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***Research paper***



# Numerical Investigation on MHD Marangoni Convective Flow of Nanofluid through a Porous Medium with Heat and Mass Transfer Characteristics

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**Abstract**

The present study investigates on a steady two-dimensional Marangoni convective flow of nanofluid through a porous medium with heat and mass transfer characteristics. The proposed mathematical model has a tendency to characterize the radiation and chemical reaction effects. The governing equations in the form of partial differential equations have been converted into ordinary differential equations through similarity transformations, which have been solved by using Runge-Kutta method via shooting technique. The characteristics of velocity, temperature and concentration boundary layers are studied for different physical parameters. The local Nusselt and Sherwood numbers are estimated and discussed for aforesaid physical parameters. It is to be noted that the Marangoni ratio parameter is improves the rate of heat transfer and decreases the mass transfer rate.

***Keywords***: *Marangoni ratio parameter, Radiation parameter, chemical reaction parameter, Thermophoresis and Brownian motion parameters.*

## 1. Introduction

The Gibbs-Marangoni effect is the mass transfer through an interface between two fluids due to a gradient of the surface tension. In the case of temperature dependence, this phenomenon may be known as thermo-capillary convection or Bénard-Marangoni convection. In fact Marangoni convection induced by the variations of surface tension gradients (i.e. temperature and concentration gradients) has important applications in crystal growth metals, nucleation vapor bubbles, spreading of thin films, semi-conductor processing, material science, the silicon wafers and soap films etc. Arafune and Hirata [1] established a rectangular double crucible system to discuss the velocity feature of surface tension driven flow caused by thermal Marangoni convection solutal Marangoni convection in In-Ga-Sb melt. They have practically proved and observed that the typical surface velocity of thermal Marangoni convection is 3-5 lesser than that of solutal Marangoni convection. This result can be studied by using Marangoni, Reynolds and Prandtl numbers. Cazabat et al. [2] discussed the dynamics of spreading of thin films driven by temperature gradients. He observed that the Marangoni film is formed by applying a thermal gradient along the direction of the flow and the temperature variation of the surface tension is fairly constant for many fluids far from the critical point, and therefore a constant temperature gradient creates a constant Marangoni surface stress. In addition, the surface tension gradient causes the interface current.

A nanofluid is a liquid which containing a dispersion of submicron of solid particles over a distinctive length scale of order 1 to 100 nm. Nanoparticles are made up of various materials like ceramics, oxide, and nitride ceramics. By the modern evaluation in engineering has led to high performance of advanced thermal systems. Moreover, the improvement can be achieved by using nanoliquids in such systems because the characteristic feature of nanofluids is thermal conductivity enhancement. This has wide range of applications such as solar water cooling, engine cooling, cooling of electronics, cooling of heat exchange, nuclear reactors and nuclear systems cooling. Buongiorno and Hu [3] bring a project on nuclear reactor to study on nanofluids and mechanism of enhanced heat transfer. They observed that nanofluids are used in water cooled nuclear systems.

Liquid to liquid interfaces can produce Marangoni boundary layer. The Characteristic of Marangoni convection usually emerges when the interface surface tension more generally liquid to liquid or liquid to air depends on the thermal distribution. Marangoni convection in fluid transport owing to their much industrial applications, such as welding, electron beam melting of metals and crystal growth. Due to the meaning, the heat and mass transfer process of boundary layer Marangoni flow has been comprehensively scrutinized several researchers. Mize and Trofimenko [4] examined the azimuthal wave number to enhance the Marangoni number and to suppress with rise in the density of the surfactant. Manjare et al. [5] experimentally studied the Marangoni flow effects. The thermal gradient with varying magnetic field and thermal conductivity on Marangoni boundary layer heat transfer characteristics are discussed by Lin et al [6]. Hayat et al [7] analyzed the aspects of viscous dissipation, Joule heating and inclined magnetic field on mixed convective Marangoni flow of non-Newtonian fluid. Aly and Ebaid [8] proposed the exact solutions for Marangoni convective flow on a porous surface using Laplace transform technique. Some important studies in this direction in-

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clude [9-12]. Recently, Sheikholeslami and Chamkha [13] discussed the Lorentz forces on nanofluid forced convection by considering Marangoni convection.

In view of these facts the present study focuses on the numerical investigation of a two-dimensional flow over the porous layers. The influence of radiation and chemical reaction effects on nanofluid with the existence of Marangoni convection is analyzed. The influence of the active controlling parameters is studied.

## 2. Mathematical Formulation:

The present analysis considers a steady two-dimensional incompressible electrically conducting Marangoni convective boundary layer flow of a nanofluid through a porous medium over a stretching sheet with slip effects. The influence of controlling parameters such as porosity parameter, radiation parameter, chemical reaction, Brownian motion parameter, thermophoresis parameter, Marangoni ratio parameter, Schmidt number and Prandtl number are studied on velocity, temperature and concentration distributions. The interface temperature is considered as a function in $x$. The governing equations for this investigation are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{B^2{}_0\delta_{el}}{\rho_f}u-\frac{\upsilon}{k'}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{(\rho c)_p}{(\rho c)_f}\left[D_B\left\{\frac{\partial C}{\partial y}.\frac{\partial T}{\partial y}\right\}+\left(\frac{D_T}{T_0}\right)\left\{\left(\frac{\partial T}{\partial y}\right)^2\right\}\right]-\frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_0}\right)\left\{\frac{\partial^2 T}{\partial y^2}\right\}-k_0(C-C_\infty) \tag{4}$$

$\sigma$ is the surface tension and it is a function of temperature and concentration

$$\sigma=\sigma_0\left[1-\gamma_c(C-C_\infty)-\gamma_T(T-T_\infty)\right] \tag{5}$$

$$\gamma_C=-\frac{1}{\sigma_0}\frac{\partial\sigma}{\partial C}\bigg|_T,\ \gamma_T=-\frac{1}{\sigma_0}\frac{\partial\sigma}{\partial T}\bigg|_C \tag{6}$$

The corresponding boundary conditions are as follows

$$u(x,\infty)=0, T(x,\infty)=T_\infty, C(x,\infty)=C_\infty,$$
$$C(x,0)=C_\infty+C_\infty X^2, X=\frac{x}{L}, v(x,0)=0,$$
$$\mu\frac{\partial u}{\partial y}\bigg|_{y=0}=-\frac{\partial\sigma}{\partial x}\bigg|=\sigma_0\left[\gamma_T\frac{\partial T}{\partial x}\bigg|_{y=0}+\gamma_C\frac{\partial C}{\partial x}\bigg|_{y=0}\right] \tag{7}$$

In order to transform the above equation in self-similar form, the following equations are defined

$$\begin{aligned}&C(x,y)=C_0X^2\phi(\eta)+C_\infty,\\&\mathrm{T}(x,y)=T_0X^2\theta(\eta)+T_\infty,\\&\varphi(x,y)=\upsilon Xf(\eta),\ \eta=y/L,\\&u=\partial\varphi/\partial y,\ \ v=-\partial\varphi/\partial x,\end{aligned} \tag{8}$$

By using equation (8) the dimensionless equations and conditions are given by:

$$f'''+ff''-f'^2-(M^2+k_p)f'=0 \tag{9}$$

$$[1-\frac{4}{3}\mathrm{R}]\theta''+P_r[f\theta'-2f'\theta]+\mathrm{N}_b\phi'\theta'+N_t\theta'^2=0 \tag{10}$$

$$\phi''+Sc\left[f\phi'-2f'\phi\right]+\frac{Nt}{Nb}\theta''-\gamma\phi=0 \tag{11}$$

$$\begin{aligned}&\theta(0)=1,\phi(0)=1,\ f(0)=0,\ f''(0)=-2(1+r),\\&f'(\infty)=0,\theta(\infty)=0,\phi(\infty)=0\end{aligned} \tag{12}$$

$$r=\frac{C_0\gamma_c}{T_0\gamma_T},$$
$$\mathrm{Pr}=\frac{\upsilon}{\alpha}, Nb=\frac{(\rho C_p)_p D_B C_0 X^2}{(\rho C_p)_f\alpha},$$
$$Sc=\frac{\upsilon}{D}, Nt=\frac{(\rho C_p)_p D_T X^2}{(\rho C_p)_f\alpha}, M=Ha^2,$$
$$Ha=B_0L\sqrt{\delta_{el}/\mu}$$

$L$ is a reference length and is defined as $L=-\frac{\mu\upsilon}{\sigma_0T_0\gamma_T}$ and

$r$ can be introduced as $r=Ma_C/Ma_T$, where

$$Ma_c=\sigma_0\gamma_cC_0L/(\alpha\mu),\ Ma_T=\sigma_0\gamma_TT_0L/(\alpha\mu)$$

## 3. Results and Discussion:

The set of ordinary differential equations (9)-(11) are solved numerically by applying Runge-Kutta fourth order with shooting technique alongside boundary conditions (12). The numerical values of the non-dimensional parameters such as porosity parameter, radiation parameter, chemical reaction, Brownian motion parameter, thermophoresis parameter, Marangoni ratio parameter, Schmidt number and Prandtl number on rate of heat and mass transfer rates have been demonstrated in Table 1.

Fig. 1 depicts that the porosity parameter on velocity distribution. It is obvious that velocity of the fluid increases significantly with increase in porosity parameter. Fig. 2 demonstrates the influence of radiation parameter on temperature profiles with in the boundary layer. It is due to fact that the fluid temperature enhances the

conduction effect of the nanofluid in the presence of radiation. Therefore, higher values of radiation parameter imply higher surface heat flux. It is found that the thermal boundary layer thickness increases with increase in radiation parameter. The influence of first order chemical reaction parameter on concentration distribution is depicted in Fig. 3. It is very interesting to notice that the solutal boundary thickness decreases.

The effect of Brownian motion parameter on concentration distribution is plotted in Fig. 4. The solutal boundary layer thickness decelerates in the liquid film with increasing values of Brownian motion parameter, which results in enhancement in the fluid temperature. Figures 5-6 are sketched to show the influence of thermophoresis parameter on temperature and concentration distributions appear on thermal and solutal boundary layer equations, that is, it is coupled with temperature function and plays strong role in determining the diffusion of heat and nanoparticles. As increase in thermophoresis parameter enhances the thermal and concentration distributions. From Figs. 7 and 8, we observed that the Marangoni ratio parameter enhances the fluid velocity, while decelerates the concentration of the fluid.

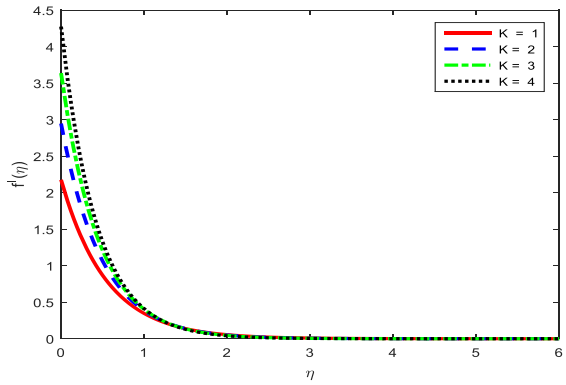



**Fig. 1**: Effect of porosity parameter on velocity profiles

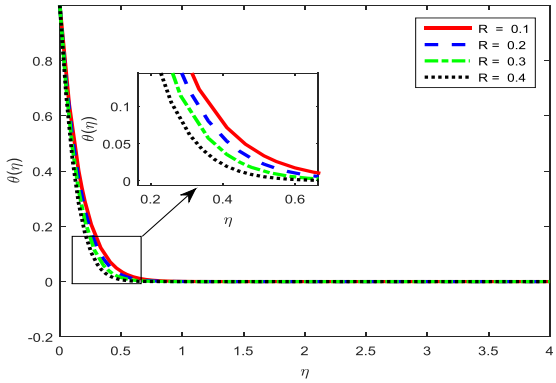



**Fig. 2:** Effect of Radiation parameter on temperature profiles

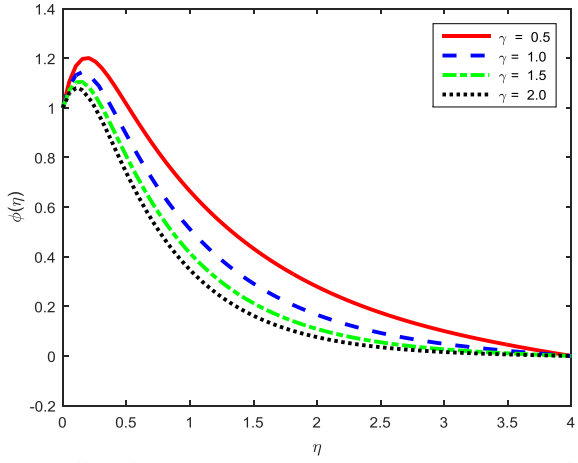



**Fig. 3:** Effect of chemical reaction parameter on concentration profiles

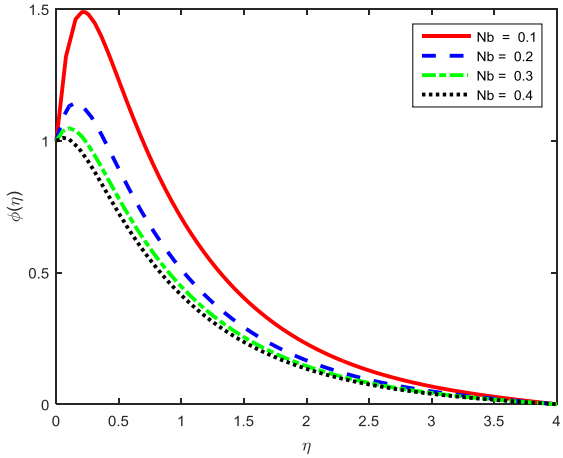



**Fig. 4:** Effect of Brownian motion parameter on concentration profiles

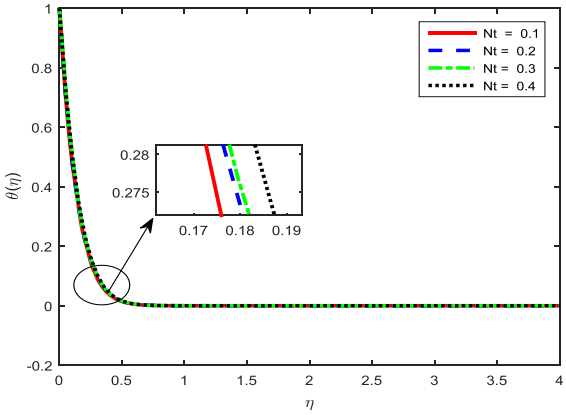



**Fig.5**: Effect of thermophoresis parameter on temperature profiles

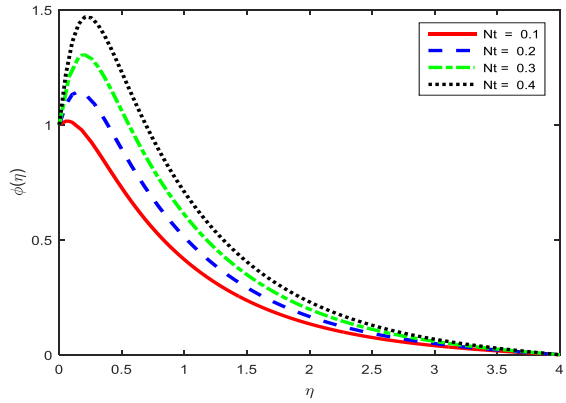



**Fig.6:** Effect of thermophoresis parameter on concentration profiles

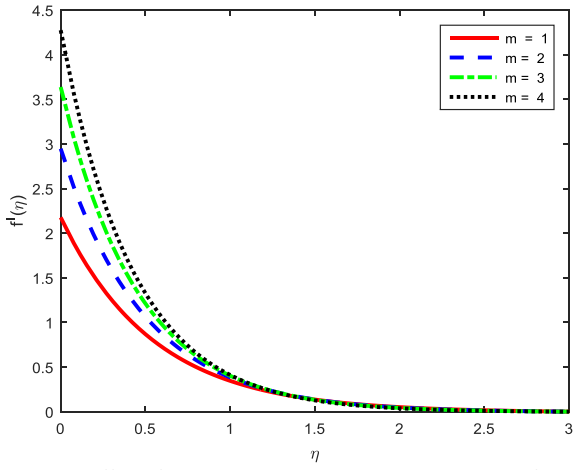



**Fig. 7**: Effect of Marangoni ratio parameter on velocity profiles

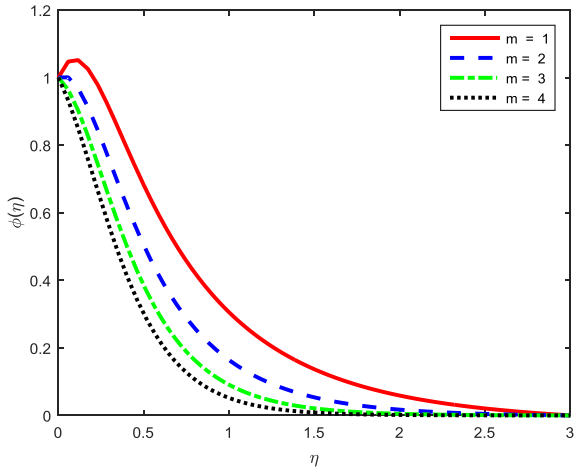



**Fig. 8**: Effect of Marangoni ratio parameter on concentration profiles

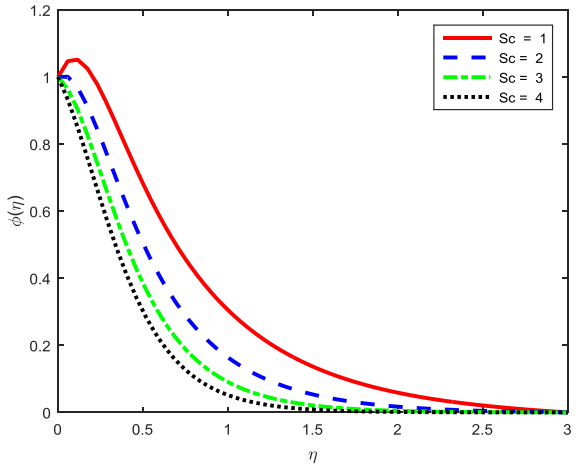



**Fig. 9**: Effect of Schmidt number parameter on concentration profiles

The influence of Schmidt number on concentration distribution is sketched in Fig.9. It is obvious that Schmidt number exponentially decrease the solutal boundary layer thickness. Figure 10 illustrates the effect of Prandtl number on temperature field. The temperature and thermal boundary layer decrease with the increase in Prandtl number meaning an increase of fluid viscosity, causes a decrease in the fluid temperature. Practically, it is quite obvious that increasing values reduce the thermal diffusivity, which therefore thins the thermal boundary layer. Further, the disturbance in the temperature profiles is quite prominent for smaller values of Prandtl number when compared with larger values. Hence Prandtl number can be used to enhance the rate of cooling.

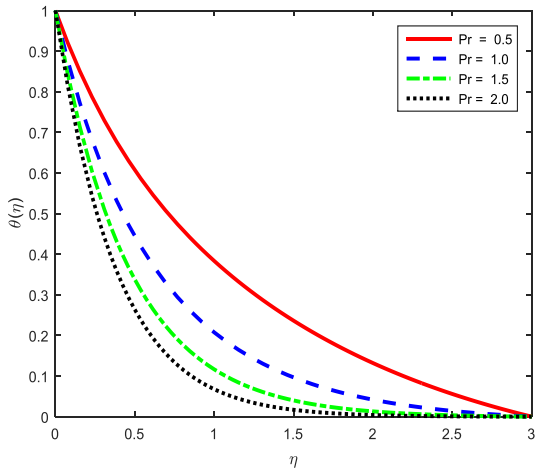



**Fig. 10:** Effect of Prandtl number parameter on temperature profiles

**Table:** he numerical values of local Nusselt number and local Sherwood number for different physical pertinent parameters.

| $R$ | $\gamma$ | $Nb$ | $Nt$ | $m$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|---|---|---|
| 0.1 | | | | | 5.648076 | -2.852768 |
| 0.2 | | | | | 6.079021 | -2.533604 |
| 0.3 | | | | | 6.612028 | -2.163925 |
| | 0.5 | | | | 6.667115 | -2.569145 |
| | 1.0 | | | | 6.612028 | -2.163925 |
| | 1.5 | | | | 6.570228 | -1.846082 |
| | | 0.1 | | | 6.702661 | -5.637277 |
| | | 0.2 | | | 6.612028 | -2.163925 |
| | | 0.3 | | | 6.522439 | -1.006564 |
| | | | 0.1 | | 6.815593 | -0.540617 |
| | | | 0.2 | | 6.612028 | -2.163925 |
| | | | 0.3 | | 6.419126 | -3.676587 |
| | | | | 1.0 | 8.416853 | -3.090015 |
| | | | | 2.0 | 9.845964 | -3.820205 |
| | | | | 3.0 | 10.973592 | -4.392509 |

The numerical values of local Nusselt number and local Sherwood number for various values of physical parameters are displayed in Table 1. It is observed that the radiation parameter enhances the rate of heat and mass transfer rates, while opposite phenomena can be obtained for thermophoresis parameter. The chemical reaction parameter decelerates the rate of heat transfer and improves the mass transfer rate as well as same phenomena can be observed for Brownian motion parameter at the wall. Marangoni ratio parameter accelerates the heat transfer rate and suppresses the mass transfer rate.

## 4. Concluding Remarks

The present computational results characterize the effect of magnetohydrodynamic Marangoni convective nanofluid over a porous medium with heat and mass transfer characteristics by using fourth order Runge-Kutta method via shooting technique. Based on the present computational investigation the following observations are made:

- The Marangoni ratio parameter decreases the concentration of the fluid.
- The thermophoresis parameter enhances the thermal and concentration boundary layer thickness.
- The radiation parameter accelerates the heat and mass transfer rates.
- The chemical reaction parameter suppresses the heat transfer rates and improves the mass transfer rate.

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