International Journal of Engineering & Technology, 7 (2.23) (2018) 328-331

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Plastic tension of porous plates due pure bending

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Abstract

The bending of the plate from rigid-plastic hardening material in the condition of plane deformation is investigated by analytical and numerical methods. Based on equations of the theory of plasticity of the porous materials the solution of the problem is lead to integration of the nonlinear differential equation regard to radial stress. The position of neutral axis is defined by numerical integration and the stress strain state components are received. Graphics of stress components and porosity changing are constructed on the height of the plate. The same problem simulated in ABAQUS software with the help finite element method. The results are compared.

Keywords: Bending; Plate; Stress; Porosity; Numerical Integration; FEM; ABAQUS

1. Introduction

The R. Hill was first, who discussed theory of plate bending for rigid-plastic materials in big deformation, when the lateral stresses caused by the curvature are influence. The position of neutral axe after bending is received [1-3]. Same problem with hardening is investigated in [4-6].The problem about bending plate in big deformation is discussed. Assumed, that material is isotropic, pores are spherical, and common (non-porous) material is behavior according to theory of plasticity of porous materials [7]. The stress components of the rigid-plastic material in general criteria of plasticity of Huber-Misses for porous material are defined. The numerical example is solved, the results are shown.
The manufactures from porous materials have wide application in different fields of industry, as load-bearing parts also. Theoretical and experimental works for plastic deformation of porous materials and solving practical problems of pressure treatment of powder materials are reviewed in [8]. During loading, the mechanical properties of porous material are changing. Researching of properties in deformation give good opportunity maximum use safety factors.

2. Problem description. basic equations

The aim of the paper is to propose a numerical method for determining the stress-strain state of a sintered body in the presence of porosity. The concept of a neutral axis is used, with respect to which the porosity of the material changes during bending: when bending, the porosity of the lower part of the plate decreases, the upper part increases. The plastic tension of the plate from homogeneous sintered porous material due pure bending is discussed. The plate is bending by the arc of a circle because of the external activity (Fig.1). In big deformation, when radius of curvature not less than 5 thickness of the plate because of big deformations, the σ_r andσ_z components are appear, and elastic deformations as compared with the plastic, may be neglected.

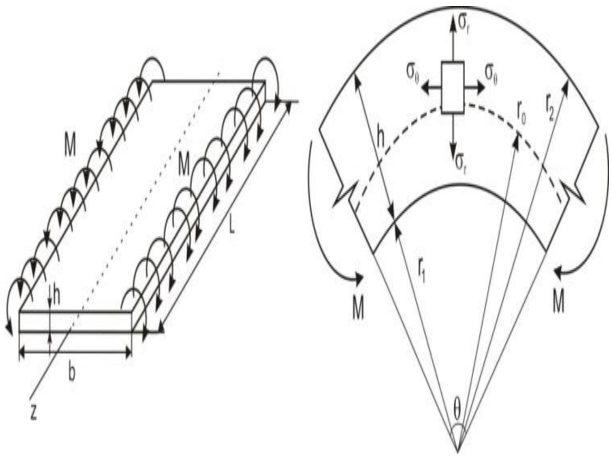

Fig. 1: Plate under Bending.

Basic equations are given in cylindrical coordinates system. The equilibrium differential equations have the following form:

$$\begin{cases} \frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\partial\sigma_{rz}}{\partial z} + \frac{\sigma_r-\sigma_\theta}{r} = 0 \\ \frac{\partial\tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + \frac{\partial\tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r} = 0 \\ \frac{\partial\tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{\theta z}}{\partial\theta} + \frac{\partial\sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0. \end{cases} \quad (1)$$

Generalized Huber-Mises'splasticity conditions for porous materials expressed through main stresses [1-9]:

$$(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2 + \\ +2\alpha^m(\sigma_1+\sigma_2+\sigma_3)^2 = 2\beta^{2n+1}{\sigma_Y}^2 \quad (2)$$

By the theory of flow of porous isotropic materials, stress-deformation relationship will be follow:

$$\begin{cases} d\varepsilon_1 = \frac{3}{2}\frac{d\lambda}{\beta^{3n}}(\sigma_1 - (1-2\alpha^m)\sigma_0) \\ d\varepsilon_2 = \frac{3}{2}\frac{d\lambda}{\beta^{3n}}(\sigma_2 - (1-2\alpha^m)\sigma_0) \\ d\varepsilon_3 = \frac{3}{2}\frac{d\lambda}{\beta^{3n}}(\sigma_3 - (1-2\alpha^m)\sigma_0). \end{cases} \quad (3)$$

where $d\lambda = \frac{d\varepsilon_{eq}}{\sigma_{eq}}$,

$$d\varepsilon_{eq} = \frac{\beta^{2n-0,5}}{3} \times \sqrt{2[(d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2]}, \quad (4)$$

$$\sigma_{eq} = \frac{1}{\sqrt{2}\beta^{n+0,5}} \times \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + 2\alpha^m(\sigma_1 + \sigma_2 + \sigma_3)^2},$$

σ_0– Average normal stress: $\sigma_0 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$,
$d\varepsilon_0$– Average deformation differences:

$$d\varepsilon_0 = \frac{1}{3}(d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3)$$

α, β– functions of porous materials, m – porous parameter, n - parameter, which reduce mechanical properties of porous material to properties of non-porous. m and n parameters are defined by experiment data on uniaxial compression of cylindrical examples from porous material.
From the relations (3) the condition of mass conservation we obtain:

$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = \frac{9d\lambda\, \alpha^m \sigma_0}{\beta^{3n}},$$

Or

$$\frac{dv}{1-v} = \frac{9\alpha^m d\lambda\, \sigma_0}{\beta^{3n}}, \quad (6)$$

wherevis current porosity of material.
Assumed, that for discussed porous material the experimental diagram $\sigma_{eq} - \int d\varepsilon_{eq}$ is known and approximated any$\sigma_{eq} = f(\varepsilon_{eq})$ function.
Tangential stresses $(\tau_{r\theta}, \tau_{rz}, \tau_{\theta z})$ are absent in pure bending, hence, σ_r,σ_θ and σ_z stress components are main, i.e. $\sigma_1 = \sigma_r, \sigma_2 = \sigma_\theta, \sigma_3 = \sigma_z$. As stresses and deformations are not depend on θ polar angle, so from equilibrium differential equations (1) will be received follow expression:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0\,, \quad (7)$$

So, the plate is deformed in plane strain conditions, i.e.$d\varepsilon_z = 0$.
From stress-deformation relation (3) we have:

$$\sigma_z = \frac{1-2\alpha^m}{2(1+\alpha^m)}(\sigma_r + \sigma_\theta) \quad (8)$$

From (4) is following, that

$$d\varepsilon_{eq} = \frac{\beta^{2n-0,5}}{3\alpha^{0,5m}}\sqrt{(1+4\alpha^m)(d\varepsilon_r^2 + d\varepsilon_\theta^2) + 2(1+4\alpha^m)d\varepsilon_r d\varepsilon_\theta}. \quad (9)$$

Substituting (8) into the (5), will be received follow formula:

$$\sigma_{eq} = \frac{\sqrt{3}}{2(1+\alpha^m)\beta^{n+0,5}} \times \sqrt{(1+5\alpha^m+4\alpha^{2m})(\sigma_r^2 + \sigma_\theta^2) - 2(1-\alpha^m-2\alpha^{2m})\sigma_r\sigma_\theta}. \quad (10)$$

From (6) follow, that

$$\frac{dv}{1-v} = \frac{9\alpha^m d\lambda\,(\sigma_r\sigma_\theta)}{2(1+\alpha^m)\beta^{3n}} \quad (11)$$

If material isrigid-plastic, then the problem is static defined and from (2), (7) and (8) equation systems the components of stresses σ_r,σ_θ and σ_z are determined.
Substituting (8) into the (2), after transformations will be received follow formula:

$$(1+5\alpha^m+4\alpha^{2m})\sigma_\theta^2 - 2(1-\alpha^m-2\alpha^{2m})\sigma_r\sigma_\theta + (1+5\alpha^m+4\alpha^{2m})\sigma_r^2 - \frac{4}{3}(1+\alpha^m)^2\beta^{2n+1}Y^2 = 0 \quad (12)$$

Solving (12) regardσ_θ, will be received:
In r∈[r1, r0]- stretched zone

$$\sigma_\theta = \frac{(1-\alpha^m-2\alpha^{2m})\sigma_r - \sqrt{A(\alpha)\beta^{2n+1}Y^2 - B(\alpha)\sigma_r^2}}{1+5\alpha^m+4\alpha^{2m}} \quad (13)$$

In∈[r0, r2]- compressed zone;

$$\sigma_\theta = \frac{(1-\alpha^m-2\alpha^{2m})\sigma_r + \sqrt{A(\alpha)\beta^{2n+1}Y^2 - B(\alpha)\sigma_r^2}}{1+5\alpha^m+4\alpha^{2m}} \quad (14)$$

Wherer0- radius of neutral layer.
Substituting expression of $\boldsymbol{\sigma_\theta}$to (7), we will received non-linear differential equation concerning$\boldsymbol{\sigma_r}$:

$$\frac{d\sigma_r}{dr} - \frac{2\alpha^m(3+2\alpha^m)\sigma_r \pm \sqrt{A(\alpha)\beta^{2n+1}Y^2 - B(\alpha)\sigma_r^2}}{(1+5\alpha^m+4\alpha^{2m})r} = 0 \quad (15)$$

Where

$$A(\alpha) = \frac{4}{3}(1 + 7\alpha^m + 7\alpha^{2m} + 13\alpha^{3m} + 4\alpha^{4m}) \quad (16)$$

$$B(\alpha) = 12\alpha^m(1+\alpha^m)^3 \quad (17)$$

Boundary conditions of the problem (Fig. 1) are follow:

In=r1,σr=0,

In r=r2,σr=0, (18)

In neutral layer (r=r0) we haveσr1=σr2. (19)

σr1=σr2condition is determinate position of neutral surface inr=r0.
The magnitude of the bending momentum per unit of width of the plate represented by $\boldsymbol{\sigma_\theta}$:

$$M = \int_{r_1}^{r_2} \sigma_\theta (r - r_0) dr. \quad (20)$$

Therefore, our problem reduces to integrate as non-linear differential equations (15) with boundary condition (18).
Note, that in big deformations current material v porosity stay unknown function from r, and complicates solution of the problem. It is might integrated equation (15) only applying numerical successive approximations method.
In particular case, for ideal rigid-plastic non-porous material ($\alpha = 0, \beta = 1$), equation (15) integrated and received known solutions [3].

3. Numerical example and results

Let discuss problem described above on the numerical method.
Let we have steel porous plate with the follow parameters: initial porosity v_0= 0.1. It contains additional Ni 4%, Cu 1.5%, Mo 0.5%,

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sintered in 20 min in the N_2 95 % and H_2 5% environment. Functions of porosity are $\alpha = v, \beta = 1 - v$. So, porosity parametersm $= 1.1$ $n = 1.4$, and $\sigma_Y = 630$MPa [9].

Plate divided on to many strips. By [4], two nearest each other strips with unit width step by step is investigated. Taking, that during the transition from the first to the second strain state increment of radial deformation $\boldsymbol{d\varepsilon_r}$and circumferential deformation$\boldsymbol{d\varepsilon_\theta}$can be considered small, and increment of circumferential deformation in transverse cross-section changes by the linear law (the hypothesis of plane sections):

$$d\varepsilon_\theta = y/r \tag{21}$$

wherey is the distance from neutral layer to current discussed layer. For this example the method of numerical integration, applied in [7], is used with one difference.Here integration starts from neutral layer, where porosity is equal to initial $\boldsymbol{v}_0$. In other layers the porosity is change because of deformation and is not known in advance. In the stretched and compressed zones numerical integration does separate by y starting from neutral layer. The calculation continuous by the successive approximations method till boundary conditions (18) will be satisfied.

Table 1: Components of the Stress-Deformation Tension in $\boldsymbol{v}_0$=0.1

r, m	σ_r, MPa	σ_θ, MPa	σ_z, MPa	$\rho = 1 - v$
0.3494	0.3569	525.7766	203.2506	0.1029
0.3474	-2.646	523.8829	201.3627	0.1029
0.3454	-5.6599	521.9841	199.4728	0.1029
0.3434	-8.685	520.0947	197.5911	0.1029
0.3414	-11.7214	518.2467	195.7408	0.1028
0.3394	-14.7695	516.5124	193.9734	0.1027
0.3374	-17.8303	515.0549	192.4054	0.1024
0.3354	-20.9056	514.248	191.3033	0.1016
0.3334	-24	515.046	191.3207	0.1
0.3334	-24	-545.6854	-221.9601	0.1
0.3314	-20.7286	-547.0716	-222.4447	0.0981
0.3294	-17.4342	-546.705	-221.596	0.0972
0.3274	-14.1215	-545.4285	-220.0714	0.0967
0.3254	-10.793	-543.7036	-218.2144	0.0965
0.3234	-7.4495	-541.7544	-216.1923	0.0964
0.3214	-4.0917	-539.6885	-214.0852	0.0964
0.3194	-0.7197	-537.5569	-211.9311	0.0964

On Fig. 2 and Fig.3 are constracted epuras for σ_r , σ_θ and σ_z stresses, and it is clear see the changing position of neutral layer (r_0=0.3334).

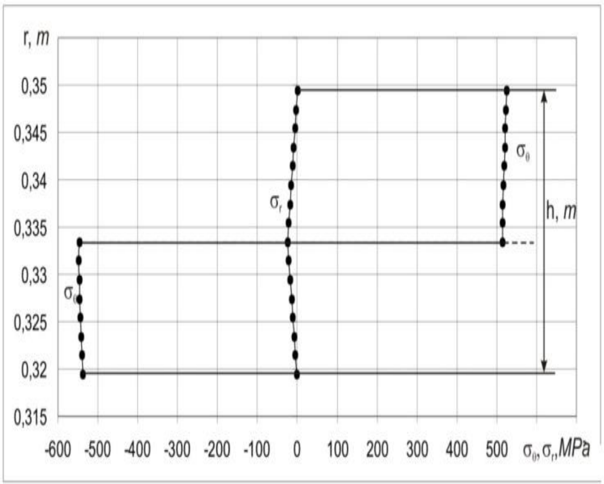

Fig. 2: Graphics of σ_r andσ_θStresses Depending Onh Height of the Plate.

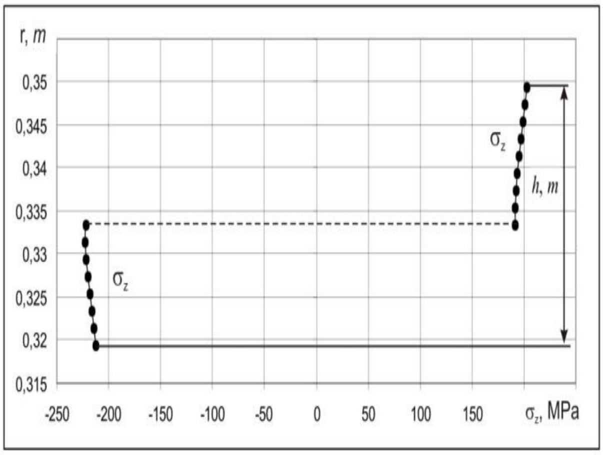

Fig. 3: Graphic of σ_zStress Depending on h Height of the Plate.

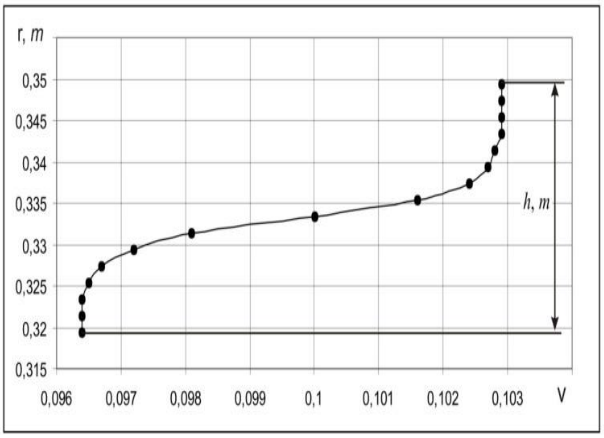

Fig. 4: Graphic for Variation of Porosity $\boldsymbol{v}$depend on h Height of the Plate.

On Fig. 4 is presented variation of porosity $\boldsymbol{v}$depending on h height of the plate.

4. FEM simulation

With the help ABAQUS finite element method software the stresses of the plate in case, when porosity of the plane is equal 0.1, is simulated and calculated.

Table 2: Properties of the Material

Density, kg*m^{-3}	Young modulus, Pa	Poison ratio	Yeld stress	Plastic strain	Porosity
7872	210·10^6	0.29	200·10^6 ÷480·10^6	0 ÷ 0.18	0.1

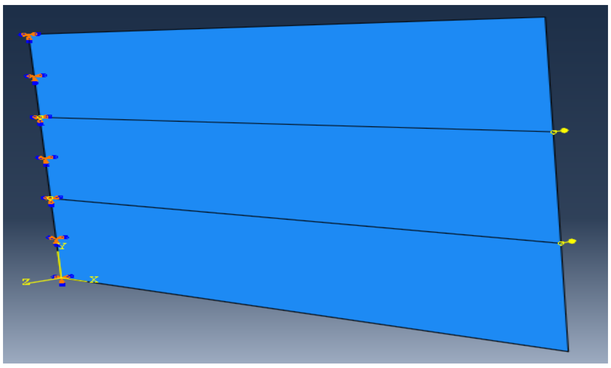

Fig. 5: Simulated in ABAQUS Model with the Boundary Conditions.

Initially, the plate with the sizes 30.0sm x 50.0 sm and 3 sm was created. Follow properties are used (Table 2).

Then, one side of the plane is fixed, the bending moment from the oposite side is applied. As result follow von Misses stresses are received (Fig. 6):

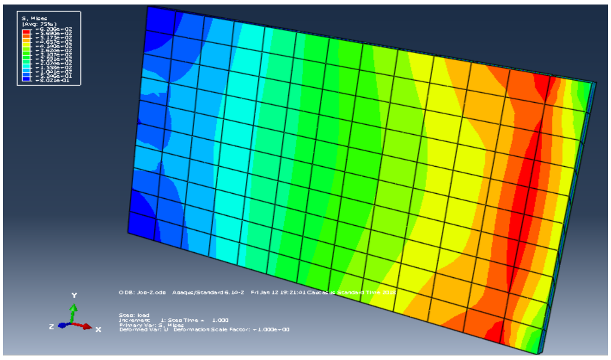

Fig. 6: Von Misses Stresses of the Porous Plane.

For analytical results compare with the simulation results we do follow. From the Table 1 first line results are input to the equation (5).

As Results, for Neutral Layer in (From Table 1)

r, m	σ_r, MPa	σ_θ, MPa	σ_z, MPa	$\rho = 1 - v$
0.3334	-24	515.046	191.3207	0.1
0.3334	-24	-545.6854	-221.9601	0.1

we have $\sigma = 620.332$ MPa.

As shown on Fig. 6, we have $\sigma = 620.602$ MPa .

5. Discussion

The problem of bending plate with porosity is solved.In numerical method for non-porous (usual) materials integration is started from lower surface, while for porous materials integration is started from neutral layer with integration step $\Delta r = 2mm$. In case of porosity$\mathbf{v}_{0=}$ 0.1 choosing increasing of deformation $\Delta\varepsilon_{eq_0} = 0.005$, all components of stress-deformation tension are received with the help of successive approximations method: the neutral layer radius ($r_0 = 0.3334$ m), internal radius ($r_1 = 0.3194$ m), outer radius ($r_2 = 0.3494$ m).

From the received results are follow, that

1) The graphics of radial and circumferential stresses have same form as in non-porous (usual) materials. σ_θIs maximum in upper and lower surfaces of the plate, σ_r is maximum on neutral layer (max $|\sigma_r| = 24$ MPa).
2) The relative density ($\rho = 1 - v$) in compressed and stretched zones accordingly are 0.0964 and 0.1029.
3) Bending problem for porous sintered material simulated in ABAQUS software and for neutral layer received the same results as analytical.

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