# Analysis of Iterated Affine Transformation Function and Linear Mapping for Content Preservation 

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#### Abstract

In image scaling contents of image can be distorted which are required to preserve using linear mapping. Geometric transformations can preserve structural properties i.e. parallelism, colinearity and orientation. It is highly desirable to preserve structural properties of image contents because human visual system is very sensitive to distortion of objects. In this paper image scaling is performed using iterative affine transformation and results show that linear mapping function applied on affine space preserve affine properties under affine transformation. A number of scaling operations are applied on image using iterative affine transformation and for each iteration linear mapping is performed to preserved object structure. Analysis present in this paper shows that in image scaling preservation of image content is possible under iterative affine transformation and linear mapping. Image artifacts can be minimize using saliency based antialiasing algorithm after affine transformation.


Keywords: Affine Transformation; Aliasing; Face Recognition; Fractal; Iterated function system; Resampling.

## 1. Introduction

In computer graphics affine transformation preserves affine property of affine space i.e. distortion of line, point and planes. Affine space generalizes the properties of Euclidian geometry in such a way that it does not include the concept of distance, angle, area and preservation of distance between points but preserve following properties:

- Parallelism- Parallel lines remain parallel.
- Co linearity- The line segments lying on the same line remain collinear.
- Ratio of Distance- Preserve ration of distance.
- Orientation- Preserve orientation of line segments.
- Convexity of sets- A convex set continues to be convex after the transformation.
Geometric transformation which can be included in affine transformation are translation, rotation, scaling, reflection, shearing and similarity transformation and any composite transformation which is combination of all the 2D transformation. Affine transformation is useful in various areas of computer science. T. Gangopadhyay [1] present study on the effect of multiple rotations on affine transformations with related trigonometric coefficients in terms of the iterated function system fractals generated by them by using unified set of equations. DandanZuo [2] proposed algorithm which utilize affine map to repair the structure of traffic signs in the real images. N. Paniphi [3] discussed various aspects related to the geometric errors observed in a satellite image and polygonal affine transformation to remove them. P. K. Mohanty [4] proposed an algorithm for designing face recognition algorithms based solely on affine transforms. C.V. Priscilla [5] proposed a method for image registration and noise reduction using affine transformation.
M. Toorani [6] introduces a secure and efficient symmetric cryptosystem based on affine transformation. Shen Wang [7] proposed a lossless JPEG2000 image steganography algorithm based on affine transformation. H. Chen [8] proposed a novel method based on affine transformation, which can correct and stitch the doppler beam sharpening image effectively. In image retargeting, images are displayed on different screen size with different aspect ratio. For preserving highly noticeable content in image researchers have found various methods. C. Chang [9] proposed an image resizing method in content aware fashion which simultaneously preserves both salient image features and line structure properties. Proposed method combines mesh deformations which control content preservation and for image resizing with similarity transforms for line features.
In Figure 4 image depicts scaling operation and linear mapping for content preservation. Output of transformation shows image artifacts which must be avoided through precise antialiasing algorithm. In this paper affine transformation is analyzed with Iterated function system to maintain affine property. Yao Xiao [13] proposed fast skewed object matching with adaptive rotation steps based on affine transformation. Lei Yu [14] proposed algorithm which focused on problem associated with imprecise location and trouble on the conventional large-screen, using projectors and Kinect with affine transformation algorithm to get exact coordinates interaction area and capture interactive action. RediaRedzuwan [15] proposed a method to for correcting position by affine matrix transformation based on image features detection, description and matching and affine transformation estimation.Xuan Li [16] proposed novel regression strategy by introducing affine transformation
In this paper, Section (2) represents utility of forward and backward mapping methods for calculating corresponding relative coordinate positions or target object in image. Section $(3,4)$ pre-
sents role contractive affine transformation to generate fractal images. In section (5) algorithm for iterated affine transformation is presented. Section (6) present result of iterated affine transformation and linear mapping for content preservation of fractal images. Section (7) present effect of iterated affine transformation over image to maintain affine properties. To analyze affine map, affine properties are justified using forward and backward linear mapping function. Section (8) represents problem of artifacts after transformation which can be removed by antialiasing algorithm [17] which is based on saliency map.


## 2. Mapping Procedure

### 2.1 Forward Method

In Forward Mapping each pixel of source image is mapped with the pixel in target image [10].
In Figure 1,forward mapping function $f(\mathrm{iu}, \mathrm{iv})$ is applied on pixel (iu, iv) in source image to map it with corresponding pixel ( $\mathrm{x}, \mathrm{y}$ ) in target image. Buffer is used to store pixel location of source image to find out precise locations of pixels after forward mapping. High computation time and speed is required to implement algorithm which incorporates this method.


Fig. 1: (a) Source Image (b) Target Image

### 2.2 Reversed Method

To map image pixels a inverse transformation function is used which maps target image pixel locations on source image. Pixel of the destination image (ix, iy) is mapped to the backward function f to the pixel $(\mathrm{u}, \mathrm{v})$ in the source image [10].


Fig. 2: (a) Target Image (b) Source Image

## 3. Affine Transformation and Image Mapping

Fractals can be generated using iterated function system [11] using following steps.

- Implement combined transformations.
- Draw any object.
- Applying geometric affine transformations on the initial object.
- Iteratively apply transformation on initial object to get new transformed object.
- Apply image mapping function, recursively to map coordinate of source fractal image to target image.
- Repeat step 5, until attractor will not generated.


## 4. Contractive Affine Transformation

A contraction mapping is a transformation where any two distinct points are brought closer together by the transformation.
A contracting affine transformation is an affine transformation that is also a contraction mapping. Contraction mappings have a very important property - they leave exactly one point fixed [12]. Following combine transformation matrix can applied on the image.
$\left[\begin{array}{l}x^{\prime} \\ y^{\prime}\end{array}\right]=s\left[\begin{array}{cc}\cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha\end{array}\right]\left[\begin{array}{l}x-x_{0} \\ y-y_{0}\end{array}\right]$
$=s\left[\begin{array}{cc}\cos \alpha\left(x-x_{0}\right)+ & \sin \alpha\left(y-y_{0}\right) \\ -\sin \alpha\left(x-x_{0}\right)+ & \cos \alpha\left(y-y_{0}\right)\end{array}\right]$

Separating the equations,
$x^{\prime}=(s \cos \alpha) x+(s \sin \alpha) y-s\left(x_{0} \cos \alpha+y_{0} \sin \alpha\right)$
$y^{\prime}=(-s \sin \alpha) x+(s \cos \alpha) y+s\left(x_{0} \sin \alpha-y_{0} \cos \alpha\right)$

This can be also written as
$x^{\prime}=a x-b y+c$
$y^{\prime}=b x+a y+d$

Where
$a=s \cos \alpha$
$b=-\sin \alpha$
The scale factor $s$ is then defined by
$s=\sqrt{a^{2}+b^{2}}$
and the rotation angle by
$\alpha=\tan ^{-1}\left(\frac{b}{a}\right)$
An affine transformation of $\mathbb{R}^{n}$ is a map $F: \mathbb{R}^{n} \rightarrow \mathbb{R}^{n}$ of the form
$F(p)=A p+q$
for all $\rho \in \mathbb{R}^{n}$, where $A$ is a linear transformation of $\mathbb{R}^{n}$.
If $\operatorname{det}(a)>0$, the transformation is orientation-preserving; if, $\operatorname{det}(a)<0$ is orientation-reversing.

## 5. Iterative Affine Transformation and Linear Mapping

In this section affine transformation is implemented and analyzed affine properties. Iterated function system is also implemented which combines for Iterative affine transformation to produce image and iterative forward and reverse image mapping method calculates relative position of each pixel of source image in to target image. After each iteration of implemented iterated function affine properties are analyzed for geometric distortion of object and their preservation.


Fig. 3: (a) Reducing image with scaling factors $S x=0.5, S y=0.5$, rotation angle $10^{\circ}$, (b) scaling factors $S x=0.5, S y=0.5$, rotation angle $10^{\circ}$, (c) Scaling factors $S x=0.3, S y=0.5$ and rotation angle $15^{0}$.

Table 1: Constructing Affine Transformation

| Figure | Matrix |  |  |  |  | Translation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{a}$ | $\mathbf{B}$ | $\mathbf{c}$ | $\mathbf{d}$ | $\mathbf{E}$ | $\mathbf{f}$ |  |
| Figure 4(a) | 0.2 | 0.231 | 0.234 | 0.2 | -1 | -2 |  |
| Figure 4(b) | 0.3 | 0.32 | 0.273 | 0.4 | -1 " | -2 |  |
| Figure 4(c) | 0.4 | 0.212 | 0.221 | 0.3 | -1 | -0.5 |  |



Fig. 4: (a) Affine transformation with scaling factors $S x=0.5 \& S y=0.5$, (b) Scaling factors $S x=0.3$ and $S y=0.5$, (c) Scaling factors $S x=0.5$ and $S y=0.5$, (d) Scaling factorsSx=0.5 and $S y=0.5$ with mapping, (d) Scaling factor $S x=0.3$ and $S y=0.5$ with image mapping.

In figure 4 (c) it is observed that affine transformation map preserves the property of parallelism, Co linearity and ratio of distance.


Fig. 5: Flowchart of affine transformation over image to reduce and expand size
Table 2: Iterated Affine transformation Function applied on image, For Figure 3(a)

| Iteration | Dimension of Source Image |  |  |  |  |  |  |  | Dimension of Target Image |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X1 | Y1 | X2 | Y2 | X3 | Y3 | X4 | Y4 | X1 | Y1 | X2 | Y2 | X3 | Y3 | X4 | Y4 |
| 1 | 0 | 0 | 600 | 0 | 600 | 600 | 0 | 600 | 0 | 0 | 500 | 0 | 500 | 500 | 0 | 500 |
| 2 | 0 | 0 | 500 | 0 | 500 | 500 | 0 | 500 | 0 | 0 | 417 | 0 | 417 | 417 | 0 | 417 |
| 3 | 0 | 0 | 417 | 0 | 417 | 417 | 0 | 417 | 0 | 0 | 347 | 0 | 347 | 347 | 0 | 347 |
| 4 | 0 | 0 | 347 | 0 | 347 | 347 | 0 | 347 | 0 | 0 | 289 | 0 | 289 | 289 | 0 | 289 |
| 5 | 0 | 0 | 289 | 0 | 289 | 289 | 0 | 289 | 0 | 0 | 241 | 0 | 241 | 241 | 0 | 241 |
| 6 | 0 | 0 | 241 | 0 | 241 | 241 | 0 | 241 | 0 | 0 | 201 | 0 | 201 | 201 | 0 | 201 |
| 7 | 0 | 0 | 201 | 0 | 201 | 201 | 0 | 201 | 0 | 0 | 167 | 0 | 167 | 167 | 0 | 167 |
| 8 | 0 | 0 | 167 | 0 | 167 | 167 | 0 | 167 | 0 | 0 | 140 | 0 | 140 | 140 | 0 | 140 |
| 9 | 0 | 0 | 140 | 0 | 140 | 140 | 0 | 140 | 0 | 0 | 116 | 0 | 116 | 116 | 0 | 116 |
| 10 | 0 | 0 | 116 | 0 | 116 | 116 | 0 | 116 | 0 | 0 | 97 | 0 | 97 | 97 | 0 | 97 |
| 11 | 0 | 0 | 97 | 0 | 97 | 97 | 0 | 97 | 0 | 0 | 81 | 0 | 81 | 81 | 0 | 81 |
| 12 | 0 | 0 | 81 | 0 | 81 | 81 | 0 | 81 | 0 | 0 | 67 | 0 | 67 | 67 | 0 | 67 |
| 13 | 0 | 0 | 67 | 0 | 67 | 67 | 0 | 67 | 0 | 0 | 56 | 0 | 56 | 56 | 0 | 56 |
| 14 | 0 | 0 | 56 | 0 | 56 | 56 | 0 | 56 | 0 | 0 | 47 | 0 | 47 | 47 | 0 | 47 |
| 15 | 0 | 0 | 47 | 0 | 47 | 47 | 0 | 47 | 0 | 0 | 39 | 0 | 39 | 39 | 0 | 39 |
| 16 | 0 | 0 | 39 | 0 | 39 | 39 | 0 | 39 | 0 | 0 | 32 | 0 | 32 | 32 | 0 | 32 |
| 17 | 0 | 0 | 32 | 0 | 32 | 32 | 0 | 32 | 0 | 0 | 27 | 0 | 27 | 27 | 0 | 27 |
| 18 | 0 | 0 | 27 | 0 | 27 | 27 | 0 | 27 | 0 | 0 | 23 | 0 | 23 | 23 | 0 | 23 |
| 19 | 0 | 0 | 23 | 0 | 23 | 23 | 0 | 23 | 0 | 0 | 19 | 0 | 19 | 19 | 0 | 19 |
| 20 | 0 | 0 | 19 | 0 | 19 | 19 | 0 | 19 | 0 | 0 | 16 | 0 | 16 | 16 | 0 | 16 |

Table 3: Forward and Backward Image Mapping For Figure 4 (a)

| Iteration | Pixel Locations in Source Image |  |  |  |  |  | Pixel Location in Target Image After Mapping |  |  |  |  |  | Xc | Yc |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X1 | Y1 | X2 | Y2 | X3 | Y3 | X1 | Y1 | X2 | Y2 | X3 | Y3 |  |  |
| 1 | 200 | 0 | 300 | 300 | 100 | 300 | 222 | 30 | 215 | 260 | 60 | 207 | 166 | 166 |
| 2 | 222 | 30 | 215 | 260 | 60 | 207 | 218 | 59 | 148 | 214 | 43 | 137 | 137 | 137 |
| 3 | 218 | 59 | 148 | 214 | 43 | 137 | 198 | 82 | 101 | 169 | 40 | 89 | 113 | 113 |
| 4 | 198 | 82 | 101 | 169 | 40 | 89 | 169 | 95 | 69 | 128 | 44 | 59 | 94 | 94 |
| 5 | 169 | 95 | 69 | 128 | 44 | 59 | 134 | 98 | 49 | 94 | 49 | 42 | 77 | 78 |
| 6 | 134 | 98 | 49 | 94 | 49 | 42 | 101 | 92 | 37 | 67 | 52 | 33 | 63 | 64 |
| 7 | 101 | 92 | 37 | 67 | 52 | 33 | 73 | 80 | 31 | 47 | 52 | 29 | 52 | 52 |
| 8 | 73 | 80 | 31 | 47 | 52 | 29 | 50 | 67 | 28 | 35 | 48 | 28 | 42 | 43 |
| 9 | 50 | 67 | 28 | 35 | 48 | 28 | 34 | 52 | 26 | 27 | 42 | 27 | 34 | 35 |
| 10 | 34 | 52 | 26 | 27 | 42 | 27 | 24 | 40 | 24 | 22 | 36 | 26 | 28 | 29 |
| 11 | 24 | 40 | 24 | 22 | 36 | 26 | 18 | 30 | 23 | 19 | 29 | 24 | 23 | 24 |
| 12 | 18 | 30 | 23 | 19 | 29 | 24 | 14 | 22 | 20 | 16 | 22 | 21 | 19 | 20 |
| 13 | 14 | 22 | 20 | 16 | 22 | 21 | 11 | 16 | 16 | 14 | 17 | 17 | 15 | 16 |
| 14 | 11 | 16 | 16 | 14 | 17 | 17 | 10 | 12 | 13 | 12 | 13 | 13 | 12 | 12 |
| 15 | 10 | 12 | 13 | 12 | 13 | 13 | 9 | 10 | 9 | 10 | 9 | 10 | 9 | 10 |
| 16 | 9 | 10 | 9 | 10 | 9 | 10 | 7 | 8 | 7 | 8 | 7 | 8 | 7 | 8 |
| 17 | 7 | 8 | 7 | 8 | 7 | 8 | 5 | 6 | 5 | 6 | 5 | 6 | 5 | 6 |
| 18 | 5 | 6 | 5 | 6 | 5 | 6 | 4 | 5 | 4 | 5 | 4 | 5 | 4 | 5 |
| 19 | 4 | 5 | 4 | 5 | 4 | 5 | 3 | 4 | 3 | 4 | 3 | 4 | 3 | 4 |
| 20 | 3 | 4 | 3 | 4 | 3 | 4 | 2 | 3 | 2 | 3 | 2 | 3 | 2 | 3 |



Table 4: Iterated Function System Code Using Matrix Compact Form for

| Iterations | Matrix |  |  |  | Translation |  | Probabil- |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | a | B | c | d | E | f | p |
| $\qquad$ | 0.00 | 0.00 | 0.00 | $\begin{gathered} \hline 0.16 \\ 0 \end{gathered}$ | $\begin{gathered} \hline 0.0 \\ 0 \\ \hline \end{gathered}$ | 0.00 | 0.010 |
| $\begin{aligned} & \text { Iteration } \\ & 2 \end{aligned}$ | $\begin{gathered} 0.85 \\ 0 \end{gathered}$ | $\begin{gathered} 0.04 \\ 0 \end{gathered}$ | $\begin{gathered} 0.04 \\ 0 \end{gathered}$ | $\begin{gathered} 0.85 \\ 0 \end{gathered}$ | $\begin{gathered} 0.0 \\ 0 \end{gathered}$ | $\begin{gathered} 1.60 \\ 0 \end{gathered}$ | 0.850 |
| $\begin{aligned} & \text { Iteration } \\ & 3 \end{aligned}$ | $\begin{gathered} 0.20 \\ 0 \end{gathered}$ | $\begin{gathered} 0.26 \\ 0 \end{gathered}$ | $\begin{gathered} 0.23 \\ 0 \end{gathered}$ | $\begin{gathered} 0.22 \\ 0 \end{gathered}$ | $\begin{gathered} 0.0 \\ 0 \end{gathered}$ | $\begin{gathered} 1.60 \\ 0 \end{gathered}$ | 0.070 |
| $\begin{gathered} \text { Iteration } \\ 4 \end{gathered}$ | $\begin{gathered} 0.15 \\ 0 \end{gathered}$ | $\begin{gathered} 0.28 \\ 0 \end{gathered}$ | $\begin{gathered} 0.26 \\ 0 \end{gathered}$ | $\begin{gathered} 0.24 \\ 0 \end{gathered}$ | $\begin{gathered} 0.0 \\ 0 \end{gathered}$ | $\begin{gathered} 0.44 \\ 0 \end{gathered}$ | 0.070 |

Above 4 Iterations are applied on image with Iterated function coefficient to generate fractal images. Geometric transformation which are including in iterations are translation, rotation, scaling. When scaling transformation is applied iteratively on image line structure properties can be preserved.

Image after 14 Iterations, (c) Fractal Image after 20 Iterations
Table 5: Code Using Scale/Rotate Form with 10 Iterations

| Iterations | Scale |  | Rotation |  |  | Translation | Probability |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 1 | 0.707 | 0.707 | 45.00 | 45.00 | 0.00 | 0.00 | 0.167 |
| Iteration 2 | 0.707 | 0.707 | -45.00 | -45.00 | 0.500 | 0.500 | 0.167 |
| Iteration 3 | 0.500 | 0.500 | 0.00 | 0.00 | -0.283 | -0.444 | 0.083 |
| Iteration 4 | 0.500 | 0.500 | 0.00 | 0.00 | .024 | 0.083 |  |
| Iteration 5 | 0.500 | 0.500 | 0.00 | 0.00 | .204 | -0.057 | 0.083 |
| Iteration 6 | 0.500 | 0.500 | 0.00 | 0.00 | 0.336 | -0.219 | 0.083 |
| Iteration 7 | 0.500 | 0.500 | 0.00 | 0.00 | 0.244 | 0.464 | 0.083 |
| Iteration 8 | 0.500 | 0.500 | 0.00 | 0.00 | -0.007 | 0.297 | 0.083 |
| Iteration 9 | 0.500 | 0.500 | 0.00 | 0.00 | 0.477 | 0.475 | 0.083 |
| Iteration 10 | 0.500 | 0.500 | 0.00 | 0.00 | 0.021 | 0.278 | 0.083 |


(b)

(c)

Fig. 7: Using above IFS code fractal Images are generated (a) Fractal Image Using 40 Iterations with 10 Fixed Points, (b) Fractal image using 100 Iterations, (c) Fractal Image using 200 Iterations

## 6. Analysis and Results

Let (wxmin, wymin) and (wxmax, wymax) be the corners of target image and (vxmin, vymin) and (vxmax, vymax) be the coordinates of source image. Let assume that aspect ratio of both source image and target image is same so,
$s x=\frac{w x 2-w x 1}{v x 2-v x 1}, s y=\frac{w y 2-w y 1}{v y 2-v y 1}$
$>$ Property 1 (Co linearity)-Three or more points which
lie on the same line (called collinear points) continue to be collinear after the transformation.

## Proof-

Let three points (a1,b1), (a2,b2), (a3,b3) be collinear. $\frac{a 2-a 1}{b 2-b 1}=\frac{a 3-a 2}{b 3-b 2}=\frac{a 3-a 1}{b 3-b 1}$
Let the coordinates of three points in view port be (a1`,b1`), (a2`,b2`), and (a3`,b3`).
$a 1^{\prime}=v x \min +(a 1-$ wxmin $) s_{x}$
$b 1^{\prime}=$ vymin $+(b 1-$ wymin $) s_{y}$
$a 2^{\prime}=$ vxmin $+(a 2-$ wxmin $) s_{x}$
$b 2^{\prime}=$ vymin $+(b 2-$ wymin $) s_{y}$
$a 3^{\prime}=$ vxmin $+(a 3-$ wxmin $) s_{x}$
$b 3^{\prime}=$ vymin $+(b 3-$ wymin $) s_{y}$
$m 1=\frac{a 2^{\prime}-a 1^{\prime}}{b 2^{\prime}-b 1^{\prime}}$

$$
\begin{equation*}
=\frac{\left\{\text { vxmin }+(a 2-\text { wxmin }) s_{x}-\text { vxmin }+(a 1-\text { wxmin }) s_{x}\right\}}{\left\{\text { vymin }+(b 2-\text { wymin }) s_{y}-\text { vymin }+(b 1-\text { wymin }) s_{y}\right\}} \tag{13}
\end{equation*}
$$

$\frac{(a 2-a 1) s_{x}}{(b 2-b 1) s_{y}}$
$m 2=\frac{a 3^{\prime}-a 1^{\prime}}{b 3^{\prime}-b 1^{\prime}}$
$=\frac{\left\{\text { vxmin }+(a 3-\text { wxmin }) s_{x}-\text { vxmin }+(a 1-\text { wxmin }) s_{x}\right\}}{\left\{\text { vymin }+(b 3-\text { wymin }) s_{y}-\text { vymin }+(b 1-\text { wymin }) s_{y}\right\}}$
$\frac{(a 3-a 1) s_{x}}{(b 3-b 1) s_{y}}$
Substituting value of $\frac{(a 3-a 1)}{(b 3-b 1)}$ from eq. (12) in eq. (15) we get
$m 2=\frac{(a 2-a 1) s_{x}}{(b 2-b 1) s_{y}}$
$\frac{a 2-a 1}{b 2-b 1}=\frac{a 4-a 3}{b 4-b 1}$
$\mathrm{m} 1=\mathrm{m} 2$
Points are collinear since one point is common and slopes are equal.
$>$ Property 2 (Parallelism) -Two or more lines which are parallel, continue to be parallel after the transformation.
Proof- Let $A B$ and $C D$ be two lines with coordinates ( $\mathrm{x} 1, \mathrm{y} 1$ ) $(x 2, y 2)$ and $(x 3, y 3)(x 4, y 4)$ respectively be parallel. The slope of AB and CD will be equal since both the lines are equal.
$\frac{a 2-a 1}{b 2-b 1}=\frac{a 4-a 3}{b 4-b 1}$
$a 1^{\prime}=$ vxmin $+(a 1-$ wxmin $) s_{x}$
$b 1^{\prime}=$ vymin $+(b 1-$ wymin $) s_{y}$
$a 2^{\prime}=$ vxmin $+(a 2-$ wxmin $) s_{x}$
$b 2^{\prime}=$ vymin $+(b 2-$ wymin $) s_{y}$
$a 3^{\prime}=$ vxmin $+(a 3-$ wxmin $) s_{x}$
$b 3^{\prime}=$ vymin $+(b 3-$ wymin $) s_{y}$
$a 4^{\prime}=v x \min +(a 4-$ wxmin $) s_{x}$
$b 4^{\prime}=$ vymin $+(b 4-$ wymin $) s_{y}$
$m 2=\frac{a 4^{\prime}-a 3^{\prime}}{b 4-b 3^{\prime}}$
$=\frac{\left\{\text { vxmin }+(a 4-\text { wxmin }) s_{x}-\text { vxmin }+(a 3-\text { wxmin }) s_{x}\right\}}{\left\{\text { vymin }+(b 4-\text { wymin }) s_{y}-\text { vymin }+(b 3-\text { wymin }) s_{y}\right\}}$
$\frac{(a 4-a 3) s_{x}}{(b 4-b 3) s_{y}}$
Substituting value of $a 4-a 3 / b 4-b 3$ from (16), we get
$m 2=\frac{(a 2-a 1) s_{x}}{(b 2-b 1) s_{y}}$
$\mathrm{m} 1=\mathrm{m} 2$
Lines are parallel since there are no common points and slope is equal.
$>$ Property 3 (Ratio of Distance) -Along a line for distinct collinear points the ratio of distance between them remains same after undergoing transformation.
Proof-Let three points (a1, b1), (a2, b2), (a3, b3) on line AB
Since the three points are collinear. Therefore slopes of line formed by considering any 2 points will be equal.
$\frac{a 2-a 1}{b 2-b 1}=\frac{a 3-a 2}{b 3-b 2}=\frac{a 3-a 1}{b 3-b 1}$
Let the coordinates of three points in target image be (a1`, b1`), ( a 2 `, $\mathrm{b} 2^{\prime}$ ), and ( $\mathrm{a} 3^{\prime}, \mathrm{b} 3^{\prime}$ ). Calculate ratio of distance by substituting coordinate position in distance formula.
In target image Ratio of distance will be:
$r 1=\frac{\sqrt{\left\{\left(a 2^{\prime}-a 1^{\prime}\right)^{2}+\left(b 2^{\prime}-b 1^{\prime}\right)^{2}\right\}}}{\left\{\left(a 3^{\prime}-a 1\right)^{2}+\left(b 3^{\prime}-b 1^{\prime}\right)^{2}\right\}}$
$r 1=\frac{\sqrt{\left\{\left((a 2-a 1) s_{x}\right)^{2}+\left((b 2-b 1) s_{y}\right)^{2}\right\}}}{\sqrt{\left\{\left((a 3-a 1) s_{x}\right)^{2}+\left((b 3-b 1) s_{y}\right)^{2}\right\}}}$
Taking b2-b1 and b3-b1 as common from numerator and denominator respectively, we get
$r 1=\frac{m \times \sqrt{\left\{(m \times s x)^{2}+s_{y}{ }^{2}\right\}}}{\sqrt{\left\{\left(m \times s_{x}\right)^{2}+s_{y}{ }^{2}\right\}}}$
$\mathrm{r} 1=\mathrm{m}$
Ratio of distance between will be
$r 2=\frac{\sqrt{\left\{\left(a 2^{\prime}-a 1^{\prime}\right)^{2}+\left(b 2^{\prime}-b 1^{\prime}\right)^{2}\right\}}}{\left\{\left(a 3^{\prime}-a 2^{\prime}\right)^{2}+\left(b 3^{\prime}-b 2^{\prime}\right)^{2}\right\}}$
$r 2=\frac{\sqrt{\left\{\left((a 2-a 1) s_{x}\right)^{2}+\left((b 2-b 1) s_{y}\right)^{2}\right\}}}{\sqrt{\left\{\left((a 3-a 2) s_{x}\right)^{2}+\left((b 3-b 2) s_{y}\right)^{2}\right\}}}$
$r 2=\frac{m \times \sqrt{\left\{(m \times s x)^{2}+s_{y}{ }^{2}\right\}}}{\sqrt{\left\{\left(m \times s_{x}\right)^{2}+s_{y}{ }^{2}\right\}}}$
$\mathrm{r} 2=\mathrm{m}$
$\mathrm{r} 1=\mathrm{r} 2$
Along a line for distinct collinear points the ratio of distance between them remains same after undergoing transformation.
$>$ Property 4 (Distance between Points) - An affine transformation does not preserve distances between points.

## Proof-

Let three points $(a 1, b 1),(a 2, b 2),(a 3, b 3)$ makes triangle in source image and after relative mapping we get corresponding points (a1`,b1`), (a2`, b2`), and (a3`, b3`).
Distance between points in source image (a1, b1), (a2, b2) is
$r 1=\sqrt{\left\{(a 1-a 2)^{2}+(b 1-b 2)^{2}\right\}}$

Distance between points in target image (a1`,b1`), (a2`,b2`)is
$r 2=\sqrt{\left\{\left((a 2-a 1) s_{x}\right)^{2}+\left((b 2-b 1) s_{y}\right)^{2}\right\}}$
$r 2=\sqrt{\left\{\left(a 2^{\prime}-a 1^{\prime}\right)^{2}+\left(b 2^{\prime}-b 1^{\prime}\right)^{2}\right\}}$

## Hence, rlır2.

> Property 5 (Area of Object) - An affine transformation does not preserve area of object.

## Proof:

Let three points (a1,b1), (a2,b2), (a3,b3) makes right triangle in source image where distance between (a1, b1) and (a2, b2) gives length and distance between (a1, a2) and (a3, b3) gives height. After relative mapping we get corresponding points (a1`,b1`), ( $\mathrm{a} 2^{`}, \mathrm{~b} 2^{`}$ ), and ( $\mathrm{a} 3^{`}, \mathrm{~b} 3^{`}$ ).
Substitute relative coordinates in formula for calculating area of triangle.
Area of right angle in source image is

Area of right triangle in target image is
A2
$=\frac{\sqrt{\left\{\left(a 1^{\prime}-a 2^{\prime}\right)^{2}+\left(b 1^{\prime}-b 2\right)^{2}\right\}} \times \sqrt{\left\{\left(a 1^{\prime}-a 2^{\prime}\right)^{2}+\left(a 3^{\prime}-b 3\right)^{2}\right\}}}{2}$

A2
$=\frac{\sqrt{\left\{(a 1-a 2)^{2}+(b 1-b 2)^{2}\right\}} \times \sqrt{\left\{(a 1-a 2)^{2}+(a 3-b 3)^{2}\right\}}}{2}$

Hence, $\mathrm{A} 1 \neq \mathrm{A} 2$

## A1

$$
=\frac{\sqrt{\left\{(a 1-a 2)^{2}+(b 1-b 2)^{2}\right\}} \times \sqrt{\left\{(a 1-a 2)^{2}+(a 3-b 3)^{2}\right\}}}{2}
$$

Table 6: Analysis of affine Properties after Iterative Affine Transformation

| Properties of Affine Map | Object of Input Image (Before Affine Transformation) |  |  |  |  |  | Coordinates of Object After Linear Mapping |  |  |  |  |  | Outcome |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X1 | Y1 | X2 | Y2 | X3 | Y3 | X 1 | Y 1 | X2 | Y'2 | X 3 | Y3 |  |
| Co linearity | 20 | 40 | 30 | 50 | 40 | 60 | 5 | 7.5 | 6.25 | 8.75 | 7.5 | 10 | Valid |
| Parallelism | 0 | 0 | 4 | 4 | 8 | 8 | 3 | 1 | 7 | 5 | 11 | 9 | Valid |
| Ratio of Distance | 20 | 40 | 30 | 50 | 40 | 60 | 5 | 7.5 | 6.25 | 8.75 | 7.5 | 10 | Valid |
| Angle or Orientation | 20 | 40 | 30 | 50 | 40 | 60 | 5 | 7.5 | 6.25 | 8.75 | 7.5 | 10 | Valid |
| Distance Between Points | 20 | 40 | 30 | 50 | 40 | 60 | 5 | 7.5 | 6.25 | 8.75 | 7.5 | 10 | Invalid |
| Area of Object | 20 | 40 | 30 | 50 | 40 | 60 | 5 | 7.5 | 6.25 | 8.75 | 7.5 | 10 | Invalid |

## 7. Antialiasing

Iterative affine transformation with image mapping produces aliasing effects due to change in angle of lines structures. A precise antialiasing algorithm is required to minimize image artifacts. A saliency based antialiasing algorithm can be implemented with iterative affine transformation for smoothening staircase effects.

(a)

(b)

(c)

Fig. 8: Iterative affine transformation and image mapping with antialiasing

## 8. Discussion and Conclusion

Affine transformation is a linear mapping function between affine space which preserves affine properties. The affine transformation corrects geometric distortions or deformations of objects present in image that occur with non-ideal camera angles. For example, in content aware image resizing line structure properties can be distorted which can be preserve with the use of affine transformation and it is also applicable in satellite imagery, face recognition, stenography, image stitching and image registration. Image mapping functions can be used to find out relative position of image pixel in source image into target image.
This paper presents analysis of affine transformation to preserve line structure properties i.e. co linearity, parallelism, ratio of distance, and convex set. Iterative Image mapping procedure is also used to map coordinate of source image with relative coordinate of target image after affine transformation to maintain affine properties. Result of geometric affine transformation produce aliasing affects which can be rectifying by anti-aliasing algorithm. For
implementation of iterative affine transformation C language is used and for removal of aliasing problem anti aliasing filters are used over image using MATLAB.

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