



# Identification, development and testing of thermal error compensation model for a headstock assembly of CNC turning centre

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## Abstract

In CNC machine tools, transient temperature variation in the headstock assembly is the major contributors for spindle thermal error. The compensation of thermal error is critical for ensuring the accuracy of machine tool. The performance of an error compensation system depends largely on the accuracy and robustness of the thermal error model. In the present work, a robust thermal error model is developed for minimizing the error in lateral direction of the spindle which significantly influences the geometrical accuracy of the workpiece. Analysis-of-variance (ANOVA) is applied to the results of the experiments in determining the percentage contribution of each individual temperature key point against a stated level of confidence. Based on the analysis of existing approaches for thermal error modeling of machine tools, an approach of LASSO (least absolute shrinkage and selection operator) is proposed in order to avoid the multi collinearity problem. The proposed method is an innovative variable selection method to remove redundant or unimportant temperature key points in the linear thermal error model and minimize the residual sum of squares. The predictive error model is found to have better robustness and accuracy in comparison to the combination of grey correlation and step wise linear regression for error compensation of CNC lathe.

**Keywords:** Analysis Of Variance (ANOVA), CNC Machine Tool, Grey Correlation Analysis (GCA), Headstock Assembly, LASSO Regression, Mean Absolute Deviation (MAD), Mean Square Error (MSE), Robustness, Standard Deviation (SD), Thermal Error.

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## 1. Introduction

The machine tool accuracy directly affects the dimensional accuracy of the machined products. The errors that affect the machine tool accuracy can be classified as geometric errors, thermal errors and cutting force-induced errors. Thermal error is caused by the accumulation of the deformation of machine tool elements, which stems from non-uniform temperature change in machine tool structure. It causes relative displacement between the work piece and the cutting tool during the machining process [1]. Thermal errors of a machine tool arise from six main sources [2], namely, heat produced in the cutting process, change of room temperature, energy lost in the mechanical system, the cooling system, operation of machine tool, and thermal memory effect. The minimization of the thermal errors of machine tools is concentrated on the three aspects [3, 4]; reduction in the heat sources, design of a thermally robust structure and compensation of thermal deformation. Among these solutions, the reduction in heat sources is not possible beyond a certain limit as friction between parts in motion would certainly generate some heat. The design of thermally robust structure has a limit to the accuracy that could be achieved. Errors like thermal and cutting force deformation cannot be completely accounted for in design. It is usually time consuming and costly and it often ends in over design of machine structure. The use of alternative materials for machine tool applications is popular amongst machine tool builders, but these methods are still incapable of catering to changes that take place in the shop floor environment on a day to day basis. Compensation after thermal deformation gains success these days both on account of its implementation as well as its cost-effectiveness [1].

The main difficulty is the error identification or thermal error modeling, since the temperature fields of a machine tool change constantly according to the working conditions, the use of coolant and the environmental conditions, etc [2]. In general, predictive thermal error model correlates the thermal error at Tool Centre Point (TCP) to the salient temperature keypoints of the machine tool. In recent years, multi linear regression analysis (MLRA), artificial neural network (ANN) theory and fuzzy Logic Theory (FLT) [7], [8] have been applied to develop the thermal error model. However, the ANN and the FLT approaches have the following disadvantages [3]:

- They are sensitive to seasonal variation.
- The thermal error models differ notably and cannot be substituted for each other.
- They require a great deal of experimental data and correspondingly a long experiment calibration time.

Even though there are a lot of methods for developing the compensation model, MLRA is widely used due to its simpler structure and better extrapolation compared to ANN and has the advantages of good predictive accuracy and robustness, less computational time, associated usage and management convenience in industry. But the problem associated with those methods is multi-collinearity when there is large collection of co-variants. In the present study, ANOVA process is carried out to determine the contribution of each temperature key point on overall thermal error. The regularized LASSO regression technique is proposed to identify the more significant temperature key points and also to develop the error model instead of adopting step-wise regression.

## 2. Methodology

The methodology for the present work is shown in Fig.1. The temperature points were selected by accounting for all the heat sources in the headstock assembly which include motor for driving the spindle and bearings which support the spindle.

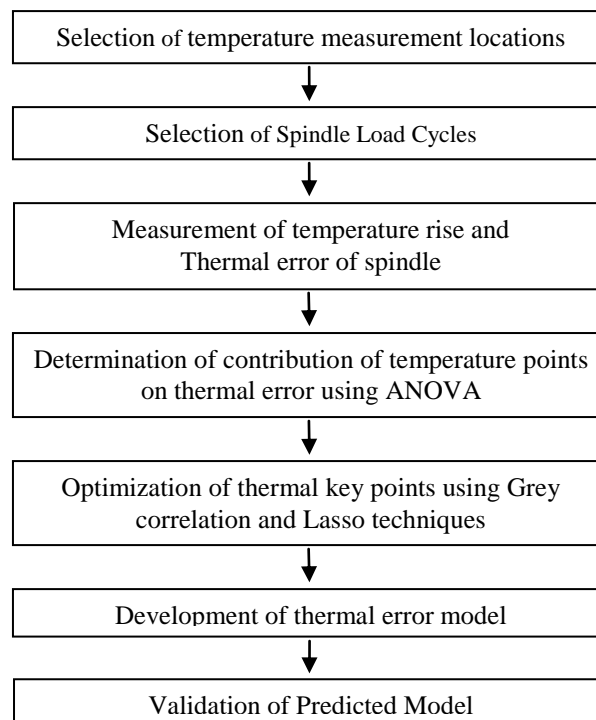


Fig. 1: Modeling Methodology

The measured temperature data are grouped according to their regression correlation coefficients, because temperature variables with high correlation coefficients represent thermal dependency on each other. The grouping results and measured thermal error data are then used to select the representative variable for each group. This step is carried out by applying a series of linear regressions. Optimal temperature group is used for thermal error modeling and to analyse the contribution of those temperature variables on overall thermal error.

## 3. Experimentation

### 3.1 Measurement of temperature and thermal error

The sensor points for temperature measurement were selected by accounting for the key heat sources in the 2-axis slant bed CNC turning center which have influence on the spindle deformation [7]. The locations for temperature

measurement include the spindle bearings, chucking cylinder and motor. The temperature sensors (PT100) were attached to the structure by heat flow paste and are insulated from the environment by foam. The details of the temperature points and their positions are shown in Fig 2.

$T_1$  = Chucking Cylinder,  $T_2$  = Hydraulic power pack of the Chucking Cylinder,  $T_3$  = Spindle Motor,  
 $T_4$  = Spindle rear bearing,  $T_5$  = Spindle front bearing,  $T_6$  = Lubricant cover of the spindle,  $T_7$  = Headstock Temperature,  
 $T_8$  = Bed underneath the spindle,  $T_9$  = Oil box machine side,  $T_{10}$  = Coolant input close to the spindle,  $T_{11}$  = Bed close to the transformer,  $T_{12}$  = Bracket of the transformer,  $T_{13}$  = Bearing of the spindle motor.

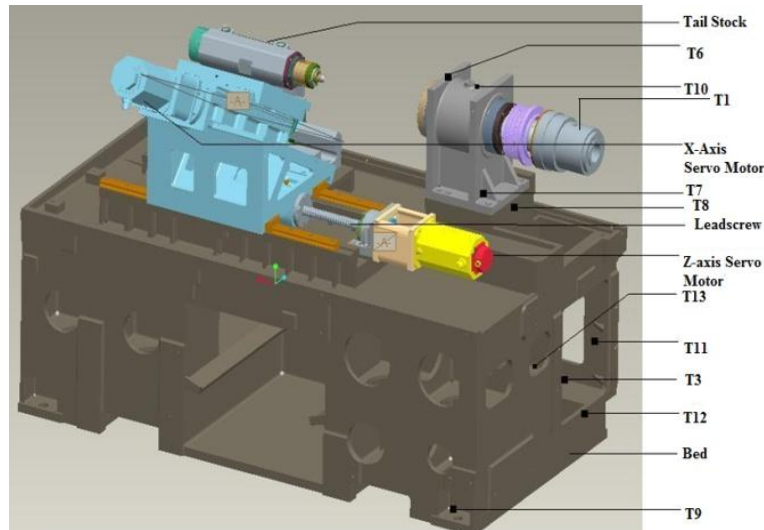


Fig. 2: Locations of temperature sensors

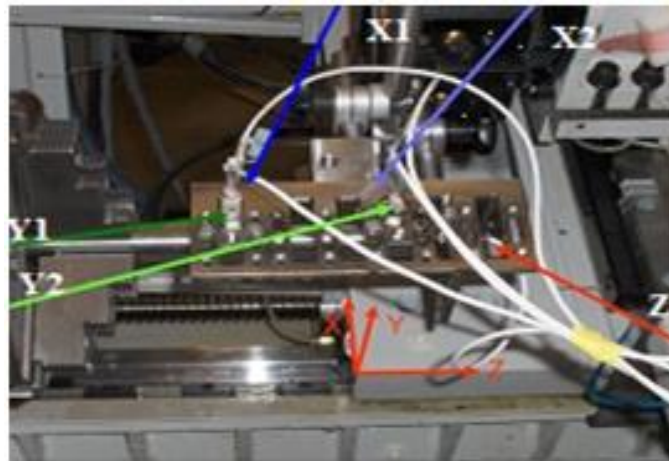


Fig. 3: Displacement sensors mounted in the fixture

In order to facilitate the measurement of spindle deformation, an invar rod was mounted on the spindle and the eddy current displacement sensors were mounted in a fixture connected to the turret as shown in Fig 3. The temperature and spindle thermal deformation were measured and recorded at a sampling interval of 5 min. Among the components of displacement, x-component of thermal error (Fig. 3) due to the spindle tilt in the lateral direction assumes greater significance in turning centre and hence it is considered in the present investigation [8].

### 3.2. Spindle load cycle

In order to investigate the thermal behavior of the CNC turning centre, two typical load cycles as shown in Figures 6 and 7 have been formulated based on the operations performed on the machine tool. The selected load cycles are of fluctuating type where spindle speed varies between 0 and 2800 rpm.

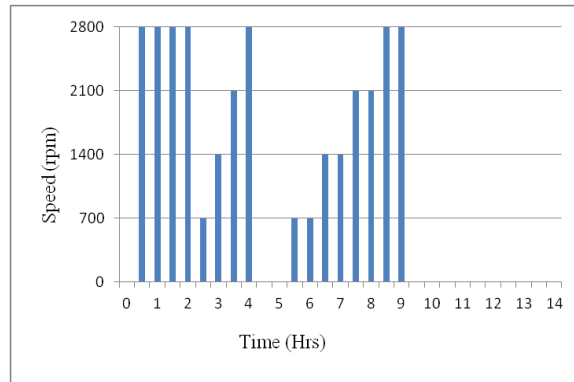


Fig. 4: Load Cycle-I

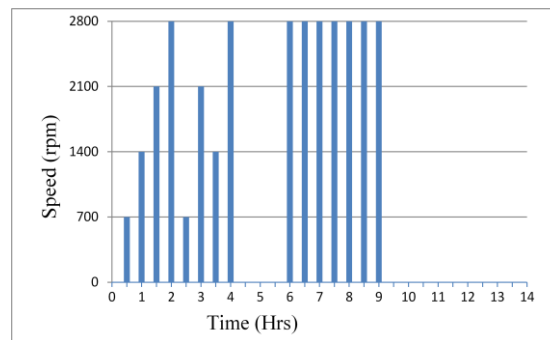


Fig. 5: Load Cycle-II

The experimental results based on Load cycle-I (Fig 4) have been used for the development of thermal error model while data obtained for load cycle-II (Fig 5) is used for validating the model. The transient variation of temperature and the resulting x-component of thermal error corresponding to load cycle-I are depicted in Figures 6, 7 and 8.

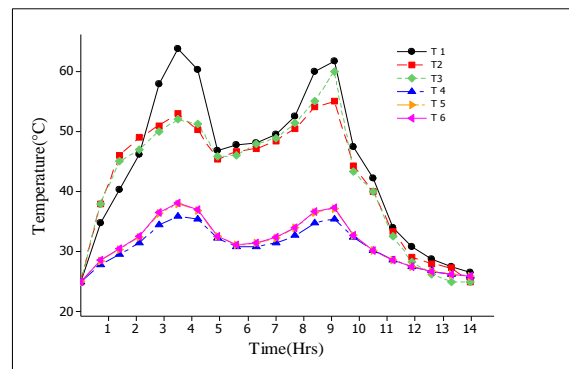


Fig. 6: Temperature Variation ( $T_1$  to  $T_6$ )

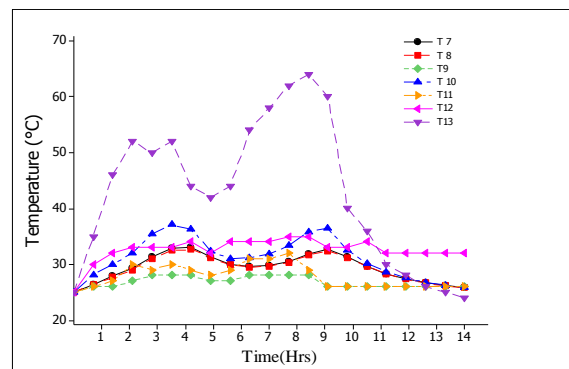


Fig. 7: Temperature Variation ( $T_7$  to  $T_{13}$ )

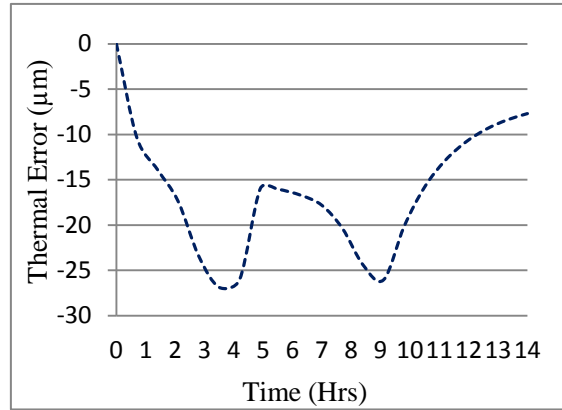


Fig. 8: Transient Variation of Thermal Error

The thermal sensor attached to the chucking cylinder records a maximum temperature of 64°C while front spindle bearing records 37°C. It is found that the temperature rise directly depends on the rotational speed of the spindle. When the spindle is suddenly stopped for a period from 4<sup>th</sup> hour to 5<sup>th</sup> hour and from 9<sup>th</sup> hour to 14<sup>th</sup> hour, the temperature falls significantly showing the sensitivity of the machine tool for internal heat generation. It is observed from the above figures that the pattern of variation of temperature as well as the thermal error is in agreement with that of the load cycle. The experimental data obtained from turning centre has been used to develop the thermal error model.

### 4. Thermal error modeling

Mathematically, a multi-linear regression model of thermal error describes the relationship between the temperature and thermal error from the identification dataset is represented as

$$\Delta = T\beta + \beta_0 \tag{1}$$

In matrix form it can be written as

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_i \end{bmatrix} = \begin{bmatrix} f_1(T_1) & \cdot & \cdot & f_j(T_1) \\ \vdots & \vdots & \vdots & \vdots \\ f_1(T_n) & \cdot & \cdot & f_j(T_n) \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_j \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_i \end{bmatrix} \tag{2}$$

Where,  $\Delta = [\Delta_1 \ \Delta_2 \ \cdot \ \Delta_i]^T$  represents the thermal error vector; T represents temperature variations vector;  $\beta$  is the regression coefficients vector; n is the number of temperature sensors and  $\beta_0$  is the scalar noise term.

#### 4.1. Analysis of variance (anova)

The knowledge of contribution of individual temperature parameters is a key in deciding the nature of control to be established on a machining process. The analysis of variance (ANOVA) is the statistical treatment most commonly applied to the results of the experiments in determining the percentage contribution of each individual temperature key point against a stated level of confidence. A general ANOVA table is shown in below.

Table 1: General ANOVA Table (for k groups, total sample size N)

Source	SS	D.F.	MSS	F
Between groups	SSG	k - 1	$\frac{SSG}{k - 1} = MSG$	$\frac{MSG}{MSE} = F$
Errors	SSE	(N-1) - (k-1)	$\frac{SSE}{N - k} = MSE$	
Total	SST	N - 1		

Where

- SS = Sum of squares.
- SST = total sum of squares.
- SSG = group sum of squares.
- SSE = sum of squares due to the errors.
- D. F. = degrees of freedom.
- MSS = mean sum of squares.
- MSE = mean square error.
- MSG = group mean squares.

F = test statistic, is the ratio of the mean sum of squares due to the differences between the group means and that due to the error.

Study of ANOVA process for a given analysis helps to determine which of the temperature key points need control. In general, Square Sum (SS) or F-value will be considered to rank the parameters which are highly influencing the response variable during the analysis of variance process [13]. If the temperature key point is highly influencing the process response, then the SS and F-value are large and they are used to rank the temperature key points. The SS and F-values obtained for thermal error model are given in Table 2.

**Table 2:** ANOVA for Thermal error

Source	Sum of Squares	d.f.	Mean Square	F	Contribution (%)
T1	0.5077	1	0.5077	0.2523	5.895
T2	4.77E-05	1	4.77e-05	2.37e-05	0.001
T3	0.2662	1	0.2662	0.1323	3.092
T4	2.3345	1	2.3345	1.1603	27.106
T5	1.9121	1	1.9121	0.9504	22.202
T6	0.8305	1	0.8305	0.4128	9.644
T7	1.0069	1	1.0069	0.5004	11.692
T8	0.8556	1	0.8556	0.4252	9.935
T9	0.0501	1	0.0501	0.0248	0.581
T10	0.7961	1	0.7961	0.3957	9.243
T11	0.0216	1	0.0216	0.0107	0.251
T12	0.0298	1	0.0298	0.0148	0.346
T13	0.0008	1	0.0008	0.0004	0.009
Error	14.083	7	2.0119	4.2806	-

From Table 2, the rear and front bearings ( $T_4$  &  $T_5$ ), head stock ( $T_7$ ), lubricant cover of the spindle ( $T_6$ ), bed underneath the spindle ( $T_8$ ), coolant input close to spindle ( $T_{10}$ ) have significant impact on overall spindle thermal error. The rear and front bearings are found to have the highest contribution of 27% and 22% on thermal error.

## 4.2. Optimization of temperature key points

The selection of the number and locations of thermal sensors plays a crucial role in thermal error compensation.

### 4.2.1. Grey correlation analysis

Grey correlation analysis is a principle theory of grey system theory, which can be applied in grey system analysis and random variables processing. The correlation between factors is represented by the similarity level of geometry which is called grey correlation degree, and the correlation degree between reference sequences and comparison sequences can be quantitatively estimated. Grey correlation degree describes the relative change between different factors in the process of system evolution, and the larger the correlation degree, the higher the similarity level [12]. Thus, the correlation degree can represent the impact of different sensors on the spindle thermal error behavior. By the calculation of improved correlation degree between thermal error and different sensors, the sensors with a relatively large correlation degree are selected since they have a larger impact on the thermal error. The steps involved in the estimation of grey correlation degree are given below.

1) Conversion of input data into dimensionless form using initialization method.

$$X_i(K) = \frac{X_i(K)}{X_i(1)} \quad (3)$$

$$\Delta_i(K) = |\delta_x(K) - X_i(K)| \quad (4)$$

$$\delta_x(K) = \text{Thermal Error}$$

$$X_i(K) = \text{Temperature data}$$

2) Grey Correlation coefficient between thermal error and sensors

$$\xi_i(K) = \frac{\text{Min}\Delta_i(K) + \rho \text{Max}\Delta_i(K)}{\Delta_i(K) + \rho \text{Max}\Delta_i(K)} \quad (5)$$

3) Mean of the correlation coefficient

$$r_i = \frac{1}{n} \sum_{k=1}^n \xi_i(K), K = 1, 2, \dots, n \quad (6)$$

4) Correlation degree with consideration of the diversity of correlation coefficients at different points

$$S(r_i) = \sqrt{\frac{1}{n} \sum_{k=1}^n (\xi_i(K) - r_i)^2} \quad (7)$$

5) Adjusted Grey Correlation Degree

$$\rho_i = \frac{r_i}{1 + S(r_i)} \quad (8)$$

**Table 3:** Results of Grey Correlation Analysis

Sensor	$\rho_i$	Relative Value of $\rho_i$	Rank
1	0.70286	0.712	7
2	0.69762	0.437	12
3	0.69871	0.492	10
4	0.70822	1.000	1
5	0.70799	0.988	2
6	0.70731	0.956	5
7	0.70791	0.984	3
8	0.7077	0.976	4
9	0.70123	0.625	8
10	0.70713	0.945	6
11	0.6895	0.000	13
12	0.69841	0.4841	11
13	0.70002	0.5625	9

The temperature key points are ranked according to the relative value of grey correlation degree. The temperature key points are selected by setting the threshold value as 0.97. From the results of grey correlation analysis as shown in Table 3, it is inferred that the four temperature key points, viz, T<sub>4</sub>, T<sub>5</sub>, T<sub>7</sub> and T<sub>8</sub> have the highest impact on the overall spindle thermal error.

**4.2.1. LASSO regression**

Lasso is a regularization technique for performing linear regression and is an alternative to stepwise regression. It includes a penalty term that constrains the size of the estimated coefficients [14]. Nevertheless, a lasso estimator can have smaller mean squared error than an ordinary least-squares estimator when applied to new data. As the penalty term increases, lasso sets more coefficients to zero. This means that the lasso estimator is a smaller model, with fewer predictors [14]. The lasso technique solves this regularization problem [14] and for a given value of  $\lambda$ , a nonnegative parameter, lasso solves the problem,

$$\text{Min} \left( \frac{1}{2N} \sum_{i=1}^N (\Delta_i - \beta_0 - T_i^1 \beta)^2 + \lambda \sum_{j=1}^b |\beta_j| \right)$$

Where

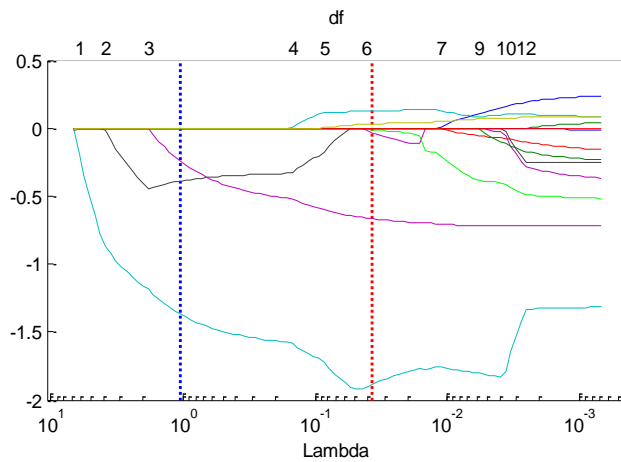
N is the number of observations.

$\Delta_i$  is the response (Thermal Error) at observation i.

T<sub>i</sub> is Temperature vector (input data) at observation i.

B represents the number of observations.

$\lambda$  is a positive regularization parameter corresponding to one value of Lambda. As  $\lambda$  increases, the number of nonzero components of  $\beta$  decreases.



**Fig. 9:** Trace Plot of Coefficients

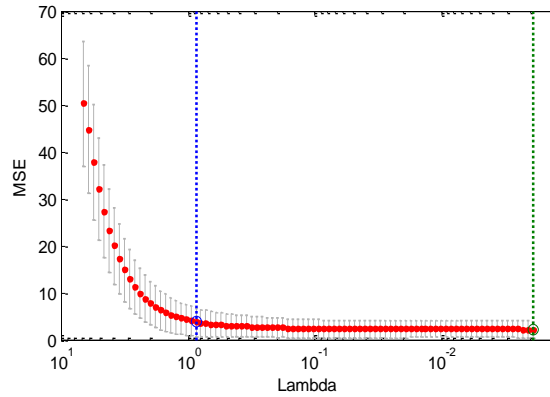


Fig. 10: Cross-Validated Mean Square Error

The figure 9 shows the nonzero coefficients in the regression for various values of regularization parameter (Lambda). Larger values of Lambda appear on the left side of the graph, meaning more regularization, resulting in fewer nonzero regression coefficients. The dashed vertical lines represent the Lambda value with minimal mean squared error (on the right), and the Lambda value with minimal mean squared error plus one standard deviation. These lines appear only during cross validation process. In the present LASSO regression, 10-fold cross validation is used. The upper part of the plot shows the degrees of freedom (df), meaning the number of nonzero coefficients in the regression, as a function of Lambda [14]. Fig 10 shows the cross validation MSE of LASSO fit, in which the circle at right side and dashed line locate the Lambda with minimal cross-validation error. The circle at left side and dashed line locate the point (at  $\lambda=10^0$ ) with minimal cross-validation error plus one standard deviation. From the figures 9 and 10, it is inferred that up to  $10^0$  the mean square error is constantly varying, after this limit the MSE is increases exponentially as lambda increases. Thus, at  $\lambda = 10^0$ , only three temperature key points ( $T_4, T_5$  and  $T_7$ ) have more regularization. It means that the impact of these key points on total thermal error is more as compared to remaining temperature key points called as redundant predictors.

Table 4: Standard deviation before and after optimization

	Technique used		No. of sensors	SD ( $\mu\text{m}$ )
	Removing redundant thermal sensors	Thermal Error Modeling		
Before Optimization	–	Ridge Regression	13	0.3741
After Optimization	Grey Correlation Analysis	Ridge Regression	4	0.5298
		Lasso Regression	3	0.4366

Based on Lasso results, it is found that the rear bearing ( $T_4$ ), front bearing ( $T_5$ ) and headstock ( $T_7$ ) temperatures are more significant temperature locations in machine tool. This paper has the same results as that of grey correlation analysis used to optimize the distribution of temperature field of machine tools. Furthermore, it reduces one more temperature variable in the thermal error model and hence significantly reduces the time for variable searching and modeling.

The thermal error model based on rear bearing ( $T_4$ ), front bearing ( $T_5$ ) and headstock ( $T_7$ ) temperatures is given below:

$$\Delta_x = 44.9 - 1.367 T_4 - 0.2412 T_5 - 0.3973 T_7 \tag{9}$$

The comparison among measured, predicted and residual thermal error for the experimental results based on load cycle-I is depicted in figure 11.

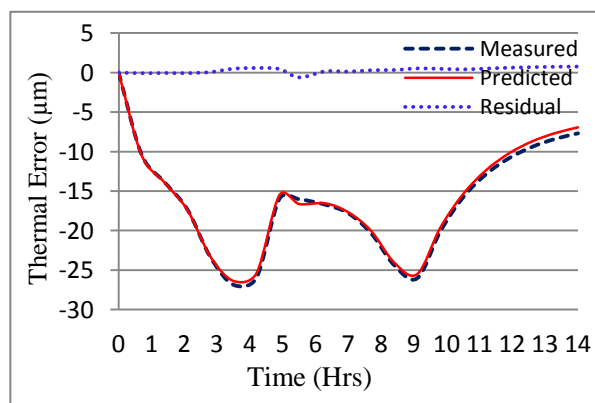


Fig. 11: Comparison Of Measured, Predicted And Residual Thermal Error



During the LASSO regression, thermal error is highly dependent on the rear bearing ( $T_4$ ), front bearing ( $T_5$ ) and headstock ( $T_7$ ) temperature key points. From comparison of ANOVA, Grey correlation analysis and LASSO results, it is established that the contribution of rear and front bearing temperature key points has more impact on total thermal error. It can be seen from Table 1 that rear and front bearings of the spindle have the highest contribution of spindle thermal error is about 27% and 22% respectively.

## 5. Testing of error model

In testing process, experimentation is carried out for the load cycle-II. In which only three temperature sensors (rear, front bearing and headstock) are pasted in respective locations for measuring transient temperature variation and thermal error model obtained from the results based on Load cycle-I is used to predict the thermal error for the data obtained by load cycle-II in order to determine the either the developed model is suitable for other experimental conditions or not. In robustness analysis, mean absolute deviation (MAD) and standard deviation (SD) are considered for performance measurement of developed model. A model is said to be better robustness if the MAD and SD of developed thermal error model are comparable, generally in the range of 0 to 1. The measured temperature and thermal error variation for the results obtained based on load cycle -II are shown in figures 12 and 13.

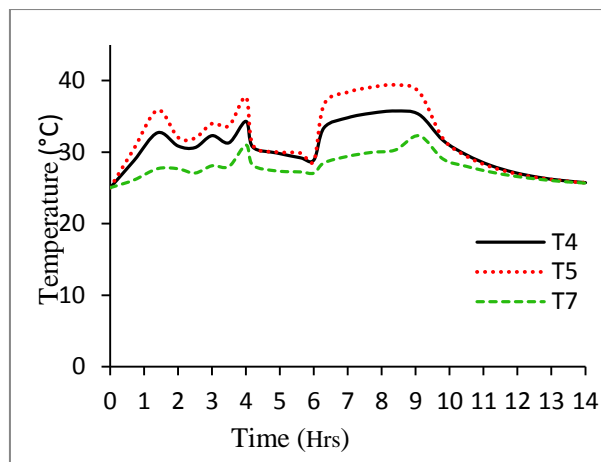


Fig. 12: Temperature Variation

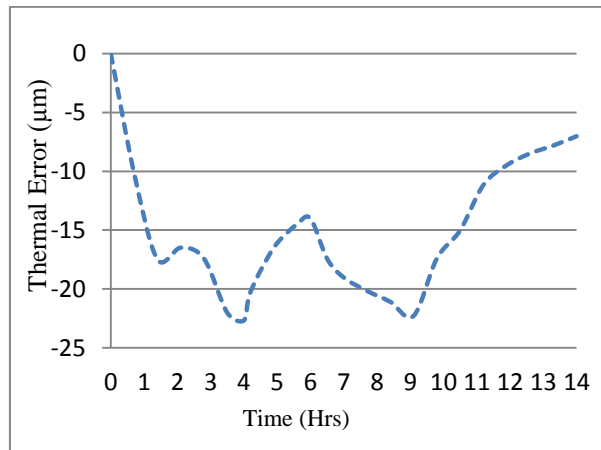


Fig. 13: Transient Variation of Spindle Thermal Error

The validated results are shown in table 5 and the comparison of measured, predicted and residual thermal error are depicted in figure 14.

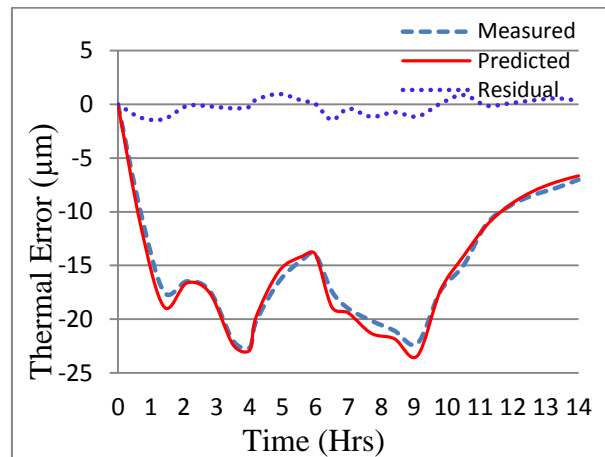


Fig. 14: Comparison of measured, predicted and residual thermal error

Table 5: Fitting accuracy of LASSO Model

Test No.	MAD ( $\mu\text{m}$ )	SD ( $\mu\text{m}$ )
I	0.366	0.4366
II	0.588	0.7426

From table 5 and Fig 16, it is inferred that the LASSO model has better robustness and accuracy as compared to least squares step wise regression approach.

## 6. Conclusion

The transient variation of temperature and thermal error in the headstock assembly of CNC turning centre has been investigated through experimentation. In the process of thermal error modeling, the application of LASSO regularized regression led to the reduction of number of temperature key points from 13 to 3. It is deduced that the contribution of the rear bearing ( $T_4$ ), front bearing ( $T_5$ ) and headstock ( $T_7$ ) temperature key points have the more impact (27.1% , 22.2% and 11.7% respectively) on total thermal error through the use of ANONA, Grey Correlation analysis as well as LASSO regression. The proposed method for the modelling thermal error can be applied for the development of real-time error compensation because it has a advantages of reduced computational time for both variable searching and modelling, plus the model developed using this method shows adequate predictive accuracy of 96.0126%.

It has been proved that the Lasso regression can fit and predict the thermal errors well with an accuracy of more than 96% at other experimental conditions.

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