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Research paper

Aligned MHD Magnetic Nanofluid Flow Past a Static Wedge

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Abstract

The problem of steady aligned MHD magnetic nanofluid flow past a static wedge is studied in this paper. The present aligned magnetic field along with constant temperature at the surface is considered. The governing partial differential equations, subject to boundary conditions are transformed into ordinary differential equations using similarity transformations. The transformed equations are then solved numerically by Keller-box method. To check the validity of the present method, numerical results for dimensionless local skin friction coefficient and rate of heat transfer are compared with results of available literature as special cases and revealed in good agreement. The influence of pertinent parameters on velocity, temperature profiles, as well as wall shear stress and heat transfer rate is displayed in graphical form and discussed. It is found that fluid velocity increases with the increase of inclined angle, magnetic parameter and thermal buoyancy parameters while decreasing for increasing in nanoparticle volume fraction. It is also noticed that magnetic parameter influences fluid velocity and temperature significantly.

Keywords: *Aligned MHD; Free convection; Magnetic Nanofluid; Static Wedge.*

1. Introduction

Conventional heat transfer fluids such as water, ethylene glycol mixtures and oil have poor thermal conductivity and thus are considered poor heat transfer fluids. These fluids find application as a cooling tool for improving manufacturing and operating costs. Many researchers have tried multiple times by suspending nano/micro particles in liquids to improve the thermal conductivities of these fluids [1, 2]. Nanofluids are formulated through ultrafine nanoparticles (< 100 nm) that are suspended in a base fluid, which can be an organic solvent or water [3]. Compared to conventional fluids, nanofluids exhibit higher conductive properties and minimum clogging, boiling as well as convective heat transfer performances [4, 5]. By integrating nanofluid with biotechnological components, nanotechnology can be applied to many potential applications covering a broad range of practical applications, including pharmaceuticals, agriculture and as biological sensors. In biotechnological applications, nanomaterials can be potentially used for a growing list of nanoparticles, nanofibres, nanostructures, nanowires and nanomachines [6]. Magnetic nanofluids constitute a special class of nanofluids that exhibit both magnetic and fluid properties It respond to applied magnetic fields and allow further manipulation of heat transfer and hydrodynamic characteristics.
Research studies in MHD boundary layer flow are helpful in many crucial engineering applications like cooling of reactors, power generators, spinning of filaments and in the polymer industry. In industrial applications, stretching occurs when filaments or sheets are exposed to cooling. However, magnetic field can be employed to control this cooling which can help in producing final products with desired shapes. Due to such significant applications, the boundary layer flow of a conducting incompressible viscous fluid was considered by [7] because of deforming elastic surface under uniformly applied magnetic field. The MHD boundary layer flow of a viscous fluid was studied by [8] beyond a stretching surface and found the same external magnetic field effect on the flow as that of viscoelasticity. The exact solutions to the boundary layer flow were obtained by [9] along a vertical plate under applied magnetic field. In this, analytical expressions were obtained in a series form for Sakiadis and Blasius-Sakiadis flows.
The fundamental solutions for wedge problems were obtained through pioneering works of [10-12], which also helped to extend the wedge problem in multiple directions. They provided solutions for governing equations related to laminar flow for conventional fluids over permeable wedges with suction, which included variable and isothermal wall temperature distributions, and determined the skin friction and heat transfer for laminar flow. In [13] studied the effect of heat generation or absorption and viscous dissipation on a viscous fluid's force convection flow in a moving wedge subject to injection/suction. They found that the presence of heat source led to higher temperature. A similarity solution method was presented by [14] to find out accurate solutions for laminar forced convection heat transfer from both uniform-flux and isothermal wedges to fluids with any Prandtl number. They proposed a simple correlation equation that could be applied for any Prandtl number or any wedge. The effects of heat generation on force convection flow were numerically studied by [15] for a wedge placed static in nanofluid. They concluded that higher temperature would result in increase in values of absorption parameter or heat generation. Recently, MHD Casson nanofluid flow was examined by [16] over a wedge with Newtonian heating. They employed the Keller-box method to solve the problem numerically and found higher skin friction resulting from Newtonian fluid when compared with Casson fluid.
The applications mentioned above in the literature and discussion were the main sources for motivation to study the steady aligned MHD magnetic nanofluid flow over a static/moving wedge. The similarity transformation was employed to convert non-linear partial differential equations into non-linear ordinary differential equations. An implicit finite difference scheme known as the Kel-

ler-box method was employed to solve the highly nonlinear transformed equations numerically [17]. A good agreement could be established when comparing few special cases with the results available in the literature.

2. Mathematical Formulation

Consider a steady, aligned magnetohydrodynamic Falkner-Skan flow in two-dimension over a moving or static wedge for water-based nanofluid with magnetic nanoparticle (Fe_3O_4). Table 1 lists the fluid's thermophysical properties and nanoparticle. The wedge's total angle is $\Omega=\lambda\pi, \lambda=\frac{2m}{m+1}$ which refers to the wedge angle parameter. The velocities at the free stream and wedge are $u_e(x)=U_\infty x^m$ and $u_w=U_w x^m$ respectively. As presented in Figure 1, to the direction of the flow direction, a variable magnetic field $B(x)=B_0 x^{\frac{m-1}{2}}$ is applied. The generated magnetic field's direction is considered normal to the surface as well as uniform. Also, the electric field because of polarisation of charges is considered negligible and the magnetic Reynolds number is assumed to be small. In addition, it is assumed that all thermo-physical properties of the fluid remain constant with the exception of density variation in the body force term as well as the thermal conductivity.

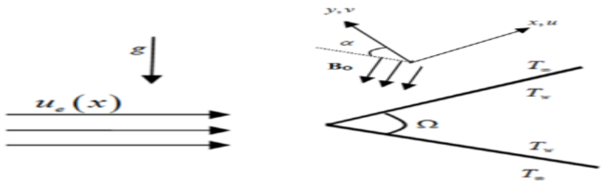

Fig. 1: Physical flow model and coordinate system.

Table 1: Thermophysical properties of the base fluid and the nanoparticle [19-20]

Physical Properties	Fluid Phase (Water)	Fe_3O_4
ρ (J/kgK)	997.1	5,200
c_p (kg/m^3)	4179	670
k (W/mk)	0.613	6
$\beta\times10^{-5}$ (K^{-1})	21	1.3
Pr	6.2	-

Based on these assumptions and the normal Boussinesq approximation, the governing boundary layer equations concerning the conservation of mass, momentum and energy can be given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\nu_{nf}\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B^2(x)}{\rho_{nf}}\sin^2\alpha\left(u_e-u\right)+\frac{(\rho\beta)_{nf}}{\rho_{nf}}g\left(T-T_\infty\right)\sin\frac{\Omega}{2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The physical boundary conditions for a problem can be represented as:

$$\begin{aligned} &u=u_w(x),\ v=0,\ T=T_w, && \text{at } y=0,\\ &u=u_e(x),\ T\to T_\infty, && \text{as } y\to\infty, \end{aligned} \tag{4}$$

where u and v signify velocity components alongside the x and y axes respectively, T represents the nanofluid temperature in the boundary layer, T_∞ signifies the ambient fluid's temperature outside the boundary layer, σ denotes the magnetic permeability, ν_{nf} and ρ_{nf} signify the kinematic and density viscosity of the nanofluid, nnf and α_{nf} represent the thermal viscosity and diffusivity respectively which can be defined as follows [18]

$$\begin{aligned} &\nu_{nf}=\frac{\mu_{nf}}{\rho_{nf}},\ \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\\ &\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\\ &(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s,\\ &(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s,\\ &\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}},\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+2\phi(k_f-k_s)} \end{aligned} \tag{5}$$

where ϕ represents the nanofluid's solid volume friction, μ_f signifies the basic fluid's viscosity, ρ_s and ρ_f represent the densities of the nanoparticle and pure fluid respectively $(\rho c_p)_s$ and $(\rho c_p)_f$ denote the specific heat parameters of the nanoparticle and basic fluid respectively $(\rho\beta)_s$ and $(\rho\beta)_f$ signify the volumetric expansion coefficient of the nanoparticle and basic fluid respectively and k_s and k_f are the thermal conductivities of the nanoparticle and basic fluid respectively. The following similarity variables are introduced:

$$\psi=\sqrt{\frac{2\nu_f x u_e}{m+1}}f(\eta),\ \eta=\sqrt{\frac{(m+1)u_e}{2x\nu_f}}y,\ \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

The relations $u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}$ define the stream function ψ to identically satisfy in (1), where ν_f represents the fluid's kinematic viscosity. The following non-linear ordinary differential equations are achieved through substituting (5) and (6) into (2) and (3):

$$\begin{aligned} &f'''+(1-\phi)^{2.5}\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left[ff'+\lambda\left(1-(f')^2\right)\right]\\ &+(1-\phi)^{2.5}M\sin^2\alpha(1-f')\\ &+(1-\phi)^{2.5}\left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda_T\theta\sin\frac{\Omega}{2}=0 \end{aligned} \tag{7}$$

$$\frac{k_{nf}}{k_f}\theta''+\Pr\left((1-\varphi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)f\theta'=0 \tag{8}$$

Under boundary conditions in (4):

$$\begin{aligned} &f(\eta)=0,\ f'(\eta)=\gamma,\ \theta(\eta)=1 \text{ at } \eta=0\\ &f'(\eta)=1,\ \theta(\eta)=0 \text{ at } \eta\to\infty \end{aligned} \tag{9}$$

where the prime symbol signifies differentiation with respect to η. Here, Prandtl number, $\Pr=\nu_f/\alpha_f$, Reynold's number is $\mathrm{Re}_x=xu_e/\nu_f$, magnetic parameter is $M=2\sigma B_o^2/\rho U_\infty(m+1)$, thermal buoyancy parameter is $\lambda_T=Gr_x/\mathrm{Re}_x^2$, Grashof number is $Gr_x=2g\beta(T-T_\infty)x^3/\nu_f^2(m+1)$ and moving wedge parameter is $\gamma=U_w/U_e$.

For this flow and heat transfer case, the important physical parameters include local Nusselt number and local skin-friction coefficient defined as follows

$$c_f = \frac{\tau_w}{\rho_f \left(u_e(x)\right)^2}, \quad Nu_x = \frac{x q_w}{k_f \left(T_w - T_\infty\right)} \tag{10}$$

where

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{11}$$

are given by shear stress and surface heat flux respectively. Employing the new similarity variables in (6) and (7) provides:

$$\begin{aligned} c_f\left(\frac{2\mathrm{Re}_x}{m+1}\right)^{\frac{1}{2}} &= \frac{1}{(1-\phi)^{2.5}} f''(0), \\ Nu_x\left(\frac{(m+1)\mathrm{Re}_x}{2}\right)^{-\frac{1}{2}} &= -\frac{k_{nf}}{k_f}\theta'(0). \end{aligned} \tag{12}$$

3. Numerical Method

The Keller-box method was used to numerically solve in (7) and (8) subject to boundary conditions in (9) as described in the work of [17]. The following four steps are involved in obtaining the solution.

(i) Reduction in (7) and (8) to first-order system.
(ii) Employ central differences to write the difference equations.
(iii) Use Newton's method to linearize the resulting algebraic equations and note it in the matrix-vector form.
(iv) Employ the block tridiagonal elimination technique to solve the linear system.

4. Results and Discussion

An implicit finite difference scheme called the Keller box method was employed to numerically compute the non-linear system of in (7) and (8). This study uses the step size of $\Delta\eta = 0.01$. Moreover, to obtain numerical results for the given boundary value problem, the MATLAB software was employed to develop an algorithm. To examine the behaviour of temperature and velocity profiles for the physical problem, numerical calculations were done for various values of parameters, including magnetic parameter M, aligned magnetic field α, thermal buoyancy parameter λ_T and nanoparticle volume fraction ϕ. For numerical computations, some of the non-dimensional values remain fixed such as $\alpha = 90°, M = 2, \phi = 0.05, \lambda_T = 1.5, \lambda = 0.2, \Omega = 60°$. In the entire analysis, these values are treated to be common, excluding the different displayed values in specific tables and figures. The case λ_T □ 1.0 relates to pure free convection, λ_T □ 1.0 relates to pure forced convection and $\lambda_T = 1.0$ relates to mixed convection.

Table 2: Comparison of $f''(0)$ values for various m values when $\mathrm{Pr} = 6.2, M = 0, \lambda_T = 0, \phi = 0$(pure fluid).

m	[21]	[22]	[23]	Present
0	0.4696	0.4696	0.4696	0.4696
1/11	0.6550	0.6550	0.6550	0.6550
1/5	0.8021	0.8021	0.8021	0.8021
1/3	0.9276	0.9276	0.9277	0.9277
1/2	-	-	1.0389	1.0389
1		1.2326	1.2326	1.2326

Table 2 provides a list of values' comparison for $f''(0)$ with various values of m when $\lambda = 0$ as reported by [21-23]. As observed from these tables, the local skin friction coefficient was seen to increase with increase in value of m. From this table, the results were found to be in a very good agreement. Therefore, it can be said that the present numerical results are very accurate.

Figure 2 depicts the impact of inclined angle on velocity and temperature profiles. It is clear from the figures that a rise in the aligned angle of magnetic direction improves the velocity profiles and reduce the temperature profiles. The rising values of the aligned angle of magnetic direction cause a rise in the magnetic field strength in the flow area.

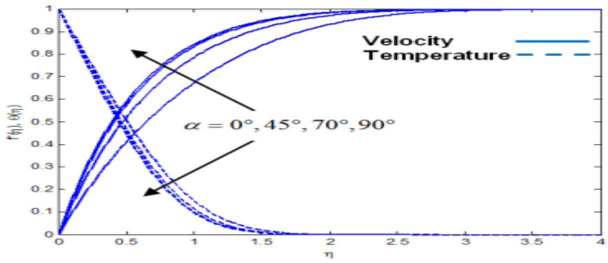

Fig. 2: Effect of aligned magnetic field parameters on the velocity and temperature profiles.

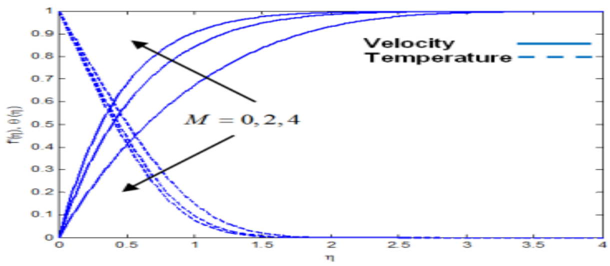

Fig. 3: Effect of magnetic strength parameters on the velocity and temperature profiles.

Figure 3 represent the impact of magnetic field parameter on temperature and velocity field. It is clear that the momentum boundary layer thickness decreases with the rising values of magnetic field parameter and also a decline occurs in the thermal boundary layer. Should the suppression of Lorentz force be less, deprecation in temperature occurs and results in the development of velocity field with progressive values for magnetic field parameters.

Figure 4 shows the effect of thermal buoyancy parameter on velocity and temperature. As λ_T increases, velocity also increases and momentum boundary layer decreases. The temperature profiles and thermal boundary layer decreases with increases in λ_T.

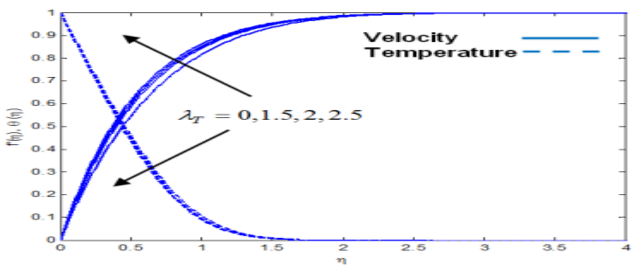

Fig. 4: Effect of thermal buoyancy on the velocity and temperature profiles.

Figure 5 shows the dimensionless velocity and temperature profiles. The friction force within the fluid can be raised by increasing nanoparticle volume fraction parameter. As a result, there is a decrease in velocity field. Increasing the volume fraction parameter results in temperature profile intensification. The thermal conductivity of the nanofluid can be improved by strengthening the nanoparticle's volume fraction due to thickening of thermal boundary layer. A key parameter is the volume fraction that plays a crucial role in enhancing fluids' heat characteristics. In many industrial processes, the nanoparticle volume fraction is modified to control the temperature.

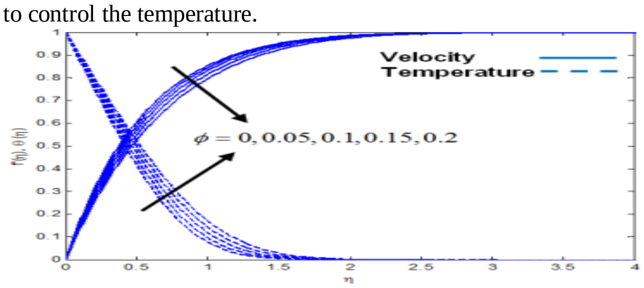

Fig. 5: Effect of volume fraction of nanoparticles on the velocity and temperature profiles.

Table 3 presents the variation occurring in skin friction coefficient and Nusselt number for different α, M, ϕ and λ_T values applicable to various wedge movements. It could be clearly seen that all parameters, $\alpha, M, \phi, \lambda_T$ had an boosting effect on the friction factor coefficients. The highest value could be seen for the case when M was modified. An enhancement was seen in the heat transfer rates with improving values of α, M, ϕ and λ_T. The highest value could be seen for the case when ϕ was varied.

Table 3: Variation in skin friction and Nusselt number coefficient for different values of $\alpha, M, \phi, \lambda_T$.

α	M	ϕ	λ_T	Skin Friction	Nusselt Number
0^0	2	0.05	1.5	1.143526	1.129088
45^0				1.592587	1.230751
70^0				1.858815	1.279309
90^0				1.932249	1.291548
90^0	0	0.05	1.5	1.143526	1.129088
	2			1.932249	1.291548
	4			2.467423	1.369058
90^0	2	0	1.5	1.791521	1.223356
		0.05		1.932249	1.291548
		0.1		2.090146	1.359083
		0.15		2.268591	1.426010
		0.2		2.471866	1.492338
90^0	2	0.05	0	1.699821	1.254008
			1.5	1.932249	1.291548
			2.0	2.007934	1.303332
			2.5	2.082806	1.314791

5. Conclusion

Some of the interesting findings include:

- Increase of α, M and λ_T resulted in rise in fluid flow.
- Increase of ϕ led to decrease in fluid velocity.
- Increase of ϕ resulted in increase in the dimensionless temperature.
- Increase of α, M and λ_T led to reduction in dimensionless temperature.
- Increase of α, M, ϕ and λ_T resulted in increase in the local skin friction coefficient.
- Increase of α, M, ϕ and λ_T led to improved rate of heat transfer.

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