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Research paper

Construction and Calculation of Reinforced Concrete Overlap with A High Spatial Work Effect

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Abstract

The article proposes the construction of a prefabricated monolithic reinforced concrete overlap consisting of beams hollow triangular section. It is shown that in such overlap the effect of spatial work is much higher than the analogous effect in traditional overlap that consists of U-shaped or T-beams and slabs. The technique of determining the forces of interaction of individual beams in the composition of the overlap is given. The technique is based on a discrete-continual method developed by the author, which is adapted to the calculation of overlaps that consist of considered beams. The technique of determining the effort between the shelf and the ribs of a beam during its bending is presented. It is based on the theory of compound rods. The algorithm of calculation taking into account the spatial work is presented as well as the principles of constructing overlaps consisting of beams hollow triangular section, taking into account the change in their rigidity as a result of cracks formation. An approach to the determination of the rigidity of beams with normal torsion fractures is given, based on the approximation of numerical experimental data.

Keywords: *approximation, bending, overlap, rigidity, spatial work, torsion.*

1. Statement of the Problem

Reinforced concrete overlaps, regardless of the method of their manufacture, are usually calculated taking into account the spatial work, which means the presence of forces and displacements along the direction of all three axes of coordinates in space. The forces between its individual elements are redistributed due to the work of the overlap [2, 4, 5, 11, 12]. This is particularly noticeable when local loads act on overlaps. Local loads distribute only to some part of the overlap. Such loads can be loads from equipment, internal partitions, premises for various purposes, etc.
Studies of many scientists have shown [4, 5] that the more spatial work is manifested, the more efficient the individual elements of the overlap are used and the more economical and reliable the overlap is. There are various ways of constructing overlaps to increase the effect of spatial work. It has been shown in various works that the spatial work of overlaps depends on the flexural and torsional rigidity of their elements, and also on the ratio of these rigidities. The use slabs or beams in overlaps, in which a large flexural rigidity, but low torsional rigidity leads to a decrease in the effect of spatial work. Very common plates of T-shaped and U-shaped sections have rather a high bending rigidity and strength, but low torsional stiffness. As a consequence, the overlap of such slabs has a low effect on spatial work.
The author of this paper [4] proposed to use hollow triangular section beams (slabs) that have the flexural rigidity almost equal rigidity of T-shaped or U-shaped section, but they have torsion rigidity tens of times higher rigidity of the mentioned beams (slabs). The disadvantage of such beams is a relatively complex manufacture in view of their closed section. In this regard, in work [4] a method of producing a hollow triangular section beams in a prefabricated monolithic variant, when the main part of the beam in a planar form concreted on manufactory floor and then transformed into their spatial hollow triangular section beam. However, in view of the inhomogeneity of such design, its calculation of the action of bending and torsion moments has its own characteristics, which must be investigated. In addition, in the above-mentioned studies, there are no mentions of constructing an overlap that consists of beams hollow triangular section and also a technique for calculating such overlaps.
In connection with the foregoing, the purpose of this article is to develop a method for calculating overlaps consisting of beams hollow triangular section, the principles of calculating such beams for bending and torsion, methods for constructing overlaps consisting of such beams, and also a technique for determining the torsional rigidity of the mentioned beams with normal cracks.

2. Determination of the Forces of Interaction of Adjacent Beams in the Composition of the Overlap

Consider the cell overlap consisting of beams (slabs) of a hollow triangular section (Fig. 1).
In the general case, an overlap consisting of *n* beams is affected by an uneven load. On each beam with number *i* (i=1…n) will act defined load q_i in this case. Various methods of spatial calculation can be applied to calculate the overlap taking into account the joint work of beams with each other. The most common method is the use of programs such as Ansys, Abacus, Lira, etc. The use of these programs is most acceptable when calculating in the elastic stage without taking into account the change in the rigidity

parameters in the result of the formation of various cracks. Discrete-continual methods are more convenient within calculating taking into account the change in the rigidity parameters.

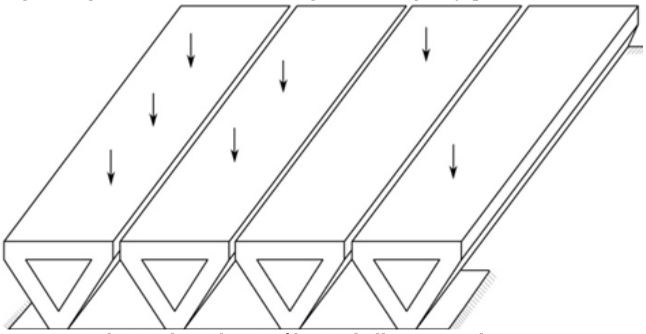

Fig.1. The overlap scheme of beams hollow triangular section

A very convenient method is the discrete-continual method by the author of the article. The modified method [4] will be used to calculate the overlap consisting of beams hollow triangular section, in which not all the internal forces arising along the lines of dissection of the overlap into individual beams will take into account. They are vertical interaction effort $P_i(x)$ on the left and $P_{i+1}(x)$ on the right, transverse bending moments $m_i(x)$ on the left and $m_{i+1}(x)$ on the right, and external load $q_i(x)$. Acting along the edges of the beam, in general case, horizontal longitudinal forces $N_i(x)$ and horizontal strut forces $H_i(x)$ (in [4]) won`t be taked into account in view of the relatively low height of the beams. In this case, the system of differential equations derived in [4] can be simplified without a noticeable decrease in the accuracy. The bending moments from the forces are denoted $p_i(x)$ through Mp_i(x). These moments are connected with efforts $p_i(x)$ of known differential dependence $Mp_i^{II}(x)=p_i(x)$. Taking into account the above, the typical line of the system of differential equations for determining unknown moments $Mp_i(x)$ and $m_i(x)$ will looks (argument x is not shown for brevity):

$$\left\{\begin{aligned} &-\frac{Mp_{i-1}}{EJ_i}+\left(\frac{1}{EJ_i}+\frac{1}{EJ_{i+1}}\right)Mp_i-\frac{Mp_{i+1}}{EJ_{i+1}}-\frac{R_i^2}{GJ_i}Mp_{i-1}^{II}-\\ &-\left(\frac{R_i^2}{GJ_i}+\frac{R_{i+1}^2}{GJ_{i+1}}\right)Mp_i^{II}-\frac{R_{i+1}^2}{GJ_{i+1}}Mp_{i+1}^{II}+\frac{R_i}{GJ_i}m_{i-1}+\\ &+\left(\frac{R_{i+1}}{GJ_{i+1}}-\frac{R_i}{GJ_i}\right)m_i-\frac{R_{i+1}}{GJ_{i+1}}m_{i+1}=\frac{Mq_{i+1}}{EJ_{i+1}}-\frac{Mq_i}{EJ_i};\\ &-\frac{R_i}{GJ_i}Mp_{i-1}^{II}+\left(\frac{R_{i+1}}{GJ_{i+1}}-\frac{R_i}{GJ_i}\right)Mp_i^{II}+\frac{R_{i+1}}{GJ_{i+1}}Mp_{i+1}^{II}+\\ &+\frac{m_{i-1}}{GJ_i}-\left(\frac{1}{GJ_i}+\frac{1}{GJ_{i+1}}\right)m_i+\frac{m_{i+1}}{GJ_{i+1}}=0 \end{aligned}\right. \qquad (1)$$

In the expression (1): R_i – distance from the center of gravity of the beam section to its edge; EI_i, GI_i – respectively, the bending and torsional rigidity of the beam; $Mq_i=Mq_i(x)$ – function of bending moments from an external load q_i.

Equations (1) are written for each section between adjacent beams. That is, if there are n beams in the overlap, then the system of equations will contain 2n-1 differential equations. These equations are most conveniently solved using the expansion of unknown functions $Mp_i(x)$, $m_i(x)$, as well as the functions of moments from an external load $Mq_i(x)$ in the Fourier series of sines.

$$Mp_i(x)=\sum_{i=1}^{n}Mp_n\cdot Sin(\alpha\cdot x);\ m_i(x)=\sum_{i=1}^{n}m_n\cdot Sin(\alpha\cdot x)$$

$$Mq_i(x)=\sum_{i=1}^{n}Mq_n\cdot Sin(\alpha\cdot x), \qquad (2)$$

where $\alpha=\pi\cdot n/l$; l – span of a beam; Mp_n, m_n - The unknown coefficients of the expansion in the Fourier series of the functions $Mp(x)$ and $m(x)$; Mq_n – known coefficients of the expansion of the external moment $Mq_i(x)$.

Substituting (2) into (1), performing differentiation and contracting by Sin (α · x), it is not difficult to obtain a system of algebraic equations for obtaining unknown coefficients of the expansion Mp_n and m_n. Than, taking into account the differential dependence $Mp_i^{II}(x)=p_i(x)$ легко получить неизвестные вертикальные силы взаимодей it is easy to obtain unknown vertical forces of interaction $p_i(x)$.

After determining unknown forces along the edges of the beams, each beam should be counted as a separate beam element, on which its external load and forces along its edges are acted $p_i(x)$ and $m_i(x)$.

Calculations show that in view of the high rigidity of the beams hollow triangular section in the transverse direction and also in the case where the beams are interconnected by welding of the embedded parts the effect of transverse moments $m_i(x)$ can also be neglected. In this case for each beam separated from the i-th beam only vertical forces $p_i(x)$ will act, as shown in Fig. 2.

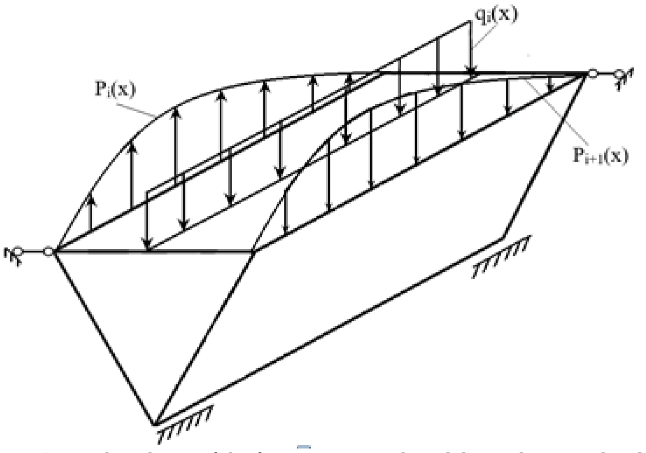

Fig. 2. The scheme of the forces acting on the i-th beam that considered

In this case, the system of differential equations will become even simpler, there will be only the unknown vertical forces of the interaction of the beams with each other $p_i(x)$. The typical line of the system of differential equations will have the form:

$$\begin{aligned} &-\frac{Mp_{i-1}}{EJ_i}+\left(\frac{1}{EJ_i}+\frac{1}{EJ_{i+1}}\right)Mp_i-\frac{Mp_{i+1}}{EJ_{i+1}}-\frac{R_i^2}{GJ_i}Mp_{i-1}^{II}-\\ &-\left(\frac{R_i^2}{GJ_i}+\frac{R_{i+1}^2}{GJ_{i+1}}\right)Mp_i^{II}-\frac{R_{i+1}^2}{GJ_{i+1}}Mp_{i+1}^{II}=\\ &=\frac{Mq_{i+1}}{EJ_{i+1}}-\frac{Mq_i}{EJ_i} \end{aligned} \qquad (3)$$

where the notation is the same as in expression (1).

Comparison of the forces determined by solving the system of equations (1) and (3) with the forces determined by the finite element method with using the Lira software package has shown a sufficiently high accuracy of the above technique. The advantage of the above approach to the spatial calculation of the overlap from the beams hollow triangular section is in next: it is possible to take into account the variation of the flexural and torsional rigidities of the beams in a result of the formation of cracks [4], because in systems (1) and (3) of rigidities EJ_i and GJ_i each of the beams are taken into account separately.

3. Determination of the Effort between the Shelf And The Inclined Ribs Within Bending the Beam of A Hollow Triangular Section

The vertical forces that capable to tear off the shelf of the beams are arisen when the beams work in the composition of the overlap along the lines of joining adjacent plates. Let as a result of spatial calculation the forces of interaction of beams with each other are determined. Consider the selected individual beam (Figure 2). The

external load acts on the beam $q_i(x)$, and also the forces of interaction with the adjacent beams on the left $p_i(x)$ and on the right $p_{i+1}(x)$.
Various methods can be used to determine the stress-strain state of the shelf and beam edges including a technique based on the theory of compound rods. For this, imagine a beam of a hollow triangular section in the form of a two-layer compound rod in which the upper rod imitates the work of the shelf, and the lower one - the work of inclined ribs. Here is an external load q_i, load on the left edge p_i and the load on the right edge p_{i+1} relative to the equivalent load acting on the upper face of the beam. The magnitude of this load $q=q_i-p_i+p_{i+1}$ (Fig. 2). It should be had in mind the signs under efforts p_i and p_{i+1}. The result is a diagram of the composite rod shown in Fig. 3.

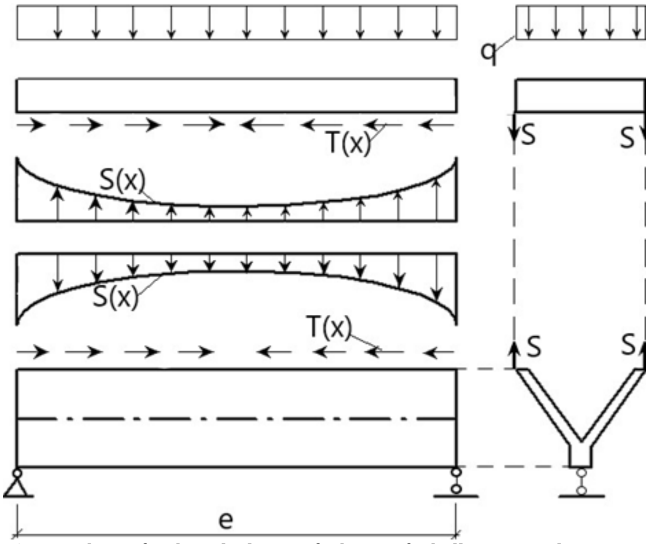

Fig. 3. Scheme for the calculation of a beam of a hollow triangular section with the application of the theory of composite rods

In general, the linear tangents act on the cut plane $T(x)$ and vertical $S(x)$ efforts, which are distributed over two shelf joint lines with inclined ribs.
A system of differential equations for a two-layer composite rod for unknown functions $T(x)$ and $S(x)$ is in [13]:

$$\begin{cases} \frac{T^{II}}{\xi} = gT + i \cdot Ms + g_0; \\ \frac{-Ms^{IV}}{\eta} = i \cdot T + k \cdot Ms + k_0 \end{cases} \quad (4)$$

where g, i, g_0, k and k_0 – parameters associated with geometric characteristics and external loads on the component rods, determined by [13]; $Ms=Ms(x)$ – bending moment function of lateral forces $S(x)$; η, ξ – respectively, the rigidity coefficients of the transverse bonds and shear bonds, representing the ratio of the force in the connection to the corresponding deformation. In this case, it can be assumed that the cross-links are the layer of the beam, which height is equal to the thickness of the upper shelf. The coefficients η and ξ should be determined by [13].
For a convenient solution of the system of differential equations (4), for any ratio of the rigidity of the upper and lower components of the rods, it is correct to use trigonometric Fourier series in sines, similarly to [3], since in this case the boundary conditions will be satisfied automatically. Then the solution for the unknown functions $T(x)$ and $S(x)$ will look:

$$T(x) = \sum_{n=1}^{\infty} T_n Sin(\alpha \cdot x); \; S(x) = \sum_{n=1}^{\infty} S_n Sin(\alpha \cdot x) \quad (5)$$

where also, as in (2), is denoted $\alpha=\pi \cdot n/l$.
Substituting (5) into system (4), taking into account the dependence $MS^{II}(x)=S(x)$ after appropriate transformations, expressions for the decomposition coefficients T_n and S_n are obtained:

$$T_n[-\alpha^6 - \xi g \alpha^4 - k\eta\alpha^2 + \xi\eta(i^2 - gk)] = B_1 M_{1,n} \quad (6)$$

$$S_n = -T_n\left(\frac{\alpha^4}{\xi i} + \frac{g\alpha^2}{i}\right) - \frac{2}{l\alpha i}\frac{qa}{EJ_1}[(-1)^n - 1] \quad (7)$$

where is denoted:

$$B_1 = -\frac{\xi\eta i + \xi\eta k a}{EJ_1}; \quad (8)$$

$M_{1,n}$ – coefficient of expansion of the bending moment from an external load into a Fourier series of sines; EJ_1, EJ_2 - bending rigidities of the upper composite rod (beam shelf) and the lower rod (inclined ribs); a, b – distance from the centers of gravity of the upper and lower rods respectively to the line of their conjugation.
Substituting (6) and (7) into (5) the required functions $T(x)$ and $S(x)$ are obtained.
After determining efforts $T(x)$ and $S(x)$ the upper component rod (shelf) should be considered as a beam on which the external load is applied q, and also efforts $T(x)$ and $S(x)$. The bottom rod (inclined V-shaped ribs) should be considered as a beam, on which only the forces $T(x)$ and $S(x)$ are acted.

4. The Algorithm for Calculating the Overlap of Beams in a Triangular Section, Taking Into Account the Spatial Work

Let consider an algorithm for the complete calculation of an overlap consisting of beams hollow triangular section.
1. Determine the initial rigidity characteristics of the section and beam reinforcement: torsional D_t and bending B rigidities of beams by the known formulas for the resistance of materials; the rigidities coefficients of the shear bonds and transverse bonds joining the shelf and the inclined edges of the beams, according to [13]. These coefficients depend on the reinforcement, thickness, and class of the concrete of the monolithic seam, as well as on the presence or absence of dowels.
2. At zero external iteration, the overlap is calculated with the rigidity B and D_t of the beams taking into account the spatial work according to the methodology given above. The result is a force of interaction p_i and p_{i+1} of beams with each other.
3. Sequentially examine each beam separately with its own external load and the interaction forces along the left and right edges (Fig. 2).
4. Determine the equivalent load on the shelf, taking into account the signs (direction) of the interaction forces $q=q_i-p_i+p_{i+1}$ and determine the deflection of the beam f_4 and angle of rotation φ_4 at this stage of calculation in the middle of the span.
5. Calculate each beam according to the theory of compound rods according to the technique given above, taking into account changes in the rigidity of the shelves and inclined edges from the formation of cracks. For this, at the zero internal iteration, it is assumed that the element works elastically. According to expression (5), taking into account (6) and (7), the forces $T(x)$ and $S(x)$ acting in the joint between the shelves and inclined ribs are determined.
6. Any known method for calculating reinforced concrete elements (for example, according to [1, 7]) calculates the time of formation of cracks, rigidities in the absence of cracks and in the places where they exist and determine the deflection of each beam element separately (shelves and inclined ribs) in the middle of the span $f_{(l/2)}$ from efforts, acting directly on the shelf and on the inclined edges (this is most convenient to do in the form of a calculation progamm).
7. Determine the equivalent bending rigidities of a conditionally elastic shelf B^1_{ekv} with constant rigidity and conditionally elastic inclined edges B^2_{ekv}. These rigidity is determined from the condition of equality of deflections $f_{(l/2)}$ in the middle of the span of a real element with cracks (as defined in paragraph 6) and deflection f_f of a fictitious element with constant bending rigidity, i.e. from the condition:

$$f_{l/2} = f_f \quad (9)$$

The deflection of a fictitious beam with conditionally constant rigidity is determined from the known formula for the resistance of materials:

$$f_f = y_{(x=\frac{l}{2})} = \iint M(x)dx \qquad \text{(with } x=l/2) \quad (10)$$

Then in view of the foregoing, equivalent rigidities$B^1_{ekv}$and B^2_{ekv} are determined:

$$B^1_{ekv} = \frac{\iint M_1(x)dx}{f_{l/2}} \ , \ \ B^2_{ekv} = \frac{\iint M_2(x)dx}{f_{l/2}} \quad \text{(with } x=l/2) \quad (11)$$

where M_i=$M_i(x)$ – function of the bending moment in this element, which for the upper rod (shelves) is composed of an external load and forces *T(x)* and *S(x)*, for the lower rod (inclined ribs) – only from efforts *T(x)* and *S(x)*, defined by the expressions (5) with tacking into account (6) and (7).
8. Substituting rigidities of (11) into characteristics of the shelves and the inclined ribs, an elastic calculation is again carried out from point 5 and new values of the equivalent (fictitious) rigidities of the beam elements B^1_{ekv} и B^2_{ekv} are obtained for the next iteration. Iterations by paragraphs 5-8 are carried out until the rigidity at the last iteration coincides with the rigidities at the previous iteration with a predetermined error. As a result, the final rigidity is obtained for the bending of the beam as a whole.
9. With the final rigidities determine the bending of the entire beam in the middle of the span f_9 and the angle of rotation in the middle of the span φ_9 by paragraph 8.
10. Next, an equivalent bending B^{tot}_{ekv} and equivalent torsional $D^{tot}_{t,ekv}$ rigidities of the entire beam are determined, considering it conditionally elastic with constant rigidity. These rigidities are determined from the condition of equality of the deflection f_9 and angle of rotation φ_9 in the middle of the span of the element with cracks in the deflection f_f and angle of rotation φ_f of a fictitious element with constant bending rigidity B^{tot}_{ekv} and constant torsional rigidity $D^{tot}_{t,ekv}$, i.e. from the conditions by paragraph 9:

$$f_9 = f_f \ \text{ and } \quad \varphi_9 = \varphi_f \quad (12)$$

The angle of rotation of a beam with constant rigidity is determined by the known formula of the resistance of materials:

$$\varphi_f = \int_0^{l/2} \frac{M_t}{D_t} dx \quad (13)$$

where M_t – torsion moment in the beam in question, which consists of the moments created by the efforts of the interaction of the beams with each other on the left $p_i(x)$ and on the right $p_{i+1}(x)$.
The deflection of a beam with constant rigidity is also determined by a known formula analogous to (10).
Taking this into account, the equivalent rigidity of the beam as a whole will be determined from the expressions:

$$B^{tot}_{ekv} = \frac{\iint M_{tot}(x)dx}{f_9} \qquad \text{(with } x=l/2) \quad (14)$$

$$D^{tot}_{ekv} = \frac{\int_0^{l/2} \frac{M_t}{D_t} dx}{\varphi_9} \quad (15)$$

Substituting rigidities according to (14) and (15) as new characteristics of the beams of the overlap B and D_t and repeat the calculation from paragraph 2. External iterations from paragraphs 2-10 are repeated until the rigidities at the last and the previous iteration match with the predetermined accuracy.

5. The Construction of A Prefabricated Monolithic Overlap Consisting of Beams Hollow Triangular Section

Let us now consider methods for constructing overlaps consisting of beams hollow triangular section. In view of the complexity of the concreting of such beams in a monolithic variant, they fabrication was proposed in [4] in two stages. At the first stage (Fig. 4), the bottom wooden pads with *1* thickness equal to half the thickness of the wall of the hollow beam are laid on a concrete surface. The width of the pads can be 50-120 mm depending on the thickness of the beam wall. The distance between the pads is selected depending on the length of the beam bay. Then the reinforcement grid *2* is laid on the pads.

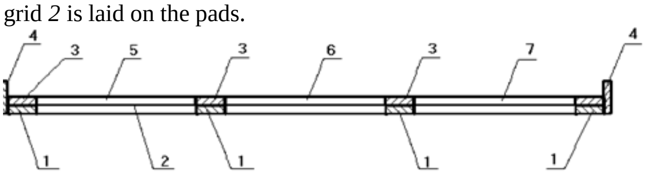

Fig. 4. The scheme of concreting the beam of a hollow triangular section

The diameter of the reinforcement grid (3-8 mm) is selected by calculating the strength of the inclined sections of the reinforced concrete beam. After that, lay the upper wooden pads *3* and the end-type board *4* are laid (its width is equal to the thickness of the beam wall). Then, between the pads, the concrete of the beam structure is laid in sections *5*, *6* and *7*. The lengths of the sections can be any, depending on the required cross-sectional dimensions of the hollow triangular beam. After the strength of the concrete is reached, the end-type board *4* and the upper pads *3* are removed. The extreme sections of the joists *5* and *7* are raised to the top. In the result, a hollow triangle is formed (Fig. 5). The extremes of the grid are overturned and tied with knitting wire. Then in the required corner of the formed triangle, the working reinforcement *8* of the beam is laid (the reinforcement is laid in the place where the stretched beam zone will be in working condition). The areas of an intersection of sections in the corners of triangle 9 are concreted with the help of tie formwork (from separate boards).

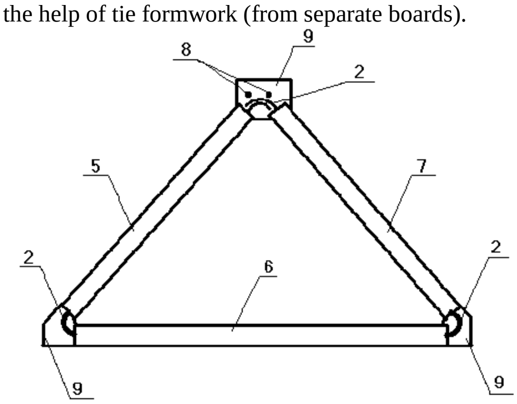

Fig. 5. Section of a prefabricated monolith reinforced concrete beam of hollow triangular shape

After the strength of the concrete sections and monolithic areas *9* is set, the beam is ready for use. The beam is turned into working position in such a way that the working reinforcement is in the stretched zone.
The section of the stretched reinforcement *8*, laid before the concreting of the monolithic sections *9*, and also the thickness of the compressed shelf (item 6 in Figure 5) is selected from the condition of the required strength and rigidity of the beam in the normal section. For ease of installation, it can be arranged the supporting diaphragms on the beam supports in the ribs.
Taking into account that the prefabricated monolithic beams of the hollow triangular cross section have their own specifics of manufacturing, it is necessary to separately consider the construction of such beams. For the production of a beam in a flat form, it is nec-

essary to lay the reinforcing grid (Fig. 4). This grid on the one hand makes it possible to create a unified structure, and on the other hand it plays the role of connections that unite the beam shelf with its ribs when the beam is operating under load.

The calculations according to the authors with using the theory of compound rods by the method given above show that for a certain rigidity of a monolithic seam the beam can be calculated as completely monolithic. In other words, if the diameter of the transverse reinforcement grid is selected in a certain way then the beam can be considered monolithic. On the other hand, a thoughtless increase in the diameter of the transverse rods of the grid will lead to an increase in the amount of reinforcement and, correspondingly, a rise in the structure cost. In addition, the transverse reinforcement grid plays the role of clamps in calculating the strength of the beam along incline sections, and also in calculating the strength for the action of torsion moments.

Longitudinal reinforcement grid on the one side unites the grid into a single whole, and on the other hand, it plays the role of additional working reinforcement. Therefore, the pitch of the longitudinal reinforcement should be assigned unequally, concentrating it at the ends of the edges located at the main working reinforcement and diluting at the level of the beam shelf.

In connection with the above, when assigning the diameter and pitch of the transverse and longitudinal reinforcement grid, the following should be considered:

1. With a known total load on the beam (hence, the maximum bending moment is known), the required amount of longitudinal working reinforcement (including both the main and longitudinal rods of the grid) must first be calculated.

2. Efforts in the seam between the upper shelf and the inclined side ribs are determined by the technique given above as a composite double-layer rod, the upper component of which is the stern of the beam and the lower one is inclined ribs. The monolithic suture is modeled by cross-links and shear links. The total shear strength of the transverse reinforcement of the mesh should not be less than the strength of the working reinforcement. This is due to the fact that the external bending moment is perceived by a pair of forces, one of which is the compression force in the shelf, and the second is the effort in the working stretched reinforcement. The strength of the shear bonds should be sufficient to perceive the force in the compressed shelf.

Given that the most rational is the beam structure in which the ultimate limit state is the thickness of the shelf height of the compressed zone, the combined force T_s, perceived by the transverse reinforcement, connecting the shelf with the ribs, should be not less than:

$$T_s \geq f_{ck} b_f \cdot h_f \ , \qquad (16)$$

where f_{ck} – design resistance of concrete; b_f, h_f - respectively, the width and thickness of the beam shelf.

If the section of the working reinforcement is less than the limiting force, perceived by shelf, the force should not be less than:

$$T_s \geq f_y \cdot A_s \ , \qquad (17)$$

where f_y, A_s – respectively, the design resistance and the area of the working reinforcement.

3. To reduce the amount of transverse reinforcement of the grid, it is possible to device the dowels on the dowels of the beam during concreting. In this case, the dowels can be formed by inserting scraps of a wooden beam. The step and proportions of the dowels can be determined by the known method of designing reinforced concrete prefabricated monolithic structures [1, 7].

4. Calculate the strength of the inclined beam sections for the action of the bending moment and shear force. The diameter and step of the transverse reinforcement grid must satisfy the strength conditions for the inclined sections.

5. The diameter and pitch of the transverse reinforcement must satisfy the conditions for the strength of the spatial sections for the action of the torsion moment.

The step of the transverse reinforcement should be the smallest of all possible, taken according to points 1-5. For strength calculations of inclined and spatial sections are recommended to use known methods, including normative [1, 6, 7, 9, 14].

Beams hollow triangular section have a high torsional rigidity. This fact makes the overlap of such beams competitive in comparison with other types of overlap. The beam of a hollow triangular section carries the functions of a beam and a plate at the same time. The width of the compressed shelf can be quite large and can be the same proportions as the width of the hollow or U-shaped prefabricated plates. At the same time, such beams have their own specifics in the arrangement of overlapping. The design of the beam means to lay a separate beam without special tools is rather difficult because in the working position the beam is laid on the corner of the triangle. However, two or more beams, connected together, are already quite a stable system. For this purpose, in the arrangement overlap before laying the first beam before the second beam must be liberated temporary braces, just as it is done when installing the reflection beams or trusses.

Combining all the beams of the overlap into a single disk can be done in several ways. The first option, when all three monolithic sections of the beams are concreted before they are installed in the overlap. Then the integration into the overlapping disk can be carried out by welding the embedded parts laid in advance. In this case, there is an increase in the cost of overlap, because at the same time, the costs of the metal of the embedded parts and the elements that unite them are increasing.

Second option. When concreting monolithic sections of beams in them, it should be arranged dowels with a certain step. After mounting the beams in the locations of the dowels, the cross bars of the meshes of the adjacent beams are combined with reinforcing steel clamps and the cobs are concreted. The step of the dowels and the diameter of the connecting clamps should be selected from the calculation of the overlap, taking into account the spatial work. In the case when the overlap is designed without taking into account the spatial work, the dowels and joining clamps can be located for constructive reasons.

Third option. In the manufacture of beams concreted only a monolithic section located at the lower edge (in working position) of the beam where the working armature is located. This, in addition, will make it easier to manufacture the beam, because immediately after the transformation of the beam from a flat state (position on the floor), the lower monolithic areas are not concreted into the position of the finished beam form (Figure 6).

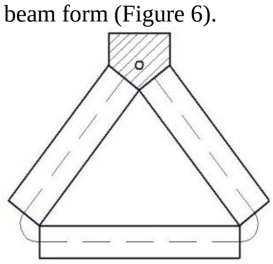

Fig. 6. Scheme concreting monolithic sections of the beam for post-erection installation

In this form, the beams are transported (in the case of manufacturing at the factory) and installed (Figure 7). The closed contour of the beam is quite rigid and, therefore, allows the transport and installation of the beams in this state.

After the installation of all the beams, the grids of adjacent beams are combined with reinforcing steel straps and the monolithic sections are concreted. Concreting of monolithic sections unites both the upper shelf of each beam with its inclined edges and all the beams into a single overlapping disk.

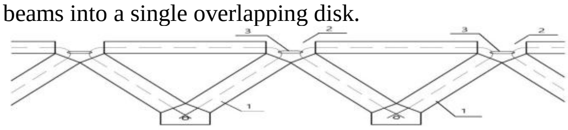

Fig. 7. The option of combining the beams in the overlap joint concreting the upper nodes of the beams: 1 – basic elements of beams; 2 – overlap areas to be concreted; 3 – clamps for combining grids of adjacent beams

The most acceptable of the options discussed above is obviously the third option, because while the cheapest (from the point of view of steel consumption) is obtained and at the same time the most reliable way to combine overlapping into a single disk.

6. Determination of Rigidity in Torsion Beams Hollow Triangular Section in the Presence of Normal Cracks

Consider a method for determining the torsional rigidity of beams hollow triangular section.

In [4, 11, 5, 12], the local load redistribution depends substantially on the same manner as from a flexural and torsional rigidities of the individual elements. This dependence is significant. Consequently, the definition of flexural and torsional rigidities is an important and urgent problem.

Despite this, most calculations for the design of various structures, including well-known powerful program complexes, such as Ansys, Abacus, Lira, are carried out without taking into account the change in torsional rigidity in result from the formation of normal cracks.

When calculating the overlap consisting of a large number of beams, modeling each beam from the volume finite elements with the inclusion of reinforcement elements is very much laborious.

For today, there is very little research on the torsional rigidity of reinforced concrete elements with normal cracks. Most of the studies on torsion in reinforced concrete are devoted to the study of the strength of such elements. Existing methods for determining deformations in torsion [6, 7, 9, 10, 14] concern only reinforced concrete elements with spatial (spiral) cracks, although experimental studies have established a significant effect of normal cracks on the torsional rigidity of reinforced concrete elements [2, 4]. In works devoted to the investigation of the torsional rigidity of reinforced concrete elements with normal cracks [2], simple types of sections are considered: a rectangle with the symmetrical reinforcement, a ring, cylindrical elements.

The resistance to movement from torsion in the section with a crack is exerted by longitudinal reinforcement. To find the pinning force in the longitudinal armature, it should be conventionally cut and a pinning force at the center of gravity of the reinforcing bar on both sides of the conditional section, oppositely directed to each other. Then, from the condition of compatibility of deformations in the cut reinforcement, determine the required pinning force of reinforcement resistance by rotating the beam section in a normal crack.

The integral evaluation of the displacement in a crack, taking into account the factors of the element's torsional rigidity (the distance between the cracks, the height of the concrete zone above the crack, the geometry of the section), is the displacement from the action of the external torque M_t. It is transferred between blocks of the element with cracks through the height of the concrete section above the cracks (Figure 8).

The most difficult part of the problem is to determine the displacements of adjacent points separated by a crack (points C and C^I on Fig. 8). In the presence of a solution to this basic problem, the further determination of the torsional rigidity of a reinforced concrete element with a crack is not difficult.

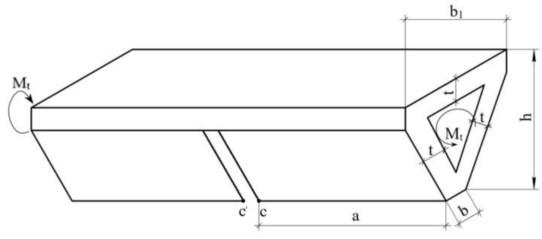

Fig. 8. The scheme of a hollow triangular section element with a normal crack

This problem can be solved both in an approximate way [2, 4], and with the use of the finite element method. One of the drawbacks of solving this problem, as mentioned above, is the condition of using a large number of volumetric finite elements, which complicates both the creation of the calculation scheme and analysis of the calculation results, especially since this is only part of the solution of the general problem of stress-strain state reinforced concrete element with normal torsional fractures [2]. The use of an elasticity theory solutions is possible not in all cases of the problem under consideration [2].

One of the ways to solve the problem of determining the displacement of crack fractured edges is as follows.

Calculations with the use of volume finite elements established that the mutual movement of the shores of the crack is a clear function of the height of the compressed zone, the height, and width of the beam section and the distance between the cracks. In this case, it is possible to obtain a formula approximating such a function.

So, for example, when the height of the compressed zone is equal to the thickness of the beam shelf of a hollow triangular section, the displacement of the crack sides (the angle of mutual rotation of the two blocks separated by a normal crack) will be a function of five variables (Figure 8):

$$\Delta=f(a,b_1,b_2,h,t). \quad (18)$$

where (Figure. 8): a – the distance between the cracks; b_1, b_2 – the width of the upper flange and the lower beam edge, respectively; h – beam height; t – thickness of plates forming a beam hollow triangular section.

Approximation of such a function of five variables can be carried out by any of the known methods, for example, by the method proposed in [8].

It should be noted that the entire database of approximation data can be obtained for specific values of the elasticity modulus E and shear modulus G of concrete. Considering that this problem is linear, to obtain displacements in an element with other values of the elasticity and shear modulus, it is easy to multiply them by the ratio of the corresponding parameters of the considered construction (or the corresponding iteration step in the nonlinear calculation) to the parameters given in the database.

If the height of the compressed zone also needs to change, then the function (18) will contain not five variables, but six, including the height of the compressed zone (the zone through which the torsion moment is transmitted from one block to another). But in any case, obtained once (although on the basis of a fairly complex set of data from a numerical experiment), this function can be used in design practice as many times as necessary.

Thus, having an instrument for determining the torsional rigidity of a reinforced concrete element with normal cracks, introducing it as one calculation block in the calculation programs, it is possible to calculate complexly repeatedly statically indeterminate systems (overlaps, bridges, etc.) with allowance for the effect of torsional rigidity on the redistribution of forces between individual elements of the system.

To obtain approximation functions of the type (18), a sufficiently large but finite number of calculations must be performed using software complexes in which the finite element method is implemented using volume finite elements (programs Ansys, Abacus, Lira, etc.). At first glance, this may seem complicated, but the advantage of this approach is obvious, because once obtained such dependencies can be used by designers and engineers as much as necessary in solving specific problems in the manner described above. The creation of a library of such approximation functions will make it possible to simplify considerably the solution of many complex problems, where the number of such elements would be much smaller than the number of finite elements using the traditional finite element method. This approach is acceptable not only for calculating the rigidity of beams hollow triangular section but also for the rigidity of elements with any cross-sectional shape. This is the advantage of the proposed approach.

7. Comparison of the Spatial Work of the Proposed and Traditional Overlaps

To compare the effect of the spatial work of overlaps consisting of beams (slabs) of a hollow triangular section and U-shaped slabs, calculations were carried out for two different overlapping fragments. For correctness of comparison, the width and thickness of the compressed shelf in both variants was adopted the same (from the point of view of the same load-bearing capacity of the reinforced concrete element for the action of the vertical load). The sections of the plates in two variants were adopted: in the form of a closed thin-walled triangle; in the form of a U-shaped thin-wall section.

The width of the plates is the same in both cases, equal to 1500 mm, height - 450 mm. The thickness *t* of all sides of the plate of the hollow triangular section and the shelf of the plate of the U-shaped section is taken equal to 50 mm. From the condition of concrete volume equality in both variants, for correctness of comparison, the thickness of the plate edges of the U-shaped section is assumed equal to 97 mm (from the condition that the sectional area of the slab longitudinal edge of the U-shaped section of the plate inclined rib hollow triangular section to be equal).

In both variants, a 7.5x12 meter overlapping cell consisting of five slabs with a width of 1.5 meters spanning 12 meters was considered. The loading by the local load of the second plate (left) and the middle plate was considered.

Due to the fact that the plate joint is adopted by welding embedded parts, the seams between the plates do not take transverse bending moments. Therefore, they were modeled in the form of cylindrical hinges, transmitting from the one plate to another only vertical forces. For this purpose, in the Lira calculation scheme, it was decided to combine linear displacements of adjacent nodes of two adjacent slabs.

For the comparison criterion, the following calculation results were accepted: maximum values of longitudinal forces N_x, N_y in the element of the shelf of the most stressed plate; maximum bending moments M_x, M_y in the shelf element; maximum deflection in the middle of overlap span $f_{l/2}$.

Table 1 shows the results of comparison of bending moments and deflections.

As can be seen from the table, the maximum bending moments in the elements of the plates shelves are more than two times different in the thickness of the overlap consisting of slabs hollow triangular section. The maximum deflections during the loading of the middle plate differ by 20 percent, and when the second plate on the left is loaded by 36 percent.

The overlap of the slabs of the hollow triangular section was also compared with the overlap of T-slabs. Moreover, the dimensions of the compressed shelves are assumed to be the same. The thickness of the edges of the T-plates is taken from the condition that the sectional area of the sloping edges of the slab hollow triangular section (equal to 194 mm) is equal to their sectional area. Table 2 shows the data for this calculation.

Table 1: Comparison of the calculating results for the overlap of five slabs with a width in 1500 mm of a hollow triangular (option # 1) and a U-shaped (option # 2) sections.

Parameter	Load on the first plate			Load on the second plate			Load on the third plate		
	#1	#2	#2/#1	#1	#2	#2/#1	#1	#2	#2/#1
M_x^{max} kHm/m	1,42	3,05	2,14	1,39	2,81	2,02	1,39	2,81	2,02
M_y^{max} kHm/m	1,36	3,34	2,45	1,4	2,2	1,57	1,39	1,74	1,25
M_{xy}^{max} kHm/m	0,99	4,25	4,26	0,82	3,79	4,62	0,78	3,66	4,67
$f_{\frac{l}{2}}^{max}$ (mm)	18,9	34,2	1,81	14,1	19,2	1,36	12,9	16,9	1,31

An even greater effect of spatial work is achieved if the width of the plates is taken equal to 750 mm. In this case, the thickness of all the sides of the plate hollow triangular section is 30 mm. The thickness of the shelf of the U-shaped plate is also equal to 30 mm, and the thickness of the longitudinal edges (also from the condition of equal volume of the concrete) is assumed equal to 39 mm.

Table 2: Comparison of the calculating results for the overlap of five slabs with a width in 1500 mm of a hollow triangular (option # 1) and T-shaped (option # 2) sections.

Parameter	Load on the first plate			Load on the second plate			Load on the third plate		
	#1	#2	#2/#1	#1	#2	#2/#1	#1	#2	#2/#1
M_x^{max} kHm/m	1,42	6,01	4,23	1,39	3,94	2,83	1,39	3,48	2,5
M_y^{max} kHm/m	1,36	6,59	4,84	1,4	4,24	3,02	1,39	3,07	2,21
M_{xy}^{max} kHm/m	0,99	13,8	13,8	0,82	17,2	20,9	0,78	16	20,4
$f_{\frac{l}{2}}^{max}$ (mm)	18,9	37,6	1,99	14,1	20,1	1,42	12,9	16,7	1,29

The above comparison shows that the use of beams (slabs) hollow triangular section substantially increases the effect of the spatial operation of overlaps that consist of such slabs. This is due to the fact that the torsional rigidity of such boards is substantially higher than the rigidity of the U-shaped or T-section slabs traditionally used in modern construction. The production of beams hollow triangular section in a prefabricated-monolithic variant proposed in this work makes it possible to significantly stanch and simplify the construction. In connection with the foregoing, the study of the stress-strain state of prefabricated monolithic reinforced concrete beams hollow triangular section, carried out in this article, is the solution of the actual task of the building science, as a result of which it becomes possible to use such beams in construction.

8. Conclusions and Prospects of Research

The method proposed in the article for calculating the overlap consisting of beams hollow triangular section makes it possible without the use of powerful programs to easily determine the interaction forces between individual beams. The method of calculating the beams for bending, based on the theory of composite rods, makes it possible to easily determine the forces between the shelf and the beams inclined edges. The algorithm for calculating the overlap, consisting of the beams proposed by the authors, allows performing a complete calculation of both the overlap and individual beams, taking into account the change in the rigidity from the formation in the cracks beams. An approach for determining the torsional rigidity of beams hollow triangular section with normal cracks, as a result of which it is possible to obtain a basis of approximation functions for the simple determination of the elements rigidity for any section with cracks within torsion is proposed. In turn, this base greatly facilitates the calculation of overlaps, taking into account the change in rigidity from the cracks formation. In the article, the principles of designing prefabricated-monolithic overlaps from the considered beams are also given, which makes it possible to create a hard disk of overlap. The comparison of overlaps consisting of the proposed beams and overlappings from traditional T-beams and U-beams (slabs) in the article shows that the maximum efforts and deflections in them are very different in favor of the proposed overlap. This allows recommending such overlaps in construction practice.

In the future, it is proposed to define functions of the type (18) for solving the reinforced concrete elements of a triangular section problems in torsion and their various sizes, extending the proposed approach to calculation taking into account the non-linear properties of reinforced concrete, and also developing a program for calculating overlaps using all the methods proposed in the article.

References

[1] ACI 318-11, Building Code Requirements for Structural Concrete and Commentary (ACI 318M-11), American Concrete Institute, Farmington Hills, 2011.

[2] Azizov, T. Effect of torsional rigidity of concrete elements with normal cracks onto special work of bridges and overlaps / T. Azizov // International science Ukrainian edition. – USA – December, 2010. – P.55-59.

[3] Azizov T.N. Ekonomichniye konstruktsii pokritiy I perekritiy / T.N. Azizov.- Uman: Almi, 2002. 64 p.

[4] Azizov T.N. Prostranstvennaya rabota zhelezobetonnyh perekrytij. Teoriya i metody rascheta [Spatial work of concrete overlaps. The theory and techniques for calculation], 2006.

[5] Baikov V.N. Precast reinforced concrete slabs under longitudinal strip loads // Jndian concrete Journal. – 1963. – August. – P.302-305

[6] Cowan H.J., Kruchenie v obychnom i predvaritel'no napriazhennom zhelezobetone [Torsion in the ordinary and prestressed reinforced concrete], Strojizdat, Moscow, 1972.

[7] EN 1992: Eurocode 2: Design of concrete structures – Part 1: General rules and rules for buildings. – Brussels, 2002. – 230 p.

[8] Kalenchuk-Porkhanova A.A. Paket programm approksimatsii funktsiy / A.A. Kalenchuk-Porkhanova // Kompyuterni zasobi, merezhi ta sistemi. — 2008. — № 7. — P. 32-38.

[9] Karpenko N.I. Obshiye modeli mekhaniki zhelezobetona / N.I. Karpenko. – Moskva, 1996. – 416 p.

[10] Luís L.F.A. Behaviour of concrete beams under torsion: NSC plain and hollow beams// Bernardo L.F.A., Sérgio M. R. Lopes. Materials and Structures. July 2008, Volume 41, Issue 6, pp. 1143-1167.

[11] M. Knauff, A. Golubińska, P. Knyziak, Tablice i wzory do projektowania konstrukcji żelbetowych z przykładami obliczeń, Wydanie II, Wydawnictwo Naukowe PWN, 2014.

[12] Pierre Lutrin, Christian Delvaux. Resultates de recherches sur des elements de plancher en beton precontraint pour betiment // Annales de l'institut Technique du Betiment et des Travaux Publics. – 1977. – Octobre. – № 354. – 54-74 p.

[13] Rzhanitsyn A.R. Sostavnyye sterzhni i plastinki / A.R. Rzhanitsyn. – Moskva: Stroyizdat, 1986. – 316 p.

[14] Wight, James K. Reinforced concrete : mechanics and design / James K. Wight, F.E. Richart, Jr., James G. Macgregor. - NJ, 2012. - 1157 p.