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Research paper



Prime Labeling of Jahangir Graphs

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Abstract

The paper investigates prime labeling of Jahangir graph $J_{n,m}$ for $n \geq 2$, $m \geq 3$ provided that nm is even. We discuss prime labeling of some graph operations *viz*. Fusion, Switching and Duplication to prove that the Fusion of two vertices v_1 and v_k where k is odd in a Jahangir graph $J_{n,m}$ results to prime graph provided that the product nm is even and is relatively prime to k. The Fusion of two vertices v_{nm+1} and v_k for any k in $J_{n,m}$ is prime. The switching of v_k in the cycle C_{nm} of the Jahangir graph $J_{n,m}$ is a prime graph provided that $nm+1$ is a prime number and the switching of v_{nm+1} in $J_{n,m}$ is also a prime graph .Duplicating of v_k, where k is odd integer and $nm + 2$ is relatively prime to k,$k+2$ in $J_{n,m}$ is a prime graph.

Keywords: *Prime labeling; Jahangir graph; Fusion; Switching and duplication.*

1. Introduction

The paper considers only finite simple undirected graph throughout. Prime labeling of a graph G is a bijection f: $V(G) \rightarrow \{1, 2, \ldots |V|\}$ such that gcd $(f(u),f(v))=1$ for each edge uv . A graph is called prime graph if it admits a prime labeling. The graph G has vertex set $V=V(G)$ and the edge set $E=E(G)$. The set of vertices adjacent to a vertex u of G is denoted as $N(u)$. For notation and terminology reference to Bondy and Murthy [1] has been made .
Prime labeling is a concept that has been introduced by Roger Entringer .Since then many researchers have studied prime labeling for different types graphs. The cycle C_n on n vertices is a prime graph was proved by Dertsky [2]. Later Fu [3] considered path P_n on n vertices to show that such graphs are prime graphs. Roger surmised during the period 1980s that all trees possess prime labeling, and what he surmised could not be confirmed as a fact till now. Sundaram [8] is one of the exponents who studied the prime labeling for planner grid. Further investigations included the development of prime labelings by authors such as Ganesan and Balamurugan [4] who developed prime labellings for Theta graphs and Meena and Vaithilingam [6] for graphs related to Helm. In addition, Prime Labeling for several fan related graphs [7] have been proved by them. As far as cycle related graphs are concerned it was Vaidhya and K.K. Kanmani [9] who proved their prime labeling. Lee [5] has been attributed with establishing the fact that wheel Wn is a prime graph iff n is even.

Definition 1.1.
Under specific conditions, when the vertices of the graphs have been demarcated with values then such phenomenon is termed as (vertex) graph labeling.

Definition 1.2.
Suppose $G = (V(G), E(G))$ is a graph possessing n vertices. A bijection $f: V(G) \rightarrow \{1, 2, \ldots n\}$ is termed as Prime labeling, when $e = uv$, hcf(f (u) ,f (v)) = 1 for each edge. When prime labeling occurs a graph is considered as prime graph.

Definition 1.3.
In a graph G, the vertices which are an independent set are a set of interdependent vertices that are nonadjacent.

Definition 1.4.
Consider u and v are two separate vertices meant for a graph G. G_1 is a novel graph that has been designed by fusing (identifying) the two vertices u and v by a sole vertex x in G_1 in such a way that each edge that has been incident with either u (or) v in G is at present incident with x in G_1.

Definition 1.5.
A vertex switching G_v of graphs G has been procured by considering a vertex v of G, deleting the total edges which are incident with v and accumulating edges combining v to each vertex that are not neighboring to$v$ in G.

Definition 1.6.
Duplication of a vertex v of a graph G produces a new graph G' by adding a vertex v′ with N (v)= N (v′).
To define the other way a vertex v′ is told to have been in duplication of v under condition that all the vertices that are beside v are at present neighbored to v′ In G′ .

Definition 1.7.
Jahangir graph $J_{n,m}$ for $n \geq 2$, $m \geq 3$ is a graph with on nm + 1 vertices comprising a certain cycle C_{nm} possessing single vertex that is additional and is beside m vertices of C_{nm} placed at a distance n between the C_{nm}. Jahangir graph $J_{2,8}$ that is visible on his tomb which is located in his mausoleum.

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2. Main Results of Prime Labeling on Jahangir Graph

Theorem 2. 1.
If *nm* is even then the Jahangir graph $J_{n,m}$ *for* $n \geq 2$, $m \geq 3$ is a prime graph.

Proof:
Let $J_{n,m}$ be a Jahangir graph. $V(J_{n,m})=\{v_1, v_2, \ldots, v_{nm+1}\}$ and $E(J_{n,m}) = \{v_i\ v_{i+1} / 1\leq i \leq nm\text{-}1\} \cup v_{nm}v_1 \cup \{v_{1+jn}v_{nm+1} / 0\leq j\leq m\text{-}1\}$ then $|V(J_{n,m})| = nm+1$ and $|E(J_{n.m})| = (n+1)m$. Here the set $\{v_i\ v_{i+1} / 1\leq i \leq nm\text{-}1\} \cup v_{nm}v_1$ represent as edges of the cycle and the set $v_{1+jn}v_{nm+1} / 0\leq j\leq m\text{-}1\}$ represent the set of edges adjacent to the vertex v_{nm+1}.
The vertex labeling of $J_{n,m}$ is $f:V(J_{n,m})\rightarrow\{1,2,\ldots, nm+1\}$ such that $f(v_i) = i+1$ for $1\leq i\leq nm$ and $f(v_{nm+1}) = 1$. It is to be noted that with $nm+1$ vertices and $nm+1$ labelings f is bijection. As '1' is relatively prime to each natural number and any two successive natural numbers are relatively prime. Therefore, for each edge $e = uv \in E(J_{n,m})$ and gcd $(f(u), f(v)) = 1$.Hence, $J_{n,m}$ proves to have undergone prime labeling. Therefore, $J_{n,m}$ is a prime graph.

Illustration 2.2.
The following graphs 1&2 indicates the Prime labeling of $J_{2,3}$ and $J_{3,4}$.

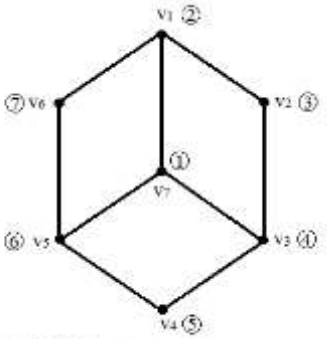


Figure 1. Jahangir graph $J_{2,3}$

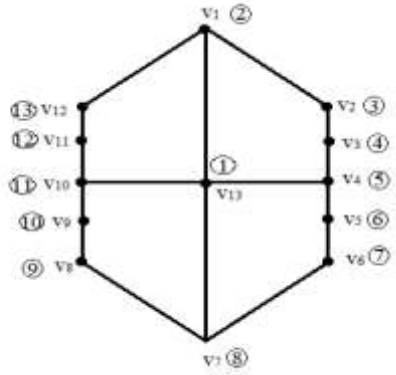


Figure 2. Jahangir graph $J_{3,4}$

Programme 2.3.
Pseudo code for the prime labeling of $J_{n,m}$ is written in 'C' programme

```
#include<stdio.h>
#include<conio.h>
int f(int);
intgcd(int,int);
intn,m;
void main()
{
inth,g,v[1000],e[1000],f1[1000],flag=1,i,j,flag1=1;;
clrscr();
printf("Enter n,m:");
scanf("%d%d",&n,&m);
for(i=1;i<=n*m;i++)
{
f1[i]=f(i);
printf("%d\n",f1[i]);
 }
for(i=1;i<n*m;i++)
 {
 g=gcd(f1[i],f1[i+1]);
printf("GCD=%d",g);
if(g!=1)
 {
flag=0;
break;
 }
 }
 h=gcd(f(n*m),f(1));
printf("h=%d",h);
for(j=0;j<=m-1;j++)
 {
 g=gcd(f(1+j*n),f(n*m+1));
printf("GCD1=%d",g);
if(g!=1)
 {
  flag1=0;
break;
 }
 }
printf("flag1=%d",flag1);
if(flag==1&&h==1&&flag1==1)
printf("\n Prime Graph");
else
printf("\n Not a Prime Graph");
getch();
}
int f(int i)
{
if(i==n*m+1)
return 1;
else
return i+1;
}
intgcd(inta,int b)
{
int r;
while(b!=0)
{
r=a%b;
a=b;
b=r;
}
return a;
}
```

Theorem 2.4.
When nm happens to be an odd number then $J_{n,m}$ will cease to be a prime graph.

Proof:
Note that the order of $J_{n,m}$ is $nm+1$. Hence, one has to use from 1 to $nm+1$ integers while labeling the vertices.In this way we have $\frac{nm+1}{2}$ odd integers. For the moment, one can allocate odd numbers to at most $\frac{nm+1}{2}$ (as nm is odd) vertices from among the said nm vertices in the cycle C_{nm}. Next one must assign a prime number to the center of the graph $J_{n,m}$ and each prime number is

odd. Therefore, one has to assign at most places $\frac{nm+3}{2}$ odd numbers to the vertices. However, because we are having $\frac{nm+1}{2}$ odd numbers with us, it is not possible. Finally, $J_{n,m}$ is not considered to be a prime graph for odd *nm*.

3. Main Results on Fusion of Vertices in the Jahangir Graph $J_{n,M}$

Theorem 3.1.
The Fusion of two vertices v_1 and v_k in a Jahangir graph $J_{n,m}$ $n\geq 2, m\geq 3$ such that nm is even and nm is relatively prime to k where k is an odd number is a prime graph**.**

Proof:
Suppose G is a graph resulting from fusion of two vertices v_1 and v_k where k is an odd number in the cycle of $J_{n,m}$ then |V(G)| = nm and |E (G) | = (n+1)m. The set of edges are the edges which are incident on v_1 and v_k are incident with the new vertex '$v_1 = v_k$' and the remaining are same.
We define the labeling f:V(G)→{1,2,3,…….nm} such that $f(v_1 = v_k) = k$ and $f(v_i) = i$ for all I and $f(v_{nm+1}) = 1$. As 'k' is an odd number so, the gcd(2,k)=1and k is relatively prime to nm and each pair of successive natural numbers are relatively prime. Therefore, hcf(f(u),f(v)) =1 for each edge e=uv∈E (G). Hence, G complies prime labeling. Therefore G is a prime graph.

Illustration 3.2.
The following graphs represent the fusion of $v_1$and v_3 in $J_{2,5}$ and v_1 and v_5 in $J_{3,4}$ respectively.

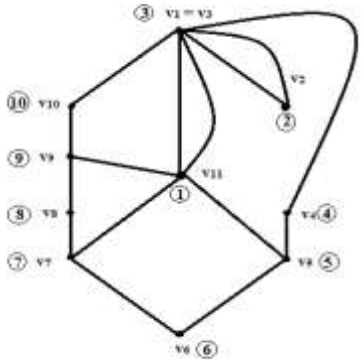


Figure 3. Fusion of v_1 & v_3 in $J_{2,5}$

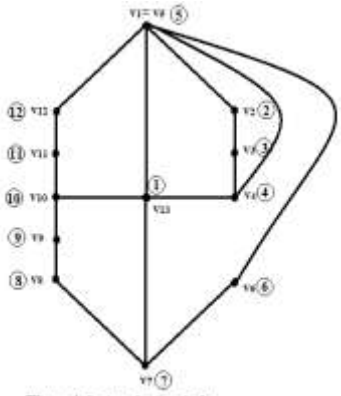


Figure 4. Fusion of v_1& v_5 in $J_{3,4}$

Remark 3.3.
The fusion of $v_1$and v_3 in $J_{2,3}$ is prime even though 6 is not relatively prime to 3.The labeling of fusion of $v_1$and v_3 in $J_{2,3}$ is shown in figure 5.

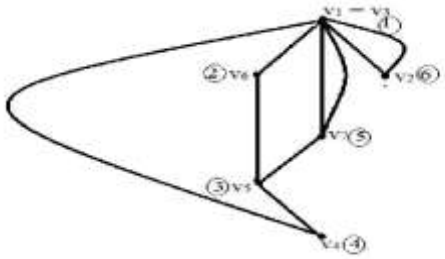


Figure 5. Fusion of v_1 and v_3 in $J_{2,3}$

Theorem 3.4.
The Fusion of two vertices v_{nm+1} and v_k for any k in a Jahangir graph $J_{n,m}$ for $n\geq 2$ and $m\geq 3$ such that nm is even is a prime graph**.**

Proof:
Suppose G is a graph resulting from fusion of two vertices v_{nm+1} and v_k in $J_{n,m}$ then |V(G)| = nm and |E (G) | = (n+1)m. The set of edges in G are the set of all edges which are in the cycle C_{nm} and the set of all edges which are adjacent to v_{nm+1}. Define the labeling f:V(G)→{1,2,3,…, nm} such that $f(v_{nm+1} = v_k) = 1$ and $f(v_{k-j}) = nm + 1 - j$ for $1 \leq j \leq k-1$; $f(v_{k+i}) = i + 1$for$1 \leq i \leq nm - k$. Note that f(u),f(v) are co-prime numbers for each edge e=uv∈E(G). Hence, G complies prime labeling. Therefore, G is a prime graph.

Illustration 3.5.
The fusion of v_{11} and v_2, fusion of v_{11} and v_4 in $J_{2,5}$ shown in the figures 6 and 7

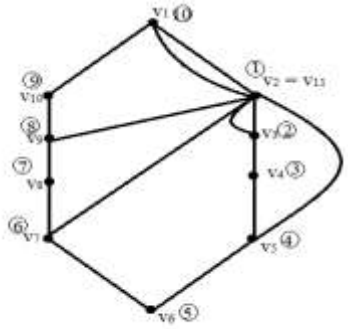


Figure 6. The fusion of v_{11} and v_2 in $J_{2,5}$

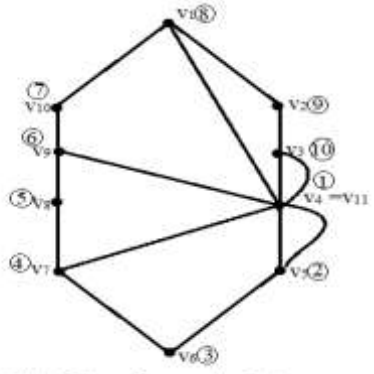


Figure 7. The fusion of v_{11} and v_4 in $J_{2,5}$

4. Main Results on Switching of Vertices in the Jahangir Graph $J_{n,M}$.

Theorem 4.1.
The switching of v_{nm+1}in the Jahangir graph $J_{n,m}$, *for* $n \geq 2$, $m \geq 3$ such that *nm* is even is a prime graph.

Proof:
Suppose G is a graph resulting from switching v_{nm+1}in $J_{n,m}$ then $|V(G)| = nm+1$ and $|E(G)| = (2n-1)m$. The set of edges in G are the set of edges in the cycle C_{nm} and the set of edges which are not adjacent to v_{nm+1}. We define the labeling $f:V(G)\rightarrow\{1,2,3,\ldots, nm+1\}$ such that $f(v_{nm+1})=1$ and $f(v_i) = i+1$ for $1\leq i\leq nm$. Note that $f(u),f(v)$ are co- prime numbers for each edge $e=uv\in E(G)$.Hence, G complies prime labeling. Therefore, G is a prime graph

Illustration 4.2.
Switching of v_{13}in $J_{3,4}$ is shown in the figure 8.

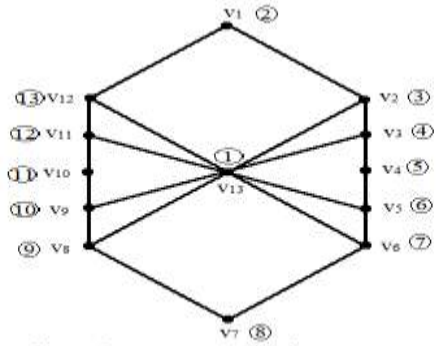


Figure 8. Switching of v13 in $J_{3,4}$

Theorem 4.3.
The switching of v_k $k\geq 1$ in the cycle C_{nm} of the Jahangir graph $J_{n,m}$ *for* $n\geq 2$, $m\geq 3$ such that $nm+1$ is a prime number, is a prime graph.

Proof:
Suppose G *is* a graph obtained by switching v_k in $J_{n,m}$ then $|V(G)| = nm+1$. The set of edges in G are the set of edges in the cycle C_{nm} which are not incident on v_k in $J_{n,m}$ are now incident on v_k and the rest of edges will remain same in $J_{n,m}$ .The required labeling $f:V(G)\rightarrow\{1,2,3,\ldots, nm+1\}$ such that $f(v_k)=nm+1, f(v_{nm+1}) = 1$ and $f(v_{k+i}) = i+1$ for $1\leq i\leq nm-k$ *and* $f(v_{k-i}) = nm+1-i$ for $1\leq i\leq k-1$. Note that $f(u),f(v)$ are co-prime numbers for each edge $e=uv\in E(G)$.Hence, G complies prime labeling. Therefore, G is a prime graph

Illustration 4.4.
The switching of v_1 and switching ofv_5 in $J_{3,4}$ are shown in the figures 9,10.

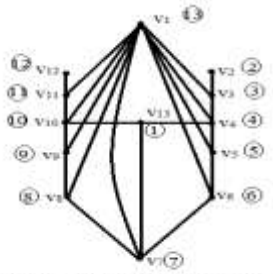


Figure 9. The switching of v_1 in $J_{3,4}$

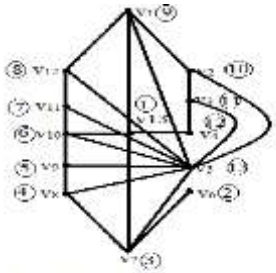


Figure 10. The switching of v_5 in $J_{3,4}$

5. Main Results on Duplication of a Vertex in the Jahangir Graph $J_{n,M}$.

Theorem 5.1.
The Duplication of v_k where k is odd integer and $nm+2$ is relatively prime to $k, k+2$ in the Jahangir graph $J_{n,m}$,for$n\geq 2, m\geq 3$ such that nm is even is a prime graph.

Proof:
Let G be a graph obtained by duplicating v_k by v_k' in $J_{n,m}$.
We define the labeling $f:V(G)\rightarrow\{1, 2, 3, \ldots, nm+2\}$ such that $f(v_{nm+1}) = 1$ and $f(v_i) = i+1$ for $1\leq i\leq nm$, $f(v_k') = nm+2$.As mentioned in the theorem 2.1, $hcf(f(u),f(v)) = 1$ for each edge $e = uv\in E(G)$.Hence,G complies prime labeling. Therefore, G is a prime graph.

Illustration 5.2.
Duplication of v_3 in $J_{3,4}$ is shown in the figure 11.

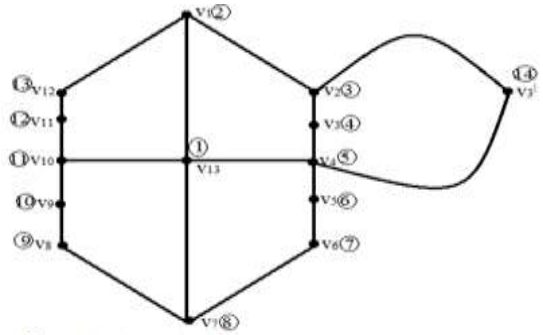


Figure 11. Duplication of v_3 in $J_{3,4}$

6. Conclusion

In this paper we checked the prime labeling of Jahangir graph $J_{n,m}$ by using 'C' Program for different n, m values which satisfy the condition nm is even. Similarly we can apply different languages for checking the labelings of different families of the graph.

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