

Dependence Measure of Daily versus Weekly Returns

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Abstract

The copula method has been popular among researchers, especially in measuring the overall dependence and extreme dependence of multivariate data. Many copula studies have been focusing on examining the correlation of bivariate daily, monthly or weekly returns to explain the co-movement between financial markets and possible financial implications on portfolio management. Differently from past studies, this paper investigates whether different frequency of bivariate data (daily and weekly returns) possesses different dependence structures. The data from Kuala Lumpur Composite Index (KLCI) and Bursa Malaysia Hijrah Shariah Index (FBMHS) for the sample period of 2008 Q1 to 2017 Q1 are used for studying the dependency. The findings from this study reveal that both daily and weekly bivariate returns have the same dependence structure but different degree of dependence. Bivariate weekly returns showed stronger dependence compared to bivariate daily returns. This paper also highlights the statistical properties of weekly and daily data. The evidence from this research draws inferences for further study that lower frequency data such as monthly or quarterly returns data may have higher degree of dependence while higher frequency data may have lower degree of dependence and different copula structure.

Keywords: Copula, KLCI, FBMHS.

1. Introduction

The study of dependency between financial markets is important as it can provide insights regarding the co-movement of markets. Traditionally, the dependence or association between two different variables is measured by the Pearson's correlation coefficient. However, the Pearson linear correlation measure is inappropriate when the variables have non-normal distribution such as financial variables that often exhibit leptokurtic distribution. This problem has been proven by past studies such as those from [6], [11] and [14]. Due to issues in linear correlation, several researchers used different methods such as the co-integration analysis to measure financial markets association in the long-run and short-run [7]; [3]. However, the co-integration analysis is not robust enough and has some limitations [10].

Copulas can be used to measure the dependency between financial markets. In fact, many researchers used them due to its flexibilities in modelling dependence. First, copulas are invariant to monotone transformations of random variables, and this property is essential since financial variables often undergo transformation. Second, copula modelling allows the multivariate distribution function to be decomposed by the marginal distribution functions and the dependence or copula function.

Many studies in finance have used copulas to measure the co-movement between markets using daily data. As an example, [12] concluded that the student's t and Gumbel-Clayton mixture copula are the most appropriate models to capture the association between BUX-PX50 and DAX-SP500 daily returns. Likewise, [1] found that the student's t copula was the best-fitted model to describe the dependence between the returns of KLCI and FBMHS indices for the period of 2000 to 2012.

In an international study, [4] found that different types of data have different dependence structures. The Clayton copula was the best model for all pairs of oil and CEE stock markets for the weekly data, but the Survival Gumbel copula fitted well for the daily data. Their study also revealed that the weekly indices have lower tail dependences than the daily indices.

There are limited studies that investigate the dependency of paired returns using weekly data, especially for the Asian stock markets. This study sets out to examine the dependence between the Malaysia portfolios which are represented by the Islamic (FBMHS) and the conventional (KLCI) index for two sets of data (weekly and daily). The outcomes of this study will give insights on whether different types of data can influence the results of dependency. Newer findings on dependency in the

Malaysian market are also provided as the data covers several important events such as the Global Financial crisis (GFC) in 2008, the Lehman Brothers collapse in 2008, the Russian Ruble crisis in 2014 and the 1MDB Scandal in 2015.

2. Materials And Methods

2.1 Data:

This study uses the daily and weekly closing prices of the FTSE Bursa Malaysia Kuala Lumpur Composite Index (KLCI) and the FTSE Bursa Malaysia Hijrah Shariah Index (FBMHS) where both indices represent conventional and Islamic stock markets. Two different types of secondary data (weekly and daily) for the period from January 2008 until Mac 2017 have sample sizes of 483 observations and 2319 observations, respectively. Due to the non-stationary and non-normality of series, price indices were transformed into return series using the following equation:

$$R_t = \log P_t - \log P_{t-1} \quad (1)$$

2.2 Marginal Model

In the copula approach, the identification of marginal model for each return series is required for estimating the dependence between pairs of returns. The dynamics of return series such as volatility is modelled by the ARMA-GARCH models with various innovation (error) distributions. The four error term distributions are normal, skewed normal, student- t , and skewed student- t . The probability density function for normal and student t distributions can be found in [9], while the mathematical expressions for the skewed normal and skew student- t distributions can be referred in [5] and [8]. The general ARMA(p,q)-GARCH(1,1) model with normal errors is given by:

$$Y_t = \alpha_0 + \sum_{i=1}^p \phi_i Y_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$\varepsilon_t = z_t \sigma_t \quad \text{where } z_t \sim N(0,1)$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (2)$$

where Y_t represent the return at time t , ε_t is a white noise process assumed to follow normal distribution with zero mean and standard deviation of 1, σ_t^2 is the variance at time t , $\alpha_1 \varepsilon_{t-1}^2$ is the ARCH component, and $\beta_1 \sigma_{t-1}^2$ is the GARCH component.

The diagnostic checks of the model are conducted by assessing the serial correlations and independence among residuals. The Ljung-Box test is used to test for independence of residuals, whereby the insignificance of Q -statistics on residuals and squared residuals indicate the independence of residuals and the model is correctly specified. The Lagrange Multiplier (LM) test

is also conducted to examine the existence of ARCH effects. A good ARMA-GARCH model is said to be an appropriate fit to the return series if no ARCH effects are detected in the residual series. The information criterion such as Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Shibata Information Criteria (SIC) and Hannan-Quinn Information Criteria (HQIC) are also computed for selecting the best-fit model. A best-fit ARMA-GARCH model to the return series should have the smallest information criterion assuming that the model has passed the diagnostic checks.

2.3 Dependence Estimation

Following the methodology adopted by past studies [1], the dependence of paired returns is estimated based on a two-step procedure. First, the marginal model for each return series is identified by fitting the appropriate ARMA-GARCH model. Second, the dependence between financial variables is estimated by using the pseudo samples, which is transformed from the standardized residuals of marginal models and estimating the copula parameters. Several copulas are considered for fitting the bivariate data, including the Gaussian, Student t , Clayton, Gumbel, Frank, Galambos and Husler-Reiss copulas. The functions of each copula function can be found in [13]. Finally, the goodness-of-fit (GOF) test is conducted to test whether the assumed copula function fits the bivariate data.

3. Results

3.1 Preliminary Analysis

The daily price indices for both FBMHS and KLCI undergo initial analysis before performing further statistical procedures. The summary statistics such as the mean, minimum, maximum, Standard Deviation (SD), Coefficient of Variation (CV), the p -values of Augmented Dickey-Fuller (ADF) test for stationarity and Jarque-Bera (JB) test for normality were calculated. Although the results are not shown here, the CVs are slightly higher for both weekly KLCI and FBMHS, indicating that weekly prices are slightly more volatile compared to the daily prices. The Shariah price index was seen to be more volatile than the conventional index.

This finding was supported by the difference between the minimum and maximum values where the FBMHS for both weekly and daily data have the largest difference compared to the KLCI, which suggest higher volatility. The insignificant value of ADF test ($p > 0.01$) indicates that the price series are non-stationary. The JB test ($p < 0.01$) shows that the distributions are non-normal. To overcome these problems, price indices are transformed into return series using equation 1. Table 1 presents the descriptive statistics for the return series of daily KLCI, weekly KLCI, daily FBMHS, and weekly FBMHS.

Table 1: Summary statistics of return series, stationary test, and normality test.

Stock (Type of data)	KLCI		FBMHS	
	Weekly	Daily	Weekly	Daily
Mean	-0.00035	-8.915e-05	-0.00031	-8.576e-05
Median	-0.00137	-2.765e-04	-0.00177	-2.932e-04
SD	0.01676	0.00732	0.01777	0.00780
Skewness	0.66770	1.25145	0.84205	1.44003
Kurtosis	5.98883	17.76729	6.21499	21.79880
ADF Test	0.01	0.01	0.01	0.01
JB Test	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16

The results in Table 1 show that both data types have negative returns, on average. The weekly data for FBMHS and KLCI have higher standard deviation compared to the daily data. In general, all return series have leptokurtic distribution. The comparison between the daily and weekly returns of both indices shows that the daily returns have heavier tail, which can be seen clearly by referring to the large differences of kurtosis value between the weekly and daily data of both indices. This finding is similar with [1] which find that value of kurtosis

greater than 12 for both FBMHS and KLCI daily returns. The skewness is greater than zero which show that the return series are positively skewed thus implying that there are more negative returns and fewer gains for the period of January 2008 to March 2017. These results also suggest the existence of positive extreme returns. The ADF test for all return series are significant, indicating the stationarity of the data. The JB test is significant at 1% level for all indices, illustrating that the data are not normal distributed.

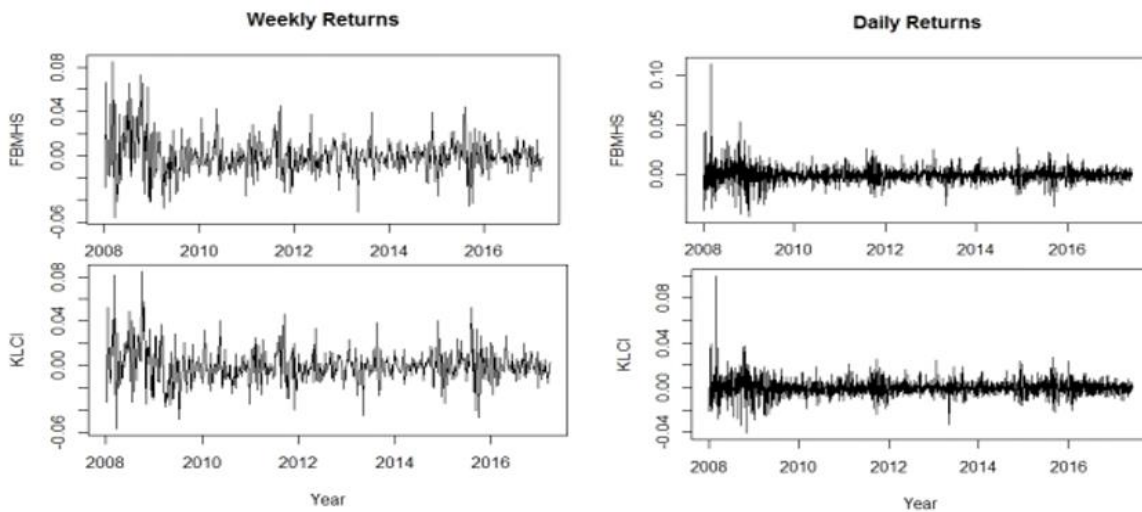
**Fig. 1:** Line plots of a) weekly and b) daily returns for FBMHS and KLCI.

Figure 1 shows the weekly and daily returns plot for both KLCI and FBMHS. The volatility clustering patterns can easily be identified in daily returns plots compared to weekly returns plot. Based on Fig. 1, volatility clusters can be seen in 2008 to 2009, late 2012, 2015 and late 2016. In general, the '1MDB Scandal 2015' slightly affect the FBMHS and KLCI performances, while the 'Russian Ruble crisis 2014' did not affect the return series.

3.2 Marginal Modeling and Dependence Estimation

For marginal modelling, we fit the ARMA-GARCH model with four error term distributions; normal, skewed normal, student- t , and skewed student- t distribution on to each return series. Table 2 shows the results for estimated ARMA-GARCH model of the daily and weekly return series.

The Ljung-Box Q-statistics for residuals and squared residuals at lag 10 and 20 have p -values greater than 0.05, implying that there is no serial correlation within the residual series.

The Lagrange Multiplier (LM) tests have insignificant value, meaning that the ARCH errors are not present in the series. By referring to the information criterion, GARCH (1,1) with student's t distribution is selected as the best marginal model for the FBMHS and KLCI weekly data, while the AR(1)-GARCH(1,1) with student's t distribution was chosen as the best marginal model for both daily data. After determining the marginal models, the standardized residuals were extracted and transformed into pseudo samples that is uniformly distributed. The pseudo samples were used to estimate the dependence between KLCI and FBMHS each for weekly and daily data.

Table 2: Summary results for selected marginal models (continued).

Stock (Type of data)	KLCI		FBMHS	
	Weekly	Daily	Weekly	Daily
Diagnostic test				
Q(10)	0.76657	0.77879	0.89107	0.77787
Q(20)	0.95948	0.94156	0.98284	0.97620
Q ² (10)	0.89117	0.65684	0.98308	0.64335
Q ² (20)	0.41733	0.72014	0.44119	0.73844
LM test	0.89422	0.73043	0.96833	0.75339

Table 3 presents the results for dependence estimation. Based on the largest *p*-value which is insignificant at 1% level, the KLCI-FBMHS pair was well fitted by the Gaussian copula for both the weekly and daily data. Figure 2 further

confirms that the dependence structure that is symmetric, which suits the description of the Gaussian copula. Another interesting result is that the KLCI-FBMHS pair have positive estimated copula parameters for both the weekly and daily data.

Table 3: Copula parameter estimated, standard error, and goodness of fit for weekly and daily data.

Copula	Weekly index			Daily index		
	Parameter	Std. Error	GOF	Parameter	Std. Error	GOF
Gaussian	0.9339	0.0070	0.4850	0.9090	0.0040	0.0175
Student's <i>t</i>	0.9303 (8.8906)	0.0050 (3.2440)	0.4660	0.9099 (8.4454)	0.0030 (1.6010)	0.0155
Clayton	6.5920	0.4550	0.0005	5.3080	0.1800	0.0005
Gumbel	4.2960	0.2280	0.1444	3.6540	0.0900	0.0005
Frank	15.3400	0.7310	0.0065	12.7300	0.2760	0.0005
Galambos	3.5870	0.2280	0.1184	2.9440	0.0900	0.0005
Husler-Reiss	4.4360	0.2580	0.1114	3.7060	0.1030	0.0005

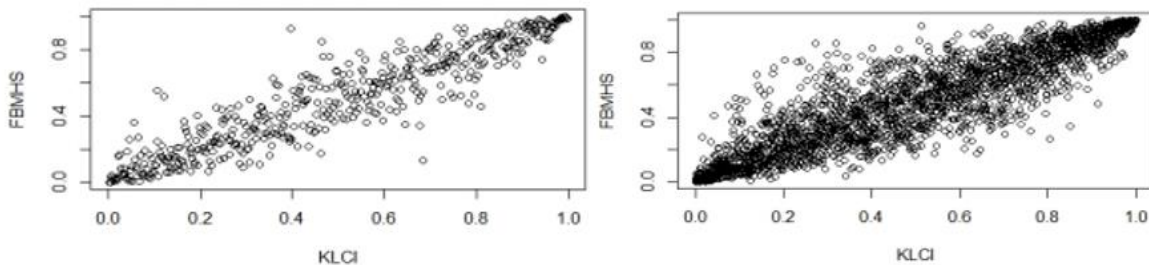


Fig. 2: Plots of pseudo observation for KLCI-FBMHS for a) weekly and b) daily data sets.

Discussion

In general, our results reveal that weekly and daily return series have similar statistical properties. However, daily returns data provides distinguishable information such as extreme returns, heavy tail and volatility clustering. In terms of marginal modelling, we found that GARCH (1,1)-Student-*t* was a good fit to the weekly return series while the AR(1)-GARCH(1,1)-Student-*t* was a good fit to the daily return series. In terms of dependency, the Gaussian copula was found suitable to describe the dependence structure for both the weekly KLCI-FBMHS returns and daily KLCI-FBMHS returns. Moreover, the copula selection for bivariate weekly data was much easier as the *p*-value of the goodness-of-fit test was obviously insignificant. However, our finding contradicts the results of [4] who obtained different best fitted copula models for different types of data. Nevertheless, the findings from this study draws inferences for further study that lower frequency data may have higher degree of dependence while higher frequency data may have lower degree of dependence and different copula structure.

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