance, the total numbers of elements of three sets *A*, *B*, and *C* which are (OOS) in *U*, formed by the equation

$$T_U = A + B + C - N_{AB} - N_{AC} - N_{BC} + N_{ABC} + R \qquad (8)$$

In general can form the total elements of finite (OOS). Let $S_i$are (OOS),such that $2 \le i \le a$ , *i*is natural finite number, then

$$T_U = \bigcup_i^a S_i - S_1 \cap S_2 - \cdots S_{a-1} \cap S_a + \bigcap_i^a S_i + R \qquad (9)$$

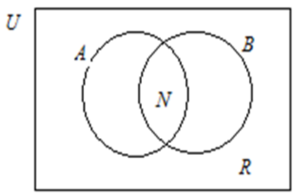

*FIG2. Illustration of overlap ordinary sets A and B.*

## 2.2. Complementary

The difference operation known as the *relative complement* [5], that means the set $A - B$ equal to relative complement of *B* respect to

the set *A*. Moreover, the set–theoretic difference between and *B* is the set of elements in *A*, but not in *B*[6].
However, the complement of sets, which are composed any overlapping ordinary sets is the *absolute complement* ty pewit respect to set $E = A \cup B$ inside universal set *U*,those the employment of this type does not contradict substance of the incomparable set definition.
The symbolism of the complement types did not have the agreement of the authors, some of them have symbolized absolute complement by $\bar{A}$[7],and another symbolized it by $A^c$ for any type.
Both types of complement should be used, that why we indicate to appropriate symbol to each type, so considered:

- $A'$, $B'$, ...Indicate to absolute complement.
- $A^c$, $B^c$, ... Indicate to relative complement.

## 2.3. Axioms

If $A§B$ then it is very clear to note that the primitive equations by using types of complement

$A' = N' - A, \quad B' = N' - B$ (10)

Or $A' = R \cup B - N$ $and$ $B' = R \cup A - N$ (11)

And $N' = R \cup (A \cup B) - N$ (12)

Subsequently;$A^c = B - N,\ B^c = A - N$ (13)

The equations (11) are equivalent of

$A^c = B - A, \quad B^c = A - B$ (14)

Or $A^c = B - N,\ B^c = A - N$ (15)

That is clear $A^c \subset A'$ (16)

$R' = R^c = A \cup B$ (17)

*Lemma* 1: If *A* and *B* are overlapping ordinary sets, then the symmetric difference under the relative complement is equal to the symmetric difference under the absolute complement.
In symbols$A^c \Delta B^c = A' \Delta B$
*Proof*: since *A* and *B* are overlapping ordinary sets then they are incomparable sets, and $A - B = A - N$ , $B - A = N$ , *N* is the nest set of $A§B$
It is clear that$A - N = B^c$ $and$ $B - N = A^c$
Then $A^c \Delta B^c = A - N \cup B - N$ ………1
From Eq. (11) $A' = R \cup B - N$ $and$ $B' = R \cup A - N$
$A' - B' = B - N, \quad B' - A' = A - N$
Then $A' \Delta B' = B - N \cup A - N$ ………2
From 1 and 2 then will be$A^c \Delta B^c = A' \Delta B'$.
The sets *A* and *B* can obey the two type of complement, because they are subset of$E = A \cup B$and subset of *U*.$R^c$And$R'$are equal to difference set$U - R$. The nest set has exclusiveness with the complement operations, that $N^c = E - N$ or $A^c \Delta B^c$ , and $N' = U - N$ .Note that a set $E = A \cup B$ called *merger of region* [8].
For any reason it may be required to contra positive involution (double complement) any set in (OOS) and confront the following different situations:
$(A')', (A^c)^c, (A^c)', (A')^c, (N^c_{AB})', (N'_{AB})^c, (R^c)', (R')^c$ .
The different characteristics of each type of operation complement almost lead to different results. If the double complement are same type that would produce universal explicit concept without addition, for instance$(A')' = (A^c)^c = A$ ,also this applies to the double complement of *N* and *B*, otherwise if the double complement contains two different types of complement operation that would get auniversal explicit concept with addition of *R* or not, for example$(A')^c$ and$(A^c)'$ are not equal. Beside the various types of the complement that are located on the sets *R* and *E*does not lead to different results.
From previous observations can illustrate by "Venn Diagrams", also note that the difference in the order of succession to the complement operations lead to different results.

Suppose that *G* be one of the sets *A*, *B* and *N* then: $(G')' = (G^c)^c = G$,$(G')^c = G$, and$(G^c)' = G \cup R$ .In following would explain details of these equations.
*Theorem*1:If *A* and *B* are overlapping ordinary sets, then the absolute complement of the relative complement of their nest set is equal to the union of the nest set and all elements around union *A* and *B* .In symbols:
If$A§B$ then $(N^c)' = N \cup R$
*Proof:*$N^c = A - N \cup B - N$
$= A^c \Delta B^c = A' \Delta B'$[Lemma.1]
$= A \cap N' \cup B \cap N'$
$= (A \cup B) \cap N'$
$(N^c)' = (A \cup B)' \cup N$, $[E = A \cup B]$
$= R \cup N$ Then $(N^c)' = R \cup N$
*Corollary* 1: Let *A* and *B* are overlapping ordinary set, then:
$(i)$ $(A^c)' = A \cup R$ $(ii)$ $(A^c)^c = (A')^c$,
$(iii)$ $(A^c)^c \subset (A^c)'$.
*Proof* (*i*)$A^c = B - N = B \cap N^c$
$(A^c)' = B' \cup (N^c)'$
$= (R \cup A - N) \cup (N \cup R)$,[Eq. (10), Theorem.1]
Then$(A^c)' = A \cup R$
*Proof* (*ii*): $A^c = B \cap N^c$ ,
$(A^c)^c = B^c \cup N = A$ ………………1
$A' = R \cup A^c$
$(A')^c = R^c \cap A$
$= (A \cup B) \cap A$, $[R^c = A \cup B)]$
$= (A \cap A) \cup (A \cap B)$
$= A \cup N_{AB}$
$= A$, $[N_{AB} \subset A]$ ………2
From 1 and 2$(A^c)^c = (A')^c$
*Proof* (*iii*) $(A^c)' = A \cup R$ , [Corollary1.*i*theorem.1] …1
$(A^c)^c = A$… 2
From 1 and 2$(A^c)^c \subset (A^c)'$.

***Note***: If substitution the set *B* instead of the set *A*, make this theorem is corrects too.

## 2.4. Overlap Operation

Overlap Operation (Denoted by symbol §), is fundamental concept in (OOS), represented by *nesting* and *splitting* of a set, or of sets. The nesting and splitting in overlap operation are correlative operations, such that the overlap operation is binary operation creates overlapping sets, by splitting the set (or sets) into incomparable sets, unique nest set and symmetric difference. In other words, the nesting set(s) by overlap operation is cutting the sets into disjoint sets, (partition as trichotomy): $N_{AB}$ , $A - N$ $and$ $B - N$, as the constraint required for finding overlapping sets. So, one can define the set $A§B$ as the following

$$A§B \stackrel{\text{def}}{=} \{x \mid x \in (A \Delta B \cup N_{AB} \cup R)\} \quad (18)$$

Such that $N_{AB} \cap A - N \cap B - N \cap R = \varnothing$

## 2.5. Overlap Ordinary System

From the previous offer; the sets *A* and *B*, which are equipped with overlap operation, are formation *mathematic system*$(E, §)$, we called this ordered pair *Overlap Ordinary System*.

## 2.6. Commutative and Associative

In examining the overlap operation found it commutative

$A§B = B§A$ (19)

To show this property considered conjunction tool$\wedge$ in (Implication.2) of overlapping ordinary sets is commutative.
Moreover, the overlap operation is associative

$$A\S(B\S C) = (A\S B)\S C \quad (20)$$

Enough to prove it show that three sets on both sides have same nest set.
*Proof*:$A\S(B\S C) \Rightarrow A\cap(B\cap C) = (A\cap B\cap C = N$.
And if $(A\S B)\S C$ are getting the same result.

### 2.7. Distribution

For any non-empty sets *A*, *B*, *C* and *D*, the following identities are hold:

$$A\S\,(B\cap C) = (A\S B)\cap(A\S C) \quad (21.a)$$
$$A\S\,(B\cup C) = (A\S B)\cup(A\S C) \quad (21.b)$$
$$(A\S B)\cap C = (A\S B)\cap(A\S C) \quad (22.a)$$
$$(A\S B)\cup C = (A\S B)\cup(A\S C) \quad (22.b)$$
$$(A\cap B)\S\,C = (A\S C)\cap(B\S C) \quad (23.a)$$
$$(A\cup B)\S\,C = (A\S C)\cup(B\S C) \quad (23.b)$$
$$(A\S B)\cap(C\S D) = (A\cap C)\,\S\,(B\cap C) \quad (24.a)$$
$$(A\S B)\cup(C\S D) = (A\cup C)\,\S\,(B\cup C) \quad (24.b)$$

To prove any identity above used theDef.2 and implication (2), for example to prove (21.a):

$$A\S\,(B\cap C) = N\subset A \wedge N\subset(B\cap C)$$
$$= (N\subset A)\wedge(N\subset B\wedge N\subset C)$$
$$= (N\subset A\wedge N\subset B)\wedge(N\subset A\wedge N\subset C)$$
$$= (A\S B)\cap(A\S C)\ .$$

### 2.8. Harmonicsets

The universal concepts can be divided into two parts; one of them is the *explicit concepts* (sets),for example "human" which contains all members in human set. The other partis *non-explicit concepts*(or *implicit concepts*), for example" non-human "which contains all members belong to another universal concepts which is not contains human, or all members underlying non-human set. It is convenient toplace both the mentioned concepts in symbols *A*,*A*′respectively.
The harmonious sets, both of them are explicit concepts (as*A* and *B*), or both of them are non-explicit concepts (as*A*′and *B*′).Then, $A\S B$and$A'\S B'$each one of them is *harmonious overlapping ordinary sets* (HOOS),$A'\S B$and$A'\S B'$each one of the mis *non-harmonious overlapping ordinary sets* (NOOS). Clearly, that the notions harmony and non-harmony are related to sets which consist (OOS), not among some (OOS) sets, that means the harmonic is adjective inside the overlap system$(E,\S)$, not between some systems. This clarification is necessary for the establishment and separate overlapping ordinary sets, in addition to accommodate all the mentioned results in this paper. Only $A\S B$ can exist in physical reality, whereas$A'\S B$, $A\S B'$and$A'\S B'$are industrial can be created under the conditions mentioned in Def.2.

## 3. Overlap Relation and Overlap Function

In classical set theory, the relation is subset of the Cartesian product set $R\subseteq A\times B = \{(a,b): a\in A\wedge b\in B\}$.
We shall denotethe*Overlap Relation* by our suggestion symbol$R_\S$, since if $A\S B$ , then$R_\S\subset A\times B$, and symbolized by

$$R_\S \overset{\text{def}}{=} \{(a,b)|\ some\ a \text{ in } A\wedge some\ b \text{ in} B\} \quad (25)$$

Noted that the domain of $R_\S$ is proper subset of *A*, and its range is proper subset of *B*.This fact is represents that the overlap relation is *partial relation*, since if $A\S B$ then the relation from *A* to *B*, (or from *B* to *A*) is

$$\left.\begin{array}{l} R_\S : N_{AB}\rightarrow B, since\ N_{AB}\subset A \\ \text{Or} \\ R_\S : N_{BA}\rightarrow A, since N_{BA}\subset B \end{array}\right\} \quad (26)$$

Sometimes the graph of the overlap relation shows that equality of the projections of all ordered pairs private inherent to (OOS), and so their nest set. Actually thesis special case, for instance if$(a_b\,, b_a)\in R_\S$ and $a_b = b_a$ then$\{a\} = \{a,b\}$.However, in generic it is impossible regarded $(a_b\,, b_a)$ unordered set. The term, which includes all the cases of the construct the graph of overlap relation is *coincidence*, since the coincidence elements is match by common character or property. For example, the nest set is a collection of the ordered pairs when every element in $domR_\S\subset A$ which is even number corresponding even element in$ranR_\S\subset B$too, regardless of their values. Based on this note one can write
$a_b \equiv b_a\ then\ (a_b\,, b_a)\in R_\S$. So the overlap relation is injective (one-to-one) and subjective (onto).

The properties of $R_\S$depend on characteristic the nest set because it's restriction relation is $R_\S/N_{AB}$,beside the formed ways of the nest set, in particular whether equality or coincidence. Indeed the nest set of incomparable sets is *the field* of overlap relation, such that the field of any relation equal to(domain$\cup$ range R = field R)[7].

Depending on the previous discussion, one can write:

- If the module of the overlapping is, coincidence by adjective (characteristic) then the overlap relation is *reflexive* and *symmetric*.
- If the module of the overlapping is coincidence by equality then the overlap relation is *reflexive* and *ant symmetric*.

### 3.1. Equivalently

In set theory the properties of relation are discussed the ordered pairs which consist of elements are belonging to at least two sets. Here we meet special case with overlapping ordinary sets, that is impossible to say that $a_b\ R_\S a_b$ because the overlap relation accordingly it's definition arise between two or more incomparable sets, not between elements, beside the overlap relation arise providedthe elements of nest set are *coincidence*.
The following definition explains the special case of $R_\S$ apparently.
*Definition* 4 (*Equivalence*)
Suppose that$a_b\ \ in\ A, b_a\ in\ B\ and\ \ c_b\ \ in\ C$ .
If overlap relation:
Reflexive, $(a_b\,, b_a\,), (b_a, a_b\,)\in R_\S$
Symmetric, $a_b \equiv b_a\ and\ b_a \equiv a_b$
Transitive, $a_b \equiv b_a\ and\ b_a \equiv c_b\ , then\ a_b \equiv c_b$
Then: $R_\S$ is equivalence relation.

### 3.2. Transitive property of overlapping ordinary sets

The transitive of (OOS) related to uniqueness of the nest set.
The following theorem explains the necessary condition in order to be $R_\S$is transitive relation.
*Theorem*2: The overlap relation is transitive if and only if the nest set is unique set for all the overlapping ordinary sets.
*Proof*: Let$N_{AB}$is the nest set of $A\S B$and$N_{BC}$ is the nest set of $B\S C$.
If$R_\S$transitive relation then *A* and *C* is$A\S C$, we have three nonempty and incomparable sets which their nest sets are$N_{AB}\ , N_{BC}\ ,$ and $N_{AC}$ , so

$N_{AB} \cap N_{BC} = N_{ABC}$ $and$ $N_{BC} \cap N_{AC} = N_{AB} \cap N_{AC} = N_{ABC}$, such that $N_{ABC}$ is nest set of $A§B§C$ .

Hence, the imposed overlapping ordinary sets have a unique nest set.

By converse way can prove that, if $A§B$, $B§C$ and $A§C$, then $R_§$ is transitive relation.

The theorem above represents that not necessarily, the overlap relation is transitive, in addition to the nest sets of overlapping ordinary sets which are more than two sets constitute overlapping ordinary sets too. See Fig.3.

As the overlap relation is one to one, onto, and partial relation then it is function, we call it *overlap function*. For any two sets which are overlapping the overlap function denoted by

$$f_§(x) \stackrel{\text{def}}{=} x\,, \qquad x \in N = A \cap B\,, \quad N \text{ is nest set} \tag{27}$$

That means $dom f_§ \subset A$ , $and$ $ran f_§ \subset B$ this in

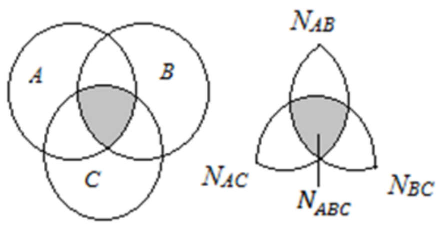


*FIG 3 (a): overlap relation is transitive*

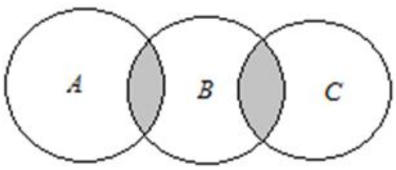


*FIG 3(b) : overlap relation is not transitive*

turn interpret that overlap function (for overlapping variables)also *a partial function* and *identity function*.This is clear from the signification, which illustrated in Def. 27, for any element in domain of the overlap function exist image in the range of the overlap function. Depending on those facts can be written

$$f_§^{-1} = f_§ \tag{28}$$

And

$$f_§^{-1}(A \cap B) = f_§^{-1}(A) \cap f_§^{-1}(B) \tag{29}$$

# 4. Induction Harmonious Overlapping Ordinary Sets

Induction (HOOS) depends on the some sources. It is found that is possible to create (HOOS) from $A§B$ , disjoint sets and only one ordinary sets. The criteria to create overlapping ordinary sets is mentioned insections2and3, and it is convenient to say; if the complement operation is required during creation (OOS),shall be only absolute complement.

## 4.1. Induction $A§B$ from an ordinary set

Let *E* is non-empty set, it is possible to split it into finite overlapping ordinary sets. The incomparable sets *A*, *B*… and the nest set *N* must each one of them contains at least one element. And let $n(§)$ the number of prober subsets of *E*, which are compose (OOS), $\lambda \geq 1$ are elements number of the nest set. The total elements of (OOS) is denoted by $T_E$, then the following inequality describes the relation between the variables $T_E$ , $n(§)$ $and$ $\lambda$

$$T_E \geq n(§) + \lambda \tag{30}$$

Such that $n(§) \geq 2$.

Or, by use combination

$$T_E \geq C_2^{n(§)} \tag{31}$$

Let $\omega_i$ any set in $n(§)$, *i* = 1,2…*m*. i.e. $E = \bigcup_{i=1}^{m} \omega_i$, *m* is the number of prober subsets of *E*.

Such that $\omega_i \not\subset \omega_{i+1}$ $and$ $\omega_{i+1} \not\subset \omega_i$… 1

And so on for $n(§) \geq 2$ prober subsets of *E*

Hence $\omega_i \cap \omega_{i+1} = \omega_{i+1} \cap \omega_i \neq \emptyset$… 2

1 and 2 are conditions of overlapping ordinary sets, they are satisfied.

Hence $\bigcap_{i=1}^{m} \omega_i = N$ , *N* is the nest set contains $\lambda \geq 1$ elements… 3

From 1, 2, and 3,it is proved that $\omega_i § \omega_{i+1}$ is overlapping ordinary sets.

*Example*: Let *U* the set of natural number is defined as the following: $U = \{x |\ x \in [8,15]\}$.To create (OOS), containing two sets,it is possible to cut two incomparable sets from *U*,

$A = \{x |\ x \in [8,12]\}$,$B = \{x |\ x \in [11,15]\}$ , such that

$A \,\Delta\, B = \{x |\ x \in \{8,10\}\} \cup \{x |\ x \in [13,15]\}$.

And $N_{AB} = \{x |\ x \in [11,12]\}$.

Perhaps the set *R*, is exist and non-empty set, otherwise the absence of *R* does not invalidate the (OOS), but invalidate the conversion of (OOS) under the influence of absolute complement. One can say that *R* is *remainder* due to the overlapping. So can add *R* to inequality (30).

## 4.2. Induction $A' § B'$ from $A§B$ and converse

*Theorem* 3: The absolute complement sets of $A§B$ is $A'§B'$, and converse.*R* is the nest set of $A'§B'$.

*Proof*: Text of the theorem can be placed in the pattern logical equivalence $A§B \Leftrightarrow A'§B'$.

### 4.2.1. To prove the implication $A§B \Rightarrow A'§B'$

Since A§B then $A \cap B = N$ nest set [Def.2]

And $A \cup B' = N'$ [De Morgan Law]

From axioms (11) and (14) the difference sets $A' - B' = B - N = A^c$ and $B' - A' = A - N = B^c$ are non-empty set.

From Def.1 and Lemma.1, the sets *A'* and *B'* are incomparable sets.

Hence, the condition *i* from Def.2is satisfied.

$A' \cap B' = R \cup B - N \cap R \cup A - N$ , [From Eq.11]

$= R \cup (A^c \cap B^c)$, [from Eq. (15) and distribution law]

$= R \cup \emptyset = R$

*R* is nest set of *A′* and *B′* [the condition *ii* from Def. 2 is satisfied]

So, $A§B \Rightarrow A'§B'$

### 4.2.2. To prove the implication $A'§B' \Rightarrow A§B$

By lemma. 1 $A^c \,\Delta\, B^c = A' \Delta B'$ and $A^c \,\Delta\, B^c = A \,\Delta\, B$

$B - N = A^c$ $and$ $A - N = B^c$ Non-empty sets

Hence *A* and *B* are incomparable sets … … 1

$A' = N' - A$ , $B' = N' - B$

$A = N \cup A$ , $B = N \cup B$ [ $De\ Morgan\ Law$ ]

$Then\ A \cap B = N$ ……………………..2

By 1 and 2 then $A§B$ that is $A'§B' \Rightarrow A§B$

Then $A§B \Leftrightarrow A'§B'$.

### 4.3. Induction $A'§B'$ from disjoint ordinary sets

Adopted method in the creation of (HOOS) sets is totally depends on the absolute complement; it found that not all disjoint sets have scalability overlapping. Such that the absolute complement of them gives way to varies arguments depending on the generality or vastness of these sets. The study results of the sufficient number of disjoint sets cases yield that there are three types of disjoint sets:

1.$A \sim B$; such that the sets are not subsets from any universal concepts, or their universal set in non-defined set. For example "*Existence*" and "*Nihilism*", they are maximum extent generality which are disjoint sets; denoted by $A_{max} \sim B_{max}$ .In this case $A = B'$, $A' = B$ and satisfies $A_{max} \sim B_{max} \Rightarrow A'_{max} \sim B'_{max}$, for these reasons it is not possible to create overlapping ordinary sets from them.

2.$A \sim B$; such that each one of them is subset from another universal concept, for instance $A \subset K, B \subset L$ and $K \sim L$ too. For example, "*Human*" and "*Rock*" disjoint sets, such that "*Human*" is subset of "*Living Creature*" and "*Rock*" is subset of "*Minerals*". This part satisfies the following properties:

$A \subset B', A' \supset B$ , $i.e. A \neq B', A' \neq B$ , for these reasons it's possible to create (OOS) from them.

3. $A \sim B$ ; both sets are subset from universal set as *M*, i.e.$A \subset M \wedge B \subset M$. Indeed this part is branch from part two,it be considered both *A* and *B* are subsets from another universal set as ,*M*.

*Rule* 1:$If\ A \sim B, A \subset K\ and\ B \subset L$ then

$$A \subset K \sim B \subset L \Rightarrow A'§B'$$

*Proof*: $A' = K \cup L - A\ or\ K' \cup L' - A$

$$B' = K \cup L - B \text{ or } K' \cup L' - B$$

It is clear that sets $A'\ and\ B'$ are incomparable sets.

$$A' \cap B' = K' \cup L' - (A \cup B)$$
$$= (K' \cup L') \cap (A' \cap B')$$

Hence$(A' \cap B') \subset (K' \cup L')$

$$A' \cap B' = K' \cup L' - (A \cup B) \subset (K' \cup L')$$

The nest set is $K' \cup L' - (A \cup B)$

$\therefore A \subset K \sim B \subset L \Rightarrow A'§B'$.

*Corollary*: If $A \sim B$ such that $A \subset M \wedge B \subset M$ , *M* ordinary set, then$A'§B'$.

*Proof*: The same of proof rule1.

## 4. Induction Non-Harmonious Overlapping Ordinary Sets

In Section.2, we found that it is possible to create $A'§B'$ from $A§B$ (Theorem. 1). In this section, we discuss creation:$A'§B$ and $A§B'$ from $A§B$ and incomparable sets. The nest sets of the mare subset of $N'$ it shown by the absolute complement.

### 4.1. Induction $A'§B$ and $A§B'$ from $A§B$

Consider two sets; *Birds* and *White things*, they are (OOS), also *non-birds* and *white things* are (OOS), the set *non-birds* contains everything except the *birds*. By same approach, the sets *Birds* and *non-white things* are (OOS).

*Theorem*4: If $A§B$ then:

$$(i)\ A'§B\ with\ nest\ set\ B - N \subset N'$$
$$(ii)\ A§B'\ with\ nest\ set\ A - N \subset N'$$

*Proof*:$(i)$ $A§B \Rightarrow (N \subset A \wedge N \subset B)$ [Implication. (2)]

$N \subset A \Rightarrow A' \subset N'$, and $B - N \subset B$, [*N* is nest set of $A§B$]

And $B - N \subset N' - A$ , $A' = N' - A$ , $B - N \subset A'$

$A'\ and\ B$ are incomparable sets

Then $N' - A \subset N' \wedge B - N \subset N'$

Hence $A' \cap B = (N' - A \cap B - N \cup N) \subset N'$

$$= B - N \subset N'$$

Then $B - N$ is a nest set of $A§B'$

*Proof* (*ii*) similar to the proof (*i*)by placing $B'$ instead of $A'$.

Also, the complement both sets of $A'§B$ and both sets of $A§B'$ which them sources $A§B$ will produce non-harmonious ordinary overlapping set:$A§B'$ and $A'§B$ respectively.

*Corollary* 1: $[A§B \Rightarrow A'§B] \Rightarrow A§B'$

*Proof*: $A§B \Rightarrow A'§B$ and

$A' \cap B = B - N \subset N'$(Theorem 4.*i*)

$$B - N \subset A' \Rightarrow A \subset (B' \cup N)$$

$A - N \subset B'$ , it is clear that $A - N \subset A$

Hence $A - N \subset A \wedge A - N \subset B' = A \cap B' = A - N$ ,

Then, $A§B'$[Theorem 4.*ii*].

*Corollary*2:$[A§B \Rightarrow A§B'] \Rightarrow A'§B$

*Proof*: by same proof of corollary1.

### 4.2. Induction $A'§B$ and $A§B'$ from comparable

*Ordinary sets*

The nesting comparable ordinary sets $A \subset B$ or $A \supset B$ are possible by absolute complement of the smallest set,otherwise if the absolute complement located on the largest set will convert them to disjoint sets,$A \subset B \Rightarrow A \sim B'$ and$A \supset B \Rightarrow B \sim A'$.

The fundamental principle of induction$A'§B$ and $A§B'$ from comparable ordinary sets is cutting subset from the smallest set, as in the following rules 1and 2.

*Rule*2: $A \subset B \Rightarrow A'§B$ and the nest set is $A - B \subset N'$

*Proof*: Let$U = (A \cup B) \cup R$, andlet*N* is non-empty set, such that

$N \subset A \subset B$, Then$A' \subset N'$ (1).

It is clear that

$B - A \subset N'$, Then$B - A \subset A' \subset N'$ (2).

*A* and $B'$ are incomparable sets

Hence $B - A \subset B$ (3).

From the results 1, 2,and 3:

$$(B - A \subset A' \wedge B - A \subset B) \subset N'$$
$$A' \cap B = B - A \subset N'$$

Then $A \subset B \Rightarrow (A' \cap B = B - A \subset N')$

And $A \subset B \Rightarrow A'§B$

The nest set is $B - A \subset N'$.

*Rule*3: $B \supset A \Rightarrow A§B'$ and the nest set is $A - B \subset N'$

*Proof:* by similarly proof ofRule2.

## 5. Separable of Overlapping Ordinary Sets

The separation of harmonious overlapping ordinary sets depends mainly on its sources, and to accomplish this procedure must be by absolute or relative complement, will probably located on both sets simultaneously or located on only one of the sets.

### 5.1. Separation of harmonious overlapping ordinary sets

Consider $A\S B$ ,it is found that their sets are separable by *relative complement* of them.
*Rule4*:If$A\S B$ then$A^c \sim B^c$
If the source of $A'\S B'$ is $A\subset K\sim B\subset L$ then can separate its sets by using the absolute complement, this procedure is accomplished through the opposite way of the nesting the disjoint sets.
*Rule 5*:$A'\S B' \Rightarrow A\subset K\sim B\subset L$
*Proof:* inconvers of proofRule.1.

### 5.2. Separation of non-harmonious overlapping ordinary sets

The source of $A'\S B$ and $A\S B'$is$A\S B$ (Theorem 4) and comparable sets (Rules.1and.2). If the source of $A'\S B$ and $A\S B'$is $A\S B$ then that is impossible to separate them by absolute complement. While using the relative complement on the $A'\S B$ will be converted into set *A*, and by same operation on $A\S B'$ will be converted into set *B*.

If$[A\S B \Rightarrow A'\S B]$then $(A'\S B)^c = A$ (32)

If $[A\S B \Rightarrow A\S B']$ then $(A\S B')^c = B$ (33)

*Proof*: by corollary (1.ii) of theorem 4 and (Equ.14)
If the source of $A'\S B$ and $A\S B'$is comparable sets that is possible to separate them by absolute complement.
***Rule*6**: $[A\subset B \Rightarrow A'\S B] \Rightarrow A\sim B'$
*Proof*: $A\subset B \Rightarrow A'\S B, B - A$ is nest set [Rule.1]

$$B - A \subset A' \wedge B - A \subset B$$
$$B - A \sim A \wedge B - A \sim B'$$
$$A \cap B' = \varnothing,\ A \sim B'$$

*Rule*7: $[A\supset B \Rightarrow A\S B'] \Rightarrow A'\sim B$
*Proof*: by similarly proof of Rule.7.

## 6. Overlapping Diagram

Overlapping diagram is a way to view translations that occur on relations due to the absolute complement. Consist of two perpendicular lines: a horizontal line denoted by $A'A - axis$, and a vertical line denoted by $BB' - axis$, in addition to two orthogonal lines;$AB - A'B'$and$BA' - AB'$.
Let $*$be any one of the relations:$\subset$, $\supset$, $\S and$ $\sim$,by putting the set$A*B$ in the first quarter, there are three movement tracks of set exist in quarters: horizontally, vertically, and diagonally movement. According to the data of this paper, we noted the following general results related overlapping diagram:

- All the movement from any quarter to other quarter in horizontal line or in vertical line will change only one set from explicit to non-explicit or vice versa.
- All the movement from any quarter to other quarter diagonally will change both explicit or non-explicit sets, each to other.
- Every two sets in any different quarters are equivalent sets. (See FIG.4)

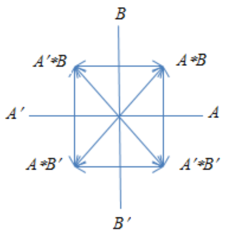

FIG4. Overlap Diagram

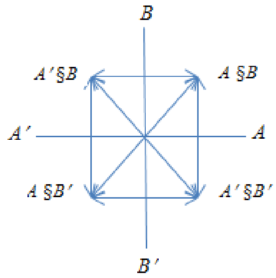

FIG4a. Overlap Daigram of $A\S B$

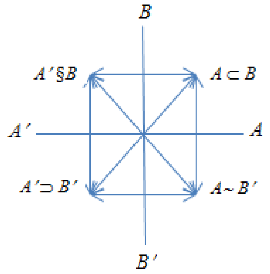

FIG4b. Overlap Daigram of $A\subset B$

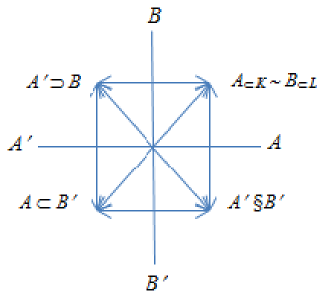

FIG4c. Overlap Daigram of $A_{\subset K}\sim B_{\subset L}$

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