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Research paper



A study on the estimation and prediction of volatility on financial assets

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Abstract

Volatility in financial markets due to changes in the internal and external financial market environment causes economic entities to increase uncertainty of economic activity, thus affecting the real economy. The volatility of stock index, interest rate, and exchange rate has a negative impact on the business performance of companies and financial institutions due to decline in the value of stocks, bonds, and derivatives held for short-term trading purposes. In this study, therefore, the KRW/USD exchange rate and the KOSDAQ index data from January 2005 to December 2017 were converted to log yield data for volatility estimation. Autocorrelation test of the error terms confirmed the partial autocorrelation function, and a Portmanteau Q-test was performed. Significant parameters were estimated by the stepwise autoregressive method. The Lagrange Multiplier test (L-M test) was used for the ARCH effect and the order of the model, and parameters were estimated by the Maximum Likelihood Method. Fit of the estimated model was found to follow the white noise according to the Portmanteau Q-test using standardized residuals. As the result, AR(1,2,3,13)-ARCH(1) model was selected as the volatility estimation model for KRW/USD exchange rate, and AR(1)-ARCH(1) model for KOSDAQ index.

Keywords: *Log yield, Volatility, Portmanteau-Q test, Lagrange Multiplier test, AR(p)-ARCH(q)*

1. Introduction

The international financial market has been rapidly cooled as the US sovereign credit rating has been downgraded, and since then has remained unstable due to the worsening European debt crisis and concerns over China's shortage of money. Global financial markets are highly volatile due to concerns over spread of the European financial crisis and the slowing US economy, which is reinforcing the preference for safe financial assets. The volatility of Korean financial market is also increasing after the global financial crisis. The volatility of financial market due to changes in the internal and external financial market environment can affect the real economy by increasing uncertainty of the economic activity from the macroeconomic viewpoint. For example, if the volatility of stock index expands, uncertainty in corporate financing tends to disrupt future investment plans. The volatility of exchange rate can be a factor to reduce international trade volume by distorting the relative price system at home and abroad, and can make uncertainty on profitability of companies increased. From the microeconomic viewpoint, volatility in stock prices, interest rates, and exchange rates has a negative impact on the performance of companies and financial institutions due to decline in the value of stocks, bonds, and derivatives, which are securities held for short-term trading purposes. Therefore, it is necessary to estimate and forecast volatility by minimizing uncertainty of economy in a macroeconomic framework, and through risk management that reflects risks from price fluctuations in advance in a microeconomic framework. The volatility of financial assets has been studied over a long period of time. Anderson & Bollerslov (1998) estimated volatility using the daily rate of return on exchange rate and compared the predictive power of estimates obtained through the GARCH model using the 5-minute rate of return [1]. Akgiray (1989) showed that the GARCH model was superior to the ARCH and EWMA model in predicting US stock index volatility [2], Jorion (1995) studied usefulness of information on the future realization volatility of foreign exchange options [3], and Ghysels et al. (2003) showed that absolute returns were superior to squared returns in predicting volatility using DM/USD returns [4]. Xiao and Aydemir (2007) compared various modified GARCH models using daily yield data [5]. Kim & Kwon (2011) developed a forecasting algorithm for the exchange rate confidence interval during the economic crisis [6]. Kim & Kim (2012) studied fit of the time series model for the KRW/USD exchange rate [7]. Kim (2013) analyzed the volatility of KRW/CNY, KRW/USD, and KRW/JPY exchange rates using the FIGARCH model, the GJR model, and the EGARCH model [8]. Jang (2013) conducted an exploratory data analysis on the Korean exchange rate through a recurrence plot [9], Kwon & Lee (2014) analyzed the KOSPI200 futures, which are time series data similar to exchange rate, using the VECM and GARCH models [10], and Lee & Chun (2016) predicted the volatility of CNY using the deep learning [11]. In this study, volatility was estimated and predicted by the AR(p)-ARCH(q) model, which is an autoregressive conditional heteroscedasticity model, using the log yield data of the KRW/USD exchange rate and the KOSDAQ index.

2. Research method

2.1. Data conversion

Data used for this study are the KRW/USD rate and KOSDAQ index from 2005 to 2017 provided by the Korea Economic Statistics System (ECOS). In order to estimate the volatility of financial assets, the original data was convert to the log yield data as shown

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in (Equation 1) to provide financial phenomenon information with more return data than the original data and to solve the asymmetric problem of probability distribution. When value of the financial asset at time t is Z_t,

$$R_t = \frac{z_t - z_{t-1}}{z_{t-1}} = \ln\frac{z_t}{z_{t-1}} \quad (1)$$

is the log yield of the asset occurred in period $[t-1,\ t]$.

2.2. Volatility model

The model for estimating the volatility of financial assets is divided into two models: the ARCH (Autoregressive Conditional Heteroscedasticity) model and the GARCH (Generalized ARCH) model. The ARCH model is a seemingly unrelated regression model at ν_t, but the present value of ν_t, is affected by the quadratic form of past values (Equation 2) [12].

$$y_t = \mathbf{X}_t'\boldsymbol{\beta} + \varepsilon_t \quad (2)$$

$$\varepsilon_t = \phi_1\varepsilon_{t-1} + \cdots + \phi_p\varepsilon_{t-p} + \nu_t$$

$$\nu_t = \sigma_t a_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\nu_{t-1}^2 + \cdots + \alpha_q\nu_{t-q}^2$$

where $a_t \sim N(0,1)$ (a white noise), and σ_t^2 is a variance in the model, so $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, \cdots, p$ must be satisfied in order to have a positive value. When the volatility σ_t^2 follows the ARCH model, σ_t^2 becomes larger as the past impact $(\nu_{t-1}, \cdots, \nu_{t-q})$ becomes larger, which also means a larger current impact (ν_t) due to $\nu_t = \sigma_t a_t$. And the GARCH model is a model proposed to overcome the problem of efficiency decrease due to increase of the number of parameters in the ARCH model (Equation 3) [13]. In the (Equation 2), if $\sigma_t^2 = \alpha_0 + \alpha_1\nu_{t-1}^2 + \cdots + \alpha_q\nu_{t-q}^2$ is modified as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}\alpha_i\,\nu_{t-i}^2 + \sum_{j=1}^{q}\delta_j\,\sigma_{t-j}^2 \quad (3)$$

σ_t^2 follows the GARCH(p, q) model. The GARCH$(0, q)$ model is the ARCH(q) model, where $a_t \sim N(0,1)$(a white noise), and $\sigma_t^2 > 0$, so $\alpha_0 > 0$, $\alpha_i \geq 0$, $\delta_j \geq 0$, $\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\delta_j < 1$ must be satisfied.

2.3. Model-setting and diagnosis

The method of estimating and predicting the volatility model using the log yield data is generally as follows.

[Procedure 1] Set up an autoregressive model for the error terms.
[Procedure 2] Perform additional autocorrelation tests on residuals.
[Procedure 3] Understand the volatility of variance of error term and set up the model.
[Procedure 4] Diagnose the model and forecast.

In procedures 1 and 2, the partial autocorrelation function (PACF) or the Portmanteau Q-statistic is used to test the autocorrelation of the error term. In procedure 3, the presence of heteroscedasticity of the error term is determined using the Portmanteau Q-statistic or the Lagrange Multiplier Test (LM-statistic) [14]. If there is a conditional heteroscedasticity, the order of the ARCH model is set to q when $LM_{q+1} - LM_q$ is no longer significant by using the fact that $LM_{q+1} - LM_q$ approximates the $\chi_\alpha^2(1)$ distribution among the orders until the LM-statistic value is significant [15]. When the ARCH effect is significant and the order of the model is determined, the parameter is estimated by the maximum likelihood estimation method. Then the model is diagnosed to predict the volatility of financial assets using the Portmanteau Q-statistic to determine whether the autocorrelation exists in the residual obtained after model fitting.

3. Results

3.1. Volatility clustering

After converting the exchange rate and KOSDAQ index data to log yield data and reviewing the movement of second moment, Figures 1 and 2 show the volatility clustering tendency. This means that autocorrelation exists in the error terms.

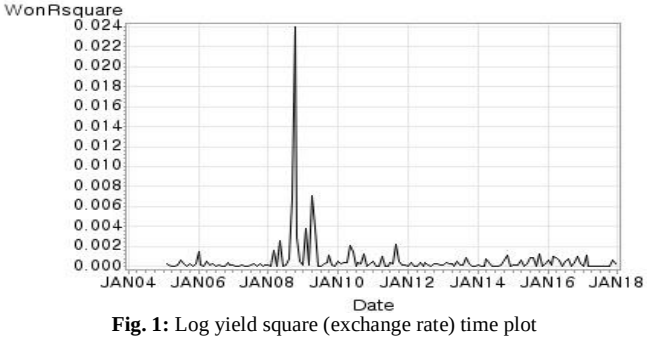


Fig. 1: Log yield square (exchange rate) time plot

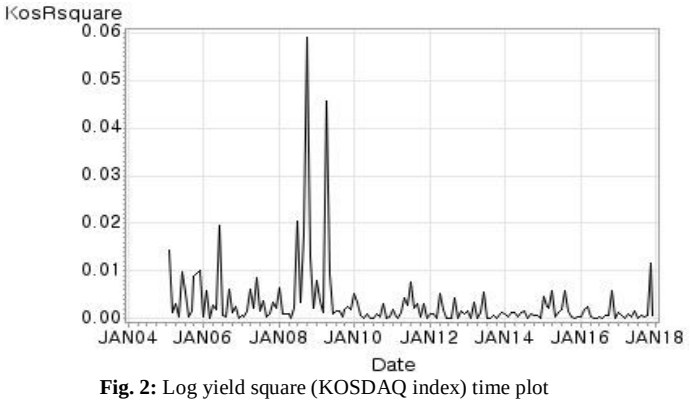


Fig. 2: Log yield square (KOSDAQ index) time plot

3.2. Autocorrelation and conditional heteroscedasticity test of the error term ε_t

Autocorrelation test using the Portmanteau Q-test of log yield (exchange rate, KOSDAQ index) was significant in all time lags. Partial autocorrelation test using PACF of the log yield of exchange rate was significant up to the time lag 13, and KOSDAQ index up to the time lag 5. In order to test whether there is additional autocorrelation in the residuals of the model with 13th and 5th order autocorrelation, we performed the Portmanteau Q-test using $\hat{\nu}_t$. As the result, there was no autocorrelation at the significance level $\alpha = 0.05$. As shown in Table 1.

Table 1: Autocorrelation test

Autocorrelation Check for White Noise							
Log yield (exchange rate)				Log yield (KOSDAQ index)			
Lag	ChiSq	DF	P>Chisq	Lag	ChiSq	DF	P>Chisq
6	2.57	6	0.8610	6	11.90	6	0.0641
12	10.35	12	0.5852	12	15.19	12	0.2314
18	12.20	18	0.8368	18	25.08	18	0.1227
24	20.48	24	0.6691	24	28.43	24	0.2424

Also, autocorrelation test using Portmanteau Q-test of $\widehat{\nu_t^2}$, square of the residuals obtained from the autoregressive model of the error terms, was found to be significant at all time lags 1 to 12, at the significance level $\alpha = 0.05$. This indicated that there was the ARCH effect. Therefore, ARCH(q) model of log yield (exchange rate) using the LM-statistic was set to the ARCH(1) model because LM2-LM1=0.3933 < $\chi_{0.05}^2(1) = 3.841$ as shown in Table 2,

and ARCH(q) model of log yield (KOSDAQ index) was set to the ARCH(1) model because LM2-LM1=0.02 < $\chi^2_{0.05}(1) = 3.841$.

Table 2: Conditional heteroscedasticity test

	Log yield (exchange rate)		Log yield (KOSDAQ index)	
Order	LM	Pr > LM	LM	Pr > LM
1	15.9352	<.0001	10.3972	0.0013
2	16.3285	0.0003	10.4172	0.0055
3	16.3305	0.0010	16.2328	0.0010
4	18.3353	0.0011	16.7854	0.0021
5	18.5944	0.0023	16.8095	0.0049
6	24.9429	0.0003	33.5643	<.0001
7	25.7799	0.0006	33.6883	<.0001
8	27.2992	0.0006	33.8090	<.0001
9	27.3108	0.0012	34.1245	<.0001
10	28.1250	0.0017	34.6819	0.0001
11	28.1661	0.0031	34.6964	0.0003
12	28.2345	0.0051	38.8424	0.0001

Also, the AR(1,2,3,13) model was estimated by the stepwise auto-regressive method except for the insignificant parameters in log yield (exchange rate) following the AR(13) model, and the AR(1) model was estimated in log yield (KOSDAQ index) following the AR(5) model.

3.3. Model fit and diagnosis

The results of estimating the parameters that fit the AR(1,2,3,13)-ARCH(1) model for log yield (exchange rate) and the AR(1)-ARCH(1) model for log yield (KOSDAQ index) were all significant at the various significance levels as shown in Table 3.

Table 3: Model fit

Parameter Estimates					
Log yield (exchange rate)					
Variable	DF	Estimate	S.E	T Value	Pr > |t|
AR1	1	-0.4634	0.1117	-4.15	<.0001
AR2	1	0.3787	0.0697	5.43	<.0001
AR3	1	-0.1930	0.0583	-3.31	0.0009
AR13	1	0.2119	0.0495	4.28	<.0001
ARCH0	1	0.000301	0.0000441	6.83	<.0001
ARCH1	1	0.2860	0.0833	3.43	0.0006
Log yield (KOSDAQ index)					
AR1	1	-0.3058	0.0909	-3.36	0.0008
ARCH0	1	0.002440	0.000260	9.37	<.0001
ARCH1	1	0.2067	0.0440	0.85	0.0428

As a result of the autocorrelation test after model fitting, it was judged that there was no more autocorrelation at the significance level $\alpha = 0.05$ as shown in Table 4.

Table 4: Autocorrelation after fitting model

Autocorrelation Check for White Noise							
Log yield (exchange rate)				Log yield (KOSDAQ index)			
Lag	ChiSq	DF	P>Chisq	Lag	ChiSq	DF	P>Chisq
6	2.33	6	0.8865	6	10.60	6	0.1014
12	12.75	12	0.3877	12	14.25	12	0.2850
18	16.64	18	0.5418	18	25.29	18	0.1172
24	22.59	24	0.5442	24	28.74	24	0.2302

Therefore, the final estimated AR(1,2,3,13)-ARCH(1) model of the log yield (exchange rate) data is as shown in (Equation 4),

$$\hat{\varepsilon}_t = 0.4634\widehat{\varepsilon_{t-1}} - 0.3787\widehat{\varepsilon_{t-2}} + 0.1930\widehat{\varepsilon_{t-3}} - 0.2119\widehat{\varepsilon_{t-13}} + \hat{v}_t \quad (4)$$

$$\hat{v}_t = \hat{\sigma}_t a_t$$

$$\widehat{\sigma_t^2} = 0.000301 + 0.2860\widehat{v_t^2}$$

and the AR(1)-ARCH(1) model of the log yield (KOSDAQ index) data in (Equation 5).

$$\hat{\varepsilon}_t = 0.3058\widehat{\varepsilon_{t-1}} + \hat{v}_t \quad (5)$$

$$\hat{v}_t = \hat{\sigma}_t a_t$$

$$\widehat{\sigma_t^2} = 0.002440 + 0.2067\widehat{v_t^2}$$

And the 95% confidence interval and the predicted values using the estimated models are shown in Figures 3 and 4. In each figure, the solid line (blue) shows the 95% confidence interval and the dotted line (red) shows the predicted values.

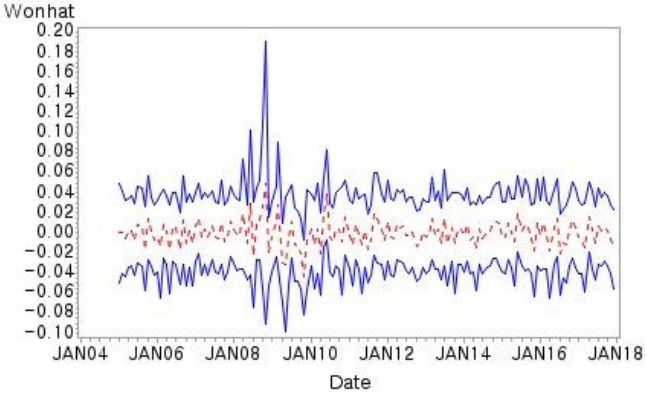


Fig. 3: 95% Confidence interval and forecast of log yield (exchange rate)

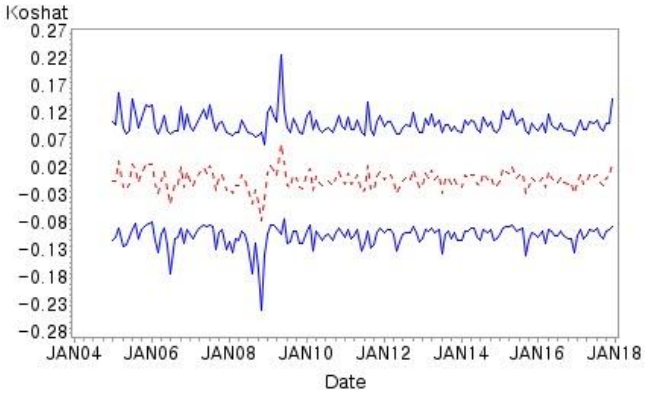


Fig. 4: 95% Confidence interval and forecast of log yield (KOSDAQ index)

4. Conclusion

The global financial market is experiencing high volatility due to the European fiscal crisis and the US economic slowdown. The volatility of the Korean financial market is also much higher than the usual financial crisis. The importance of estimating volatility for financial assets is increasing in terms of the minimization of uncertainty and pricing and risk management of financial assets. In this study, we estimated and forecasted the volatility of KRW/USD exchange rate from the macroeconomic point of view and KOSDAQ index from the microeconomic point of view by converting them into the log yields respectively. The models that analyze the characteristics of economic and financial time series data and predict the conditional volatility are various depending on the number and condition of parameters. In this study, both the KRW/USD exchange rate and the KOSDAQ index volatility estimation models were estimated to be based on the p-order auto-regressive model, and the variance of error terms on the ARCH(q) model. Then it was confirmed that there was no autocorrelation and heteroscedasticity after testing the models. Therefore, the AR(p)-ARCH(q) model, which is the estimation model this study, is expected to be used as a basic research data for analyzing the volatility of financial assets and the ripple effects of financial market policies. And, since the rate of return is a concept of variance and is known to have a linear relationship with risk, we leave studies about ARCH-M model, which is a capital assets pricing model that maintains a linear relationship between risk and return, GARCH-M model, and VECM model, which is a model of the

relationship between KRW/USD exchange rate and KOSDAQ index as future research tasks.

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