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Research paper

Forecasting of Agroindustrial Complex Efficiency in the Region: Adaptive and Rational Expectations

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Abstract

The main task of the modern Russia´s economic activity is to simulate a new type of organizational and economic structure of the agroindustrial complex (hereinafter – the AIC) in the region, since the structure of the regional AIC, which developed in the post-Soviet years, has proved to be ineffective in market economic conditions. The need to anticipate the probabilistic outcome of events in the future has never been more urgent than today. This is due to the high degree of uncertainty in emerging events in society, the complexity of production control systems, and the increasing volume of information. A clear understanding of the possible state of the AIC in the future is only possible with precise forecasting methods. However, the forecasting methods are little used by managers of agricultural enterprises. As a rule, the decisions are made intuitively; thus, there is an inadequate assessment of the existing situation, based on the subjective assessment of an expert, but not on an assessment of realistic data from the mathematical apparatus. Purpose of the research is to study forecasting methods based on the hypothesis of adaptive and rational expectation and, based on the data on the operational efficiency of the Lipetsk region's AIC, to show the mechanism for their application, as well as draw conclusions about the expediency of their application to assess the region's AIC performance. Methods. The article examines the methods of regression analysis of time series forecasting, based on hypotheses about adaptive and rational expectations. As an economic series of dynamics, statistical data on the performance of the Lipetsk region (profitability level, indicators of technological efficiency of production output) are used. Results. The mechanism for building models of adaptive and rational expectations has been studied. Based on the data on the operational efficiency of the Lipetsk region's AIC, the mechanism of their widespread use has been shown. Their advantages and disadvantages have been revealed.

Keywords: *model of adaptive expectations; model of rational expectations, regression analysis; growth curves; level of profitability; indicators of technological efficiency of agricultural production.*

1. Introduction

In recent years, the mechanism of adaptive expectations and rational expectations has been widely used by domestic and foreign scientists to verify the economic models that describe the actual economic processes. The use of the adaptation principles in economic forecasting was enshrined in the early 50s of the XXth century. The exponential smoothing method proposed by R. G. Brown underlies the first adaptation models. Further, such foreign scientists as R. Veyd, D. Mat, J. Box, I.I. Perelman, Ch. Kholt, G. Jenkins, P. Harrison, D. Ward, G. Teyl, S. Wedge, R. Markland, P.R. Winters, R.F. Mayer, I.A. Muller, D. Trigg, A. Lich, M.L. Shown, U. Chou, S. Roberte, R. Rid, A. Rao, A. Shapiro etc., as well as domestic scientists, namely Yu. P. Lukashin, Ye.M. Levitskiy, A.G. Ivakhnenko, A.A. Frenkel, Ye.M. Chetyrkin, V.V. Vensel, V.P. Borodyuk, V.V. Davnis, P.A. Ivashchenko, A.S. Korkhin, etc. were engaged in developments in this field [1–10]. The adaptive approach has been developed in three areas: the first is aimed at increasing the complexity of adaptive predictive models; the second is to improve the adaptive mechanism of forecasting models; and the third implements the approach of sharing adaptive principles and other forecasting methods. The works of E.M. Levitsky and V.V. Davnis are devoted to the development of models for sharing the adaptive forecasting and other forecasting methods. However, these forecasting models do not sufficiently take into account the properties of economic systems. Therefore, there is a need to adapt these models to the reality. One of the ways to adapt forecasting is based on a combination of extrapolation and subjective evaluations. In this area studies by J. Armstrong, R. Clemen, I. Mahud, V.I. Tinyakova and others are well known. Many works of foreign scientists such as Jongwoo Kim, Willem H. Boshoff were devoted to the AIC area. [1, 11–17]. However, this apparatus is not sufficiently developed and adapted to the Russian agricultural sector.

2. Methods

2.1. The Algorithm of Forming the Adaptive and Rational Expectations' Models Has Been Studied

The hypothesis of adaptive expectations is based on the study of past values of a variable. This means that taking into account past values, the economic agents get the forecast for the future, but it falls behind the actual data, since the adaptation to the new situation is gradual. In some economic processes (for example, the dependence of the wage level on the inflation level) the backlog and adaptive expectations' models accurately describe these economic processes.

The rational expectations' hypothesis is based on the assumption that in order to form their expectations, economic agents use all current information and knowledge rather than rely only on the past experience. It also assumes that economic agents have access to all information reflecting the real state of affairs. Such information should give an adequate idea of the economy but also predict the actions of the government, the Central Bank of the country, regional authorities, etc.

1. The adaptive expectations' model formation mechanism is as follows.

The authors consider the model of the following type:

$$y_t = a + b \cdot x^*_{t+1} + \varepsilon_t, \tag{1}$$

where y_t is the actual value of the resultant sign; and

x^*_{t+1} is the expected value of factor sign.

The mechanism for generating expectations in this model is as follows:

$$x^*_{t+1} - x^*_t = \alpha \cdot (x_t - x^*_t) \tag{2}$$

or

$$x^*_{t+1} = \alpha \cdot x_t + (1-\alpha) \cdot x^*_t, \tag{3}$$

where $0 < \alpha < 1$.

Thus, the expected value of the factor variable x_t^* in the period t is the arithmetic average weighted of its actual and expected values in the previous period. In each $t + 1$ time period, the expectations are adjusted by some fraction α of the difference between the actual value of the factor sign and its expected value in the previous period. In the model, the α parameter is called the expectations' ratio. If α tends to 1, the higher is the probability of the event occurrence, and if α tends to zero, this demonstrates the sustainability of the current trends. If $\alpha = 0$, "conditions prevailing today" will continue for all future periods of time. The expected future values of indicators will coincide with their values of the current periods" [3].

The authors insert into model (1) the ratio (3) instead of x^*_{t+1}:

$$\begin{aligned} y_t &= a + b \cdot (\alpha \cdot x_t + (1-\alpha) \cdot x^*_t) + \varepsilon_t = \\ &= a + \alpha \cdot b \cdot x_t + (1-\alpha) \cdot b \cdot x_t + \varepsilon_t \end{aligned} \tag{4}$$

If model (1) exists for period t, it will also exist for period $(t-1)$. Thus, in the period $(t-1)$ the authors get:

$$y_t = a + b \cdot x^*_t + \varepsilon_{t+1} \tag{5}$$

Multiply (5) by $(1-\alpha)$:

$$(1-\alpha) \cdot y_{t-1} = (1-\alpha) \cdot a + (1-\alpha) \cdot b \cdot x^*_t + (1-\alpha) \cdot \varepsilon_{t-1} \tag{6}$$

Subtract (6) from (4) term by term:

$$(1-\alpha) \cdot y_{t-1} = a - (1-\alpha) \cdot a + a \cdot b \cdot x_t + \varepsilon_t - (1-\alpha) \cdot \varepsilon_{t-1} \tag{7}$$

or

$$y_t = \alpha \cdot a + a \cdot b \cdot x_t + (1-\alpha) \cdot y_{t-1} + u_t, \tag{8}$$

where $u_t = \varepsilon_t - (1-\alpha) \cdot \varepsilon_{t-1}$.

An autoregression model has been obtained. Having determined its parameters, one can easily go to the initial model (1).

Model (1) includes the expected values of the factor variable, which cannot be obtained empirically. Model (8) includes only the actual values of the variables. Model (1) is called the long-term function of the model of adaptive expectations; it characterizes the dependence of the effective attribute on the expected values of the factor sign. Model (8) is called the short-term function of the model of adaptive expectations, which describes the dependence of the result on the actual values of the factor.

2. The rational expectations' model formation mechanism is as follows:

The theory of rational expectations is based on the concepts of John Math, Robert Lucas [*Lucas Robert*, 1967] and Thomas Sargent [*Hansen, Sargent*, 1991]. According to it, the rational expectations' hypothesis is based on the assumption that in order to form their expectations, individual experts use all current information and knowledge rather than rely only on the past experience.

The authors consider the model of the following type:

$$y_t = a + b \cdot x_{t-1} + c \cdot z_{t-1} + \varepsilon_t, \tag{9}$$

where $z_{t-1,}$ is the exogenous variable that affects $x_{t,}$ ε_t is the random error.

At time t nothing is known about the current values of x_t and z_t, therefore the lagged values x and z are used on the right side of the equation. At time $(t-1)$ the experts form their expectations regarding x_t as per the equation (9),

$$F(x_t, l_{t-1}) = a + b \cdot x_{t-1} + c \cdot z_{t-1}, \tag{10}$$

where l_{t-1} is the information available at time $t-1$.

The value $F(x_t, l_{t-1})$ is the rational expectations regarding the variable x_t. This gives

$$y_t - F(x_t, l_{t-1}) = \varepsilon_t\,. \tag{11}$$

Thus, ε_t is the variable prediction error y_t. The hypotheses $F(\varepsilon_t) = 0$ are accepted, and ε_t are unpredictable. If ε_t could be predicted, the experts could simply rebuild the original equation so that the error becomes unpredictable.

It is assumed that the variable y_t is determined by the equation

$$y_t = \omega_0 + \omega_1 x^*_t + u_t,. \tag{12}$$

where the value x_t^* is formed on the basis of the rational expectations' hypothesis, that is

$$x^*_t = F(x^*_t, l_{t-1})\,. \tag{13}$$

To assess the model (12), you first need to estimate the values of rational expectations, and only then get estimates of the parameters ω_0 and ω_1.

The rational expectations' model assessment procedure is as follows:

1. The parameters of equation (9) are assessed, and the following models are obtained:

$$x_t = \hat{a} + \hat{b} \cdot x_{t-1} + \hat{c} \cdot z_{t-1} + u_t\ , \tag{14}$$

$$\hat{x}_t = \hat{a} + \hat{b} \cdot x_{t-1} + \hat{c} \cdot z_{t-1}\,. \tag{15}$$

2. These values $\hat{x}_t$ were taken as an approximation $x^*_t = F(x^*_t, l_{t-1})$, they were inserted into equation (12) instead of x_t^*, and the estimates of the LSM parameters ω_0 and ω_1 were found.

3. Results

3.1. Practical Implementation of Models of Adaptive and Rational Expectations on the Example of the Lipetsk Region's AIC

The growth of production and profitability of agricultural products is the main indicator of the activities of agricultural producers in the Lipetsk region. Table 1 presents the data on the level of profitability (unprofitability) of agricultural products sold in the Lipetsk region, % [18, 19].

Table 1: Profitability (unprofitability) level of agricultural products sold in the Lipetsk region, % [18]

Year	Quarter	X_t	Profitability level (Y_t)	Year	Quarter	X_t	Profitability level (Y_t)
2004	1st	1	28.1	2010	3rd	27	15.86
	2nd	2	16.49		4th	28	9.3
	3rd	3	14.43	2011	1st	29	11.5
	4th	4	9.11		2nd	30	9.52
2005	1st	5	12.37		3rd	31	8.94
	2nd	6	11.08		4th	32	12.96
	3rd	7	10.55	2012	1st	33	10.08
	4th	8	6.64		2nd	34	11.43
2006	1st	9	12.78		3rd	35	13.86
	2nd	10	8.66		4th	36	11.19
	3rd	11	10.87	2013	1st	37	6.43
	4th	12	7.47		2nd	38	2.76
2007	1st	13	9.08		3rd	39	7.06
	2nd	14	11.27		4th	40	8.72
	3rd	15	16.59	2014	1st	41	20.82
	4th	16	14.04		2nd	42	26.13
2008	1st	17	14.93		3rd	43	26.8
	2nd	18	14.72		4th	44	27.41
	3rd	19	13.22	2015	1st	45	33.05
	4th	20	12.22		2nd	46	33.39
2009	1st	21	23.33		3rd	47	33.5
	2nd	22	18.91		4th	48	28.4
	3rd	23	16.42	2016	1st	49	30.5
	4th	24	14.01		2nd	50	22
2010	1st	25	18.08		3rd	51	23.67
	2nd	26	15.43		4th	52	26.21

The authors demonstrated the implementation of the mechanism of adaptive and rational expectations with an example. As a baseline, the time series of an indicator of the agricultural producers efficiency in the Lipetsk region (profitability level) will be used.

3.1.1. Time Series Analysis

The authors determined the autocorrelation ratio of the levels of the time series Y_t. Table 2:

Table 2: Autocorrelation ratio of the first, second, third and fourth orders

Autocorrelation function	
Lag	Autocorrelation ratios.
1	0.842
2	0.733
3	0.577
4	0.493

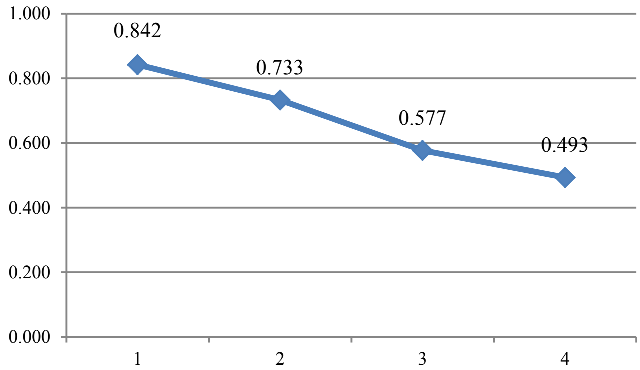

Fig. 1: The graph of the autocorrelation function (correlogram) of the time series

The authors identified anomalous observations by the Irwin method Table 3:

Table 3: Calculations of the parameter λ_t.

X_t	1	2	3	4	5	6	7	8	9
Y_t	28.1	16.5	14.4	9.1	12.4	11.1	10.6	6.6	12.8
$(y_t - \bar{Y})^2$	155.0	0.7	1.5	42.7	10.7	20.9	26.0	81.1	8.2
λ_t		**1.5**	0.3	0.7	0.4	0.2	0.1	0.5	0.8
X_t	10	11	12	13	14	15	16	17	18
Y_t	8.7	10.9	7.5	9.1	11.3	16.6	14.0	14.9	14.7
$(y_t - \bar{Y})^2$	48.8	22.8	66.9	43.1	19.2	0.9	2.6	0.5	0.9
λ_t	0.5	0.3	0.4	0.2	0.3	0.7	0.3	0.1	0.0
X_t	19	20	21	22	23	24	25	26	27
Y_t	13.2	12.2	23.3	18.9	16.4	14.0	18.1	15.4	15.9
$(y_t - \bar{Y})^2$	5.9	11.8	59.0	10.6	0.6	2.7	5.9	0.0	0.0
λ_t	0.2	0.1	**1.4**	0.6	0.3	0.3	0.5	0.3	0.1
X_t	28	29	30	31	32	33	34	35	36
Y_t	9.3	11.5	9.5	8.9	13.0	10.1	11.4	13.9	11.2
$(y_t - \bar{Y})^2$	40.3	17.2	37.6	45.0	7.2	31.0	17.8	3.2	19.9
λ_t	0.8	0.3	0.3	0.1	0.5	0.4	0.2	0.3	0.3
X_t	37	38	39	40	41	42	43	44	45
Y_t	6.4	2.8	7.1	8.7	20.8	26.1	26.8	27.4	33.1
$(y_t - \bar{Y})^2$	85.0	166.1	73.8	48.0	26.7	109.9	124.4	138.3	302.8
λ_t	0.6	0.5	0.5	0.2	**1.5**	0.7	0.1	0.1	0.7
X_t	46	47	48	49	50	51			
Y_t	33.4	33.5	28.4	30.5	22.0	23.7			
$(y_t - \bar{Y})^2$	314.8	318.7	162.6	220.6	40.3	64.3			
λ_t	0.0	0.0	0.7	0.3	**1.1**	0.2			

With probability P=0.95 ($\lambda_t \leq 1.1$), the observations 2, 21, 41, 50 were abnormal, and with probability P=0.99 ($\lambda_t \leq 1.6$), no anomalies were observed. With probability P=0.99, the oscillations were insignificant, therefore, smoothing the series could be avoided.

The authors identified the trends in the development of the studied indicator. The verification of the presence of a trend in the series under study came down to testing the hypothesis of equality of the average two normally distributed sets.

The authors verified the variances' equality hypothesis using the Fisher criterion Table 4:

Table 4: The result of a two-sample F-test for dispersion

	Variable 1	Variable 2
Average	13.9	18.1
Variance	22.7	94.7
Observations	26	26
df	25	25
F	0.24	
P(F<=f) one-sided	0.00	
F critical one-sided	0.51	

By analyzing the results of the two-sample F-test to verify the variance's equality hypothesis, the authors concluded that the corrected sample variance was significantly different; therefore, there was a linear trend.

The authors conducted a visual analysis of a number of dynamics and build trends (graphs of the time dependencies of series) (Table 5, Figure 2).

Table 5: Types of trend models and their assessment

Trend model type	Trend equation	Determination ratio (R^2)
Linear	$y = 0.25x + 9.22$	0.24
Logarithmic	$y = 2.34\ln(x) + 8.97$	0.07
Power	$y = 10.3x^{0.1065}$	0.03
Exponential	$y = 10.19x^{0.0125x}$	0.14
Polynomial (n=6)	$y = -2E-0.7x^6 + 2E-0.5x^5 - 0.0006x^4 - 0.0089x^3 + 0.55x^2 - 6.45x + 30.54$	0.79

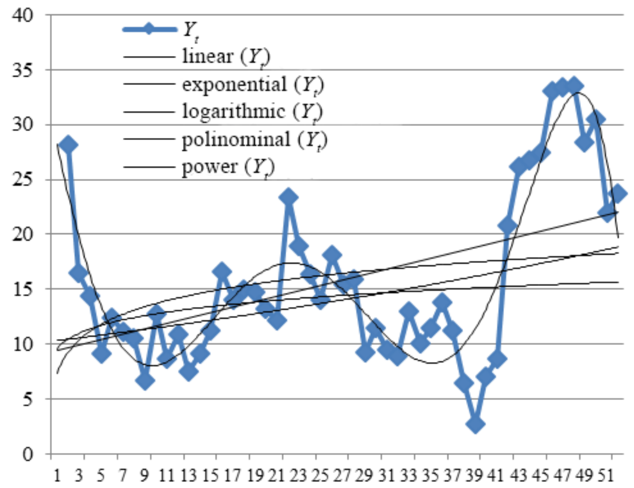

Fig. 2: Visual analysis of a dynamics series

The listed trend equations poorly described time series (Table 5), since the values of determination ratio were low.
To describe the time series, the authors used the adaptive and rational expectations' models.

3.1.2. Adaptive expectations' model

According to the model (14), the authors built a short-term adaptive expectations' model, which described the dependence of the result on the actual values of the factor Table 6, Table 7.

Table 6: Baseline data of the adaptive expectations' model

X_t	2	3	4	5	6	7	8	9	10
Y_t	16.49	14.43	9.11	12.37	11.08	10.55	6.64	12.78	8.66
Y_{t-1}	28.1	16.49	14.43	9.11	12.37	11.08	10.55	6.64	12.78
X_t	11	12	13	14	15	16	17	18	19
Y_t	10.87	7.47	9.08	11.27	16.59	14.04	14.93	14.72	13.22
Y_{t-1}	8.66	10.87	7.47	9.08	11.27	16.59	14.04	14.93	14.72
X_t	20	21	22	23	24	25	26	27	28
Y_t	12.22	23.33	18.91	16.42	14.01	18.08	15.43	15.86	9.3
Y_{t-1}	13.22	12.22	23.33	18.91	16.42	14.01	18.08	15.43	15.86
X_t	29	30	31	32	33	34	35	36	37
Y_t	11.5	9.52	8.94	12.96	10.08	11.43	13.86	11.19	6.43
Y_{t-1}	9.3	11.5	9.52	8.94	12.96	10.08	11.43	13.86	11.19
X_t	38	39	40	41	42	43	44	45	46
Y_t	2.76	7.06	8.72	20.82	26.13	26.8	27.41	33.05	33.39
Y_{t-1}	6.43	2.76	7.06	8.72	20.82	26.13	26.8	27.41	33.05
X_t	47	48	49	50	51	52			
Y_t	33.5	28.4	30.5	22	23.67	26.21			
Y_{t-1}	33.39	33.5	28.4	30.5	22	23.67			

Table 7: Estimation of parameters of adaptive expectations' model

Regression Statistics		Variance analysis						
Multiple R	0.87		*df*	*SS*	*MS*	*F*	Significance *F*	
R-square	0.75	Regression	2.00	2,265.60	1,132.80	72.03	0.00	
Normalized R-square	0.74	Residual	48.00	754.91	15.73			
Standard error	3.97	Total	50.00	3020.51				
Observations	51.00							
	Ratios	Standard error	t-statistics	P-Value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Y-intersection	1.01	1.38	0.73	0.47	-1.76	3.78	-1.76	3.78
Variable X 1	0.12	0.04	2.79	0.01	0.03	0.21	0.03	0.21
Variable X 2	0.73	0.08	8.99	0.00	0.57	0.89	0.57	0.89

The short-term function of the adaptive expectations' model was $y_t = 1.01 + 0.12 \cdot x_t + 0.73 \cdot y_{t-1} + \varepsilon_t$ (Figure 3).

The authors assessed the parameters *α* = 0.27, *b* = 0.44, *α* =3.74 and got the long-term function of the adaptive expectations' model $y_t = 3.74 + 0.44 \cdot x^*_{t+1} + \varepsilon_t$ (Figure 3).

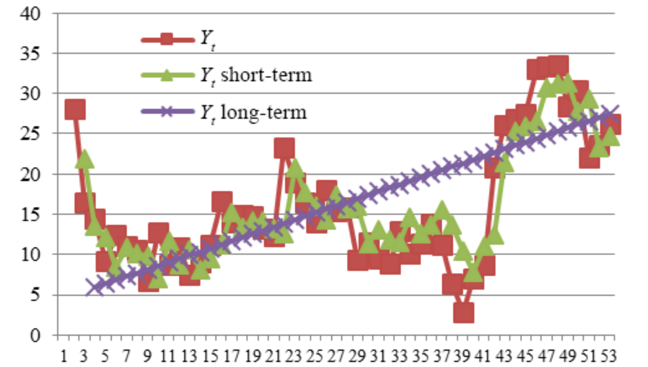

Fig. 3: Short-term and long-term forecast of the level of profitability of the Lipetsk region's agricultural products using the adaptive expectations' model

The authors assessed the quality of the built model. To do this, the authors checked the residuals of the regression for the equality of the mathematical expectation to zero, the lack of autocorrelation, and compliance with the normal law.
To check the autocorrelation in residuals, the authors used the Durbin-Watson criterion (*DW)*, the value of which was calculated by the following formula (Table 8):

$$DW = \frac{\sum_{t=1}^{n-1}(\varepsilon_{i+1} - \varepsilon_t)^2}{\sum_{t=1}^{n}\varepsilon_t^2} \quad (16)$$

$DW = 1{,}453.6 / 726.82 = 2$.

Table 8: Calculation of regression residuals and Durbin-Watson statistics

X_t	Y_t	$\tilde{Y}_{t\ short.}$	ε_t	ε_t^2	$(\varepsilon_t - \varepsilon_{t-1})^2$
1	2	3	4	5	6
1			0.00	0.00	
2	28.1		0.00	0.00	0.00
3	16.49	21.883	-5.39	29.08	29.08
4	14.43	13.5277	0.90	0.81	39.63
5	9.11	12.1439	-3.03	9.20	15.49
6	12.37	8.3803	3.99	15.92	49.33
7	11.08	10.8801	0.20	0.04	14.36
8	10.55	10.0584	0.49	0.24	0.09
9	6.64	9.7915	-3.15	9.93	13.27
10	12.78	7.0572	5.72	32.75	78.75
11	8.66	11.6594	-3.00	9.00	76.08
12	10.87	8.7718	2.10	4.40	25.99

13	7.47	10.5051	-3.04	9.21	26.35
14	9.08	8.1431	0.94	0.88	15.78
15	11.27	9.4384	1.83	3.35	0.80
16	16.59	11.1571	5.43	29.52	12.97
17	14.04	15.1607	-1.12	1.26	42.95
18	14.93	13.4192	1.51	2.28	6.92
19	14.72	14.1889	0.53	0.28	0.96
20	13.22	14.1556	-0.94	0.88	2.15
21	12.22	13.1806	-0.96	0.92	0.00
22	23.33	12.5706	**10.76**	115.76	137.36
23	18.91	20.8009	-1.89	3.58	160.03
24	16.42	17.6943	-1.27	1.62	0.38
25	14.01	15.9966	-1.99	3.95	0.51
26	18.08	14.3573	3.72	13.86	32.60
27	15.43	17.4484	-2.02	4.07	32.96
28	15.86	15.6339	0.23	0.05	5.04
29	9.3	16.0678	-6.77	45.80	48.91
30	11.5	11.399	0.10	0.01	47.18
31	9.52	13.125	-3.61	13.00	13.73
32	8.94	11.7996	-2.86	8.18	0.56
33	12.96	11.4962	1.46	2.14	18.69
34	10.08	14.5508	-4.47	19.99	35.22
35	11.43	12.5684	-1.14	1.30	11.10
1	2	3	4	5	6
36	13.86	13.6739	0.19	0.03	1.75
37	11.19	15.5678	-4.38	19.17	20.83
38	6.43	13.7387	-7.31	53.42	8.59
39	2.76	10.3839	-7.62	58.12	0.10
40	7.06	7.8248	-0.76	0.58	47.05
41	8.72	11.0838	-2.36	5.59	2.56
42	20.82	12.4156	8.40	70.63	115.95
43	26.13	21.3686	4.76	22.67	13.27
44	26.8	25.3649	1.44	2.06	11.06
45	27.41	25.974	1.44	2.06	0.00
46	33.05	26.5393	6.51	42.39	25.75
47	33.39	30.7765	2.61	6.83	15.19
48	33.5	31.1447	2.36	5.55	0.07
49	28.4	31.345	-2.95	8.67	28.09
50	30.5	27.742	2.76	7.61	32.52
51	22	29.395	**-7.40**	54.69	103.08
52	23.67	23.31	0.36	0.13	60.14
53	26.21	24.6491	1.56	2.44	1.44
				726.82	1,453.60

With the level $DW \to 2$, there was a rather weak positive correlation between ε_t and ε_{t-1} ($r \to 0$). The critical values of the Durbin-Watson test (*DW*) were $d_l = 1.5$, $d_u = 1.59$. Since the values of the Durbin-Watson test (*DW*) were in the range of $0 \geq DW \geq 2$ and the condition $du < DW \geq 2$ was satisfied, thus the hypothesis H_0 about the absence of autocorrelation was accepted.

The regression residual graph was as follows (Figure 4).

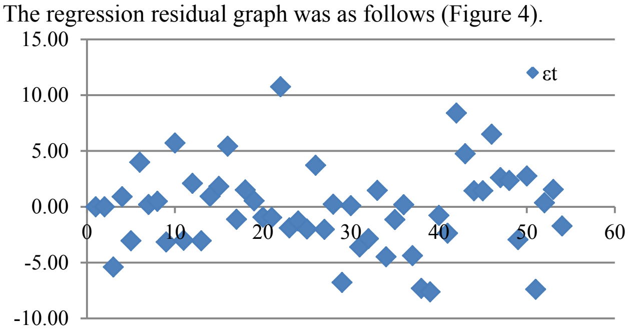

Fig. 4: Regression residual graph

Thus, according to this criterion, the model could be considered adequate.

The authors checked the correspondence of a number of residues to the normal distribution and determined the proximity to the relevant parameters of the normal distribution law to the asymmetry and kurtosis ratios. The authors used the *RS* criterion, calculated the unbiased standard deviation

$$S_\varepsilon = \sqrt{\frac{1}{n-1}\sum \varepsilon_t^2}, \qquad (17)$$

$S_\varepsilon = 3.85$, and found

$$RS = \frac{(\varepsilon_{\max} - \varepsilon_{\min})}{S_\varepsilon}, , \qquad (18)$$

$RS = (10.76 - (-7.4))/3.85 = 4.71$. The values of the RS criterion fell between the critical values of 3.63 and 5.35; therefore, the hypothesis of the normal distribution was accepted. The model as per this criterion could be considered adequate, and a number of residues – corresponding to the normal law. The calculation of the average value for a number of residues gave the result $\bar{e}$ = -0.014. Thus, the hypothesis of equality of the mathematical expectation to zero was satisfied.

As a result, the authors have come to the conclusion that the model could be considered statistically adequate by the *DW* test and the *RS* criterion; therefore, it could be used to build predictive estimates.

3.1.3. Rational Expectations' Model

The authors assessed the model (12) according to the above assessment procedure. As an exogenous variable z_t the authors took the inflation rate of food products presented in Table 9.

Table 9: Dynamics of the consumer prices for food products (month to previous month, %) [19]

Year	Quarter	Increase in food prices (z_t)	Year	Quarter	Increase in food prices (z_t)
2004	1st	3.6	2010	3rd	-2.7
	2nd	2.5		4th	1.7
	3rd	1.7	2011	1st	2.3
	4th	3.3		2nd	2.4
2005	1st	5.1		3rd	0.7
	2nd	2.5		4th	1.9
	3rd	0.4	2012	1st	1.9
	4th	2.1		2nd	1.6
2006	1st	6.2		3rd	1.1
	2nd	0.8		4th	1.7
	3rd	-0.2	2013	1st	4
	4th	1.9		2nd	3.5
2007	1st	2.5		3rd	0.6
	2nd	3.5		4th	6.5
	3rd	1.8	2014	1st	10.6
	4th	6.8		2nd	0
2008	1st	5.6		3rd	-0.6
	2nd	5.4		4th	3.4
	3rd	0.6	2015	1st	2.3
	4th	3.9		2nd	0.9
2009	1st	5		3rd	-0.7
	2nd	1.9		4th	2.1
	3rd	-1.1	2016	1st	-2.7
	4th	0.3		2nd	1.7
2010	1st	3.7		3rd	2.3
	2nd	1.5		4th	2.4

The baseline data of the model will be presented in the form Table 10.

Table 10: Baseline data of the adaptive expectations' model

No.	1	2	3	4	5	6	7	8	9
X_t	28.1	16.5	14.4	9.1	12.4	11.1	10.6	6.6	12.8
X_{t-1}		28.1	16.5	14.4	9.1	12.4	11.1	10.6	6.6
Z_{t-1}		3.6	2.5	1.7	3.3	5.1	2.5	0.4	2.1
No.	10	11	12	13	14	15	16	17	18
X_t	8.7	10.9	7.5	9.1	11.3	16.6	14.0	14.9	14.7
X_{t-1}	12.8	8.7	10.9	7.5	9.1	11.3	16.6	14.0	14.9
Z_{t-1}	6.2	0.8	-0.2	1.9	2.5	3.5	1.8	6.8	5.6
No.	19	20	21	22	23	24	25	26	27
X_t	13.2	12.2	23.3	18.9	16.4	14.0	18.1	15.4	15.9

X_{t-1}	14.7	13.2	12.2	23.3	18.9	16.4	14.0	18.1	15.4
Z_{t-1}	5.4	0.6	3.9	5.0	1.9	-1.1	0.3	3.7	1.5
No.	28	29	30	31	32	33	34	35	36
X_t	9.3	11.5	9.5	8.9	13.0	10.1	11.4	13.9	11.2
X_{t-1}	15.9	9.3	11.5	9.5	8.9	13.0	10.1	11.4	13.9
Z_{t-1}	2.8	4.2	4.7	0.2	-2.7	1.7	2.3	2.4	0.7
No.	37	38	39	40	41	42	43	44	45
X_t	6.4	2.8	7.1	8.7	20.8	26.1	26.8	27.4	33.1
X_{t-1}	11.2	6.4	2.8	7.1	8.7	20.8	26.1	26.8	27.4
Z_{t-1}	1.9	1.9	1.6	1.1	1.7	4.0	3.5	0.6	6.5
No.	46	47	48	49	50	51	52		
X_t	33.4	33.5	28.4	30.5	22.0	23.7	26.2		
X_{t-1}	33.1	33.4	33.5	28.4	30.5	22.0	23.7		
Z_{t-1}	10.6	0.0	-0.6	3.4	2.3	0.9	-0.7		

1. By assessing the equation parameters (9) (Table 9), the authors obtained the model of the following type: $x_t = 2.3 + 0.8\ x_{t-1} + 0.1\ z_{t-1} + u_t$ and value $\hat{x}_t = 2.3 + 0.8x_{t-1} + 0.1z_{t-1}$

Table 11: Estimation of parameters of adaptive expectations' model

Regression Statistics			Variance analysis					
Multiple R	0.8			df	SS	MS	F	Significance F
R-square	0.7		Regression	2.0	2,148.6	1,074.3	59.1	0.0
Normalized R-square	0.7		Residual	48.0	871.9	18.2		
Standard error	4.3		Total	50.0	3020.5			
Observations	51.0							
	Ratios	Standard error	t-statistics	P-Value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Y-intersection	2.3	1.4	1.6	0.1	-0.5	5.2	-0.5	5.2
Variable X 1	0.8	0.1	10.6	0.0	0.7	1.0	0.7	1.0
Variable X 2	0.1	0.3	0.5	0.6	-0.4	0.7	-0.4	0.7

2. The authors took the values $\widehat{x_t}$ as approximation $x_t^* = F(x_t, l_{t-1})$, inserted them into the equation (12) instead of x_t^* (Table 12), and found the assessment of LSM parameters ω_0 and ω_1 (Table 13).

Table 12: Baseline data of the adaptive expectations' model

No.	1	2	3	4	5	6	7	8	9
$\widehat{x_t}$	26.1	16.3	14.5	10.3	13.3	11.9	11.1	8.1	13.8
Y_t	28.1	16.5	14.4	9.1	12.4	11.1	10.6	6.6	12.8
No.	10	11	12	13	14	15	16	17	18
$\widehat{x_t}$	9.6	11.3	8.8	10.2	12.2	16.3	14.9	15.5	15.3
Y_t	8.7	10.9	7.5	9.1	11.3	16.6	14.0	14.9	14.7
No.	19	20	21	22	23	24	25	26	27
$\widehat{x_t}$	13.4	13.0	22.4	18.3	15.8	14.0	17.8	15.3	15.9
Y_t	13.2	12.2	23.3	18.9	16.4	14.0	18.1	15.4	15.9
No.	28	29	30	31	32	33	34	35	36
$\widehat{x_t}$	10.6	12.5	10.2	9.3	13.3	11.0	12.1	13.9	11.9
Y_t	9.3	11.5	9.5	8.9	13.0	10.1	11.4	13.9	11.2
No.	37	38	39	40	41	42	43	44	45
$\widehat{x_t}$	7.9	4.8	8.3	9.8	20.1	24.5	24.6	26.0	31.2
Y_t	6.4	2.8	7.1	8.7	20.8	26.1	26.8	27.4	33.1
No.	46	47	48	49	50	51	52		
X_t	30.0	30.0	26.3	27.9	20.7	21.8	24.3		
Y_t	33.4	33.5	28.4	30.5	22.0	23.7	26.2		

Table 13: Estimation of parameters of adaptive expectations' model

Regression Statistics			Variance analysis					
Multiple R	1.0			*df*	*SS*	*MS*	*F*	Significance *F*
R-square	1.0		Regression	1.0	3,161.8	3,161.8	20,094.9	0.0
Normalized R-square	1.0		Residual	50.0	7.9	0.2		
Standard error	0.4		Total	51.0	3,169.6			
Observations	52.0							
	Ratios	Standard error	t-statistics	P-Value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Y-intersection	-3.0	0.1	-20.7	0.0	-3.3	-2.7	-2.7	-3.3
Variable X 1	1.2	0.0	141.8	0.0	1.2	1.2	1.2	1.2
Variable X 2	-3.0	0.1	-20.7	0.0	-3.3	-2.7	-2.7	-3.3

The equation (12) will be as follows $y_t = -3 + 1.2x_t^* + u_t$ (Figure 5).

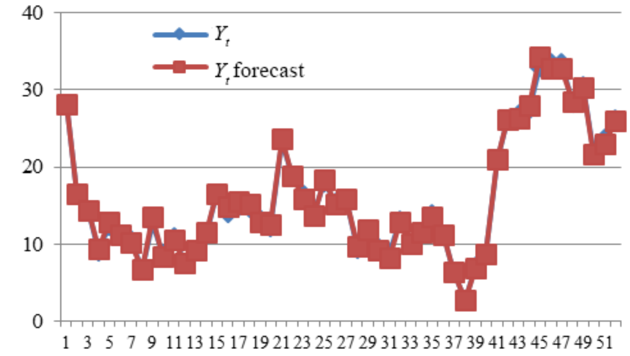

Fig. 5: Forecast of the level of profitability of the Lipetsk region's agricultural products using the rational expectations' model

The authors assessed the quality of the built model. To do this, the authors checked the residuals of the regression for the equality of the mathematical expectation to zero, the lack of autocorrelation, and compliance with the normal law.

The Durbin-Watson criterion was $DW = 12.61/7.68 = 1.6$ (Table 14).

Table 14: Calculation of regression residuals and Durbin-Watson statistics

X_t	Y_t	$\bar{Y}_{t\ short.}$	ε_t	ε_t^2	$(\varepsilon_t - \varepsilon_{t-1})^2$
1	2	3	4	5	6
1	28.1	28.2	0.06	0.00	
2	16.49	16.5	0.00	0.00	0.00
3	14.43	14.3	-0.11	0.01	0.01
4	9.11	9.3	0.22	0.05	0.11
5	12.37	12.9	0.49	0.24	0.07
6	11.08	11.1	0.06	0.00	0.19
7	10.55	10.3	-0.29	0.09	0.13
8	6.64	6.7	0.04	0.00	0.11
9	12.78	13.5	0.68	0.46	0.40
10	8.66	8.5	-0.20	0.04	0.77
11	10.87	10.5	-0.40	0.16	0.04
12	7.47	7.5	0.00	0.00	0.16
13	9.08	9.2	0.08	0.01	0.01
14	11.27	11.5	0.23	0.05	0.02
15	16.59	16.5	-0.12	0.01	0.12
16	14.04	14.8	0.77	0.59	0.79
17	14.93	15.5	0.55	0.30	0.05
18	14.72	15.2	0.52	0.27	0.00
19	13.22	12.9	-0.29	0.08	0.65
20	12.22	12.5	0.29	0.08	0.33
21	23.33	23.7	0.35	0.12	0.00
22	18.91	18.8	-0.13	0.02	0.23
23	16.42	15.8	-0.62	0.38	0.24
24	14.01	13.7	-0.35	0.12	0.07
25	18.08	18.3	0.19	0.04	0.29
26	15.43	15.3	-0.16	0.03	0.12
1	2	3	4	5	6
27	15.86	15.9	0.06	0.00	0.05
28	9.3	9.7	0.37	0.14	0.10
29	11.5	11.9	0.43	0.19	0.00
30	9.52	9.2	-0.32	0.10	0.56
31	8.94	8.1	**-0.81**	0.65	0.24
32	12.96	12.9	-0.10	0.01	0.50
33	10.08	10.1	0.04	0.00	0.02
34	11.43	11.5	0.04	0.00	0.00
35	13.86	13.6	-0.28	0.08	0.10
36	11.19	11.1	-0.04	0.00	0.06
37	6.43	6.4	0.01	0.00	0.00
38	2.76	2.8	0.00	0.00	0.00
39	7.06	6.9	-0.13	0.02	0.02
40	8.72	8.7	-0.05	0.00	0.01
41	20.82	21.0	0.21	0.04	0.07
42	26.13	26.2	0.06	0.00	0.02
43	26.8	26.4	-0.45	0.20	0.26
44	27.41	28.0	0.56	0.31	1.01
45	33.05	34.3	**1.20**	1.44	0.41
46	33.39	32.8	-0.62	0.39	3.33
47	33.5	32.8	-0.73	0.53	0.01
48	28.4	28.4	0.02	0.00	0.56
49	30.5	30.3	-0.20	0.04	0.05
50	22	21.7	-0.34	0.11	0.02
51	23.67	23.0	-0.63	0.40	0.09
52	26.21	26.0	-0.18	0.03	0.20
53	e	15.77	0.00	7.86	12.61

With the level *DW* →2, there was a rather weak positive correlation between ε_t and ε_{t-1} (*r* →0). The critical values of the Durbin-Watson test (*DW*) were d_l =1.5, d_u= 1.59. The values of the Durbin-Watson test (*DW*) were in the range of 0 ≥*DW* ≥2, and the condition

du < *DW* ≥2 was satisfied, therefore, the hypothesis H_0 about the absence of autocorrelation was accepted.

The regression residual graph is as follows (Figure 6).

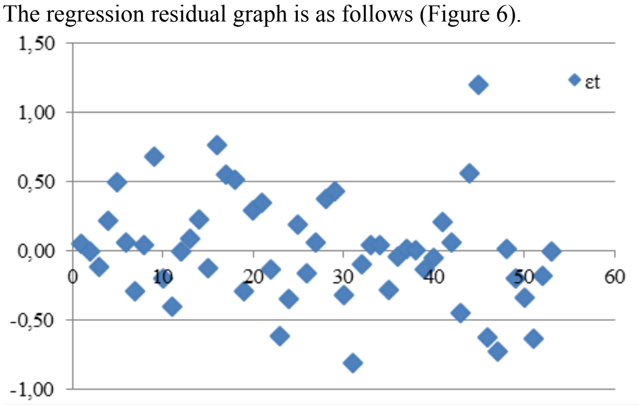

Fig. 6: Regression residual graph

Thus, according to this criterion, the model could be considered adequate.

The *RS* criterion was calculated as follows. The unbiased standard deviation S_ε = 0.4, thus, *RS* = (1.2 – (–0.81))/0.4 = 5.01. The values of the *RS* criterion fell between the critical values of 3.63 and 5.35; therefore, the hypothesis of the normal distribution was accepted. The model as per this criterion could be considered adequate, and a number of residues – corresponding to the normal law. The calculation of the average value for a number of residues gave the result $\bar{e} = -0.001$. Thus, the hypothesis of equality of the mathematical expectation to zero was satisfied.

As a result, the authors have come to the conclusion that the model could be considered statistically adequate by the *DW* test and the *RS* criterion; therefore, it could be used to build predictive estimates.

4. Discussion

It should be noted that the adaptive expectations' model provides good results, especially when forecasting in the long term. However, it should be noted that the α parameter cannot remain unchanged in equation (2) throughout the selected period, since the economic system is not stationary and undergoes constant changes. Therefore, it is necessary to constantly adapt this parameter. For adaptation, for example, the function can be used, where α reflects the dependence on any variable, reflecting bursts and declines of the economic system, or on differences in the calculated and real data.

The rational expectations' hypothesis is based on the connection between the subjective views of economic agents and the actual behavior of the economic system (5). The left part of equality (5) is interpreted as a subjective expectation, and the right part – as an objective one. The use of formula (5) implies the following conditions:

1. The mathematical expectation of a random variable x_t^* at a given set of information l_{t-1} is unique.
2. Economic agents behave as if they knew this expectation, and identify it with their own subjective expectation with respect to x_t^*, which means the structure of the model and its parameters.

The main drawback of the theory of rational expectations is that economic agents usually do not always behave this way. Therefore, the economic agents must be experts in this field and have all the necessary information and mathematical apparatus for analysis.

5. Conclusion

The use of the regression analysis provides more accurate assessment and forecasting of the region's AIC development. Therefore, at the AIC enterprises, it is advisable to introduce an assessment procedure based on the above mechanism, and other types of models proposed by the authors should also be used [1, 11–17].

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