International Journal of Engineering & Technology, 7 (2.23) (2018) 228-231

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Heat loss analysis in a radiating slab of variable thermal conductivity

Ramoshweu S. Lebelo[1]*, Kholeka C. Moloi[2]

[1]*Department of Education, Vaal University of Technology, Vanderbijlpark, 1911, South Africa*
[2]*Department of Education, Vaal University of Technology, Vanderbijlpark, 1911, South Africa*
**Corresponding author E-mail: sollyl@vut.ac.za*

Abstract

This article investigates the transfer of heat in a stockpile of reactive materials, that is assumed to lose heat to the environment by radiation. The study is modeled in a rectangular slab whose materials are of variable thermal conductivity. The stockpile's reactive material in this context is one that readily reacts with the oxygen trapped within the stockpile due to exothermic chemical reaction. The study of the combustion process in this case is conducted theoretically by using the Mathematical approach. The differential equation governing the problem is tackled numerically by applying the Runge-Kutta Fehlberg (RKF45) method coupled with the Shooting technique. To investigate the heat transfer phenomena, some kinetic parameters embedded in the governing differential equation, are varied to observe the behavior of the temperature profiles during the combustion process. The results obtained from the temperature profiles, are depicted graphically and discussed accordingly. It was discovered that kinetic phenomena such as the reaction rate parameter, accelerates the exothermic chemical reaction. However, the radiation parameter decelerates the exothermic chemical reaction by lowering the temperature profiles.

Keywords: *Heat Transfer; Numerical Methods; Radiation; Reactive Slab; Variable Thermal Conductivity.*

1. Introduction

The study of spontaneous combustion in neglected stockpiles of reactive materials has stimulated interest of many researchers. Examples of stockpiles that spontaneously combust include those of coal, hay and wood. One of the previous studies indicated that some of the veld fires are caused by spontaneous combustion of stockpiles of reactive materials, such as the dumped rubbish in the open veld [1]. A reactive material is one that contains carbon or hydrocarbon component that readily reacts with the oxygen trapped within the stockpile, due to low-temperature oxidation reaction, also called the exothermic chemical reaction [2,3]. Previous studies indicated that about 80% of the greenhouse gases emitted to the atmosphere, which affect the climate negatively and increase the global warming, are due to the exothermic chemical reaction in stockpiles of reactive materials [4,5]. Spontaneous combustion due to exothermic chemical reaction produces heat as one of the products. If the heat produced cannot escape the stockpile, the heat will keep on accumulating to increase the temperature of the system to level where the temperature is sufficient to lead to self-ignition of the materials [6,7,8]. Thus, control measures of the stockpiles should be ensured, especially in the municipality dumping areas, to avoid self-ignition of the materials, that may lead to uncontrollable fires that may be hazardous to faunal and flora [9]. The significance of this study is to simplify the understanding of the complicated combustion process, by using the mathematical approach that considers the kinetic parameters embedded in the governing differential equation. The challenge of this theoretical approach is that one should have thorough knowledge and understanding of the thermo-physical parameters associated with the combustion process [10]. The combustion process results through nonlinear interactions of the radicals, resultantly, the differential equation governing the problem is also nonlinear and the exact solution thereof cannot be found, hence, the numerical approach to the solution is employed [11]. In this case, the RKF45 coupled with the Shooting technique were used to solve the governing equation numerically, using the Maple software. The study of heat transfer is significant in the manufacturing of industrial appliances. Some of the applications are in the combustion of solids, biomass and coal pyrolysis, cellulose materials storage, for example [12]. The studies of heat transfer and its stability in a reactive slab were conducted in [13,14,15] where the materials constant thermal was considered. In all the studies carried out, the combustion in a radiating slab with variable thermal conductivity was neglected. The mathematical formulation of the problem and the results obtained graphically are outlined in the sections that follow.

2. Mathematical formulation

The complicated combustion process is simplified by assuming a one-step irreversible chemical reaction represented by the following equation

$$\mathrm{C}_i\mathrm{H}_j + \left(i + \frac{j}{4}\right)\mathrm{O}_2 \rightarrow i\mathrm{CO}_2 + \left(\frac{j}{2}\right)\mathrm{H}_2\mathrm{O} + \text{heat}. \quad (1)$$

The slab is also assumed to lose heat to the surrounding environment by radiation. According to the Stefan-Boltzmann's law, an object's heat loss by radiation is expressed as $q = \mu\sigma(T^4 - T_0^4)$, where, T is the slab's absolute temperature, T_0 is the temperature of the environment, μ is the object's emissivity such that $0 < \mu < 1$, and, σ is the Stefan-Boltzmann constant, approximated as

$5.6703\times 10^{-8} W/m^2K^4$. The physical set-up of the problem is illustrated in Fig. 1 below

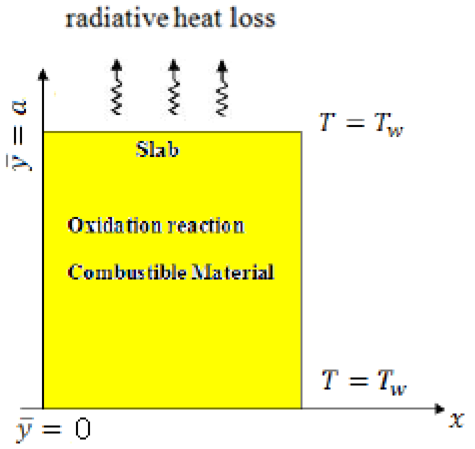

Fig. 1: The Geometry of the Problem

The ordinary differential equation governing the problem is given as [13,14,15]

$$k\frac{d^2T}{d\bar{y}^2} + Q\boldsymbol{A}\left(\frac{KT}{vl}\right)^m exp\left(-\frac{E}{RT}\right) - \mu\sigma(T^4 - T_0^4) = 0, \quad (2)$$

whose boundary conditions are

$$\bar{y} = 0; \qquad T(0) = T_0 \quad (3)$$

$$\bar{y} = a; \qquad T(a) = T_0. \quad (4)$$

The parameters are described as follows: k is the thermal conductivity of the slab, which is temperature dependent, and it is expressed as $k = \tau e^{b(T-T_0)}$, where τ is the slab's thermal conductivity at the ambient temperature T_0, b is the thermal conductivity variation parameter, Q is the heat of reaction, and, $\boldsymbol{A}$ the rate constant. K is the Boltzmann's constant, v is the vibration frequency, l is the Planck's number, E is the activation energy, and, R is the universal gas constant. The kinetic type parameter is given as m, which is assigned the following values, -2 for sensitized kinetics, 0 for Arrhenius kinetics, and 0.5 for bimolecular kinetics. Moreover, $\bar{y}$ is the slab's width.

The dimensionless parameters are introduced, to modify the governing differential equation into the form that is suitable to solve when the numerical methods are applied. The introduction of the dimensionless parameters is done as follows:

$$\theta = \frac{E(T-T_0)}{RT_0^2}, \beta = \left(\frac{vl}{KT_0^2}\right)^m \frac{b\tau RT_0^2}{QE^2Aa^2} exp\left(\frac{E}{RT_\infty}\right), \ y = \frac{\bar{y}}{a}, \quad \varepsilon = \frac{RT_0}{E},$$
$$Ra = \frac{\mu\sigma E a^2 T_0^2}{kR}, \lambda = \left(\frac{KT_0}{vl}\right)^m \frac{QAEa^2(C_w)^n}{kRT_0^2} exp\left(-\frac{E}{RT_0}\right). \quad (5)$$

The dimensionless parameters are used in Eqs. (2) – (4) to give

$$\frac{d^2\theta}{dy^2} + \beta\lambda\left(\frac{d\theta}{dy}\right)^2 + \lambda(1+\varepsilon\theta)^m e^{\theta/(1+\varepsilon\theta)} e^{-\lambda\beta\theta} - Ra((\varepsilon\theta e^{-\lambda\beta\theta} + 1)^4 - 1) = 0, \quad (6)$$

with the boundary conditions

$$y = 0; \qquad \theta(0) = 0 \quad (7)$$
$$y = 1; \qquad \theta(1) = 0. \quad (8)$$

In this case, θ is the dimensionless temperature, λ is the Frank-Kamenetskii (reaction rate) parameter, β is the thermal conductivity variation parameter, ε is the activation energy parameter, y is the slab's width, and Ra is the radiation parameter.

2.1. Numerical approach

The RKF45 coupled with the Shooting technique, which are embedded within any Mathematical software, and in this case Maple was applied, were used to solve the dimensionless Eqs. (6) – (8). The technique applied reduces the higher orders of the ordinary differential equations into the first order form. The algorithm followed is, $\theta = p_1$, $\theta' = p_2$. The Eqs. (6) – (8) are then modified to

$$p'_1 = p_2$$

$$p'_2 = -\beta\lambda(p_2)^2 - \lambda(1+\varepsilon p_1)^m e^{p_1/(1+\varepsilon p_1)} e^{-\lambda\beta p_1} + Ra((\varepsilon p_1 e^{-\lambda\beta p_1} + 1)^4 - 1) \quad (9)$$

subject to the conditions

$$p_2(0) = 0, \ p_2(1) = 0. \quad (10)$$

3. Results and discussion

In this section, the numerical solutions to the coupled Eqs. (6) – (8) are given graphically to illustrate the effects of the kinetic parameters' variations on the temperature behavior of the combustion process. The following parameters values were applied, with minor changes in some cases:
$\lambda = 0.1, m = 0.5, \beta = 0.1, \varepsilon = 0.1$, and $Ra = 1$.

3.1. Effects of the parameters on the temperature

The variation of the parameters and their effects on the temperature behavior are illustrated in Figs. 2 – 6. From Figs 2 and 3 we observe that the increase in the reaction parameter (λ) and the kinetics type parameter (m), show a corresponding increase in the profiles of the temperature. The increase in the temperature profiles means that the exothermic chemical reaction is accelerated to enhance the combustion process that progressively produce heat. As more heat is accumulated, the temperature of the system is increased, and if the heat release to the surrounding environment is not enough, the temperature elevation may come to a level where the stockpile can self-ignite. In other words, the higher levels of the said parameters lead to the thermal instability condition. A different scenario is observed from Figs. 4 to 6, where an increase in the activation energy parameter (ε), the radiation parameter (Ra), and the thermal conductivity variation parameter (β), show a decrease in the temperature profiles. The implications towards this scenario is that the higher values of the parameters under discussion, decelerate the rate of the exothermic chemical reaction. The deceleration of the exothermic chemical reactions means that the combustion process is slowed down to release less heat, and hence the decline in the temperature profiles. The slowing down of the combustion process also means that the emission of the greenhouse gases is reduced, which is advantageous to the environment.

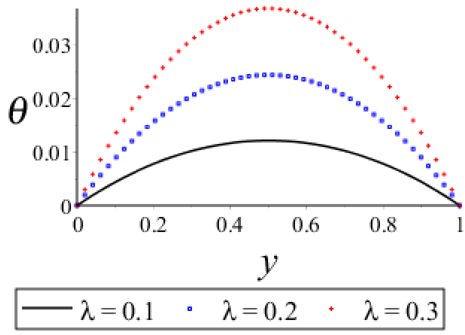

Fig. 2: λ effect on temperature

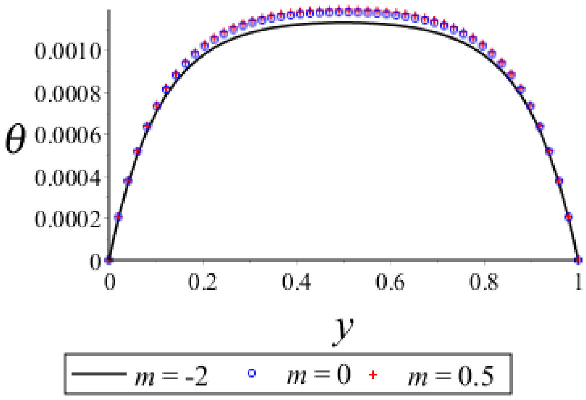

Fig. 3: *m* effect on temperature

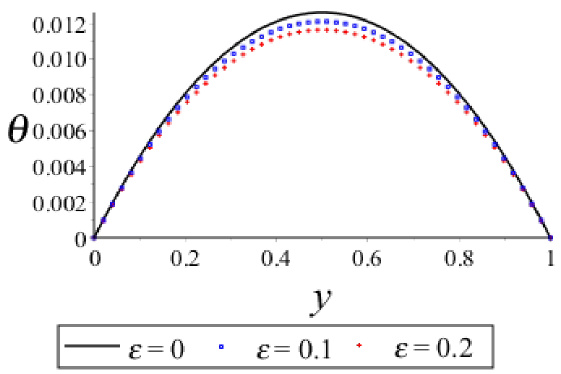

Fig. 4: ε effect on temperature

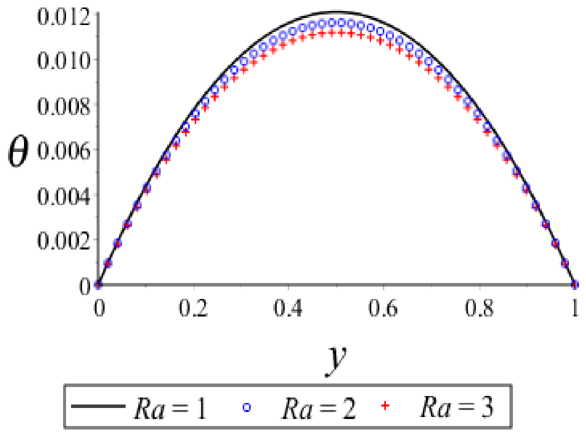

Fig. 5: *Ra* effect on temperature

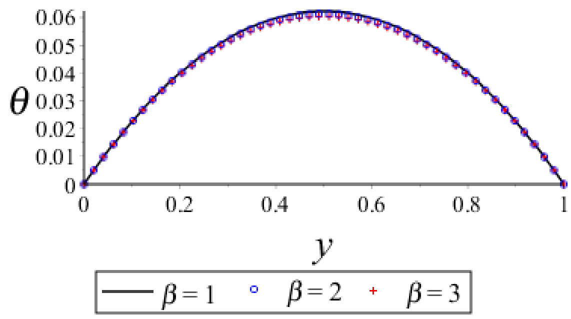

Fig. 6: β effect on temperature

4. Conclusion

The analysis of heat transfer in a reactive slab of variable thermal conductivity that loses heat to the surrounding by radiation, was conducted in this article. The study executed theoretically by using the mathematical approach only, is more advantageous, compared to the experimental one, because the theoretical approach facilitates the understanding of the complicated combustion process in a quicker, cheaper and hazardless manner. From the investigation done, it was discovered that the higher values of the rate of reaction and the kinetics type parameters, accelerate the exothermic chemical reaction to enhance the spontaneous combustion process that may lead to self-ignition of the reactive materials. If there is no proper management of the stockpiles of reactive materials, the self-ignited stockpiles may cause fires that can destroy fauna and flora. This study was able to show that phenomena such as the activation energy parameter, the radiation parameter and the thermal conductivity variation parameter, help to reduce the tendency of the reactive materials in stockpiles to self-ignite. This understanding of the process of combustion is necessary to help the environmental engineers to manufacture products that can be used on stockpiles of reactive materials to minimize the self-ignition process. This study can be extended to two-step combustion processes, such as the fuel combustion in automobiles.

Acknowledgement

The authors would like to thank the Vaal University of Technology for availing facilities to produce this work.

References

[1] Lebelo RS (2016), Thermal stability investigation in a reactive sphere of combustible material. *Advances in Mathematical Physics* 2016, 1 – 9.

[2] Monazam ER, Shadle LJ & Shamsi A (1998), Spontaneous combustion of char stockpiles. *Energy & Fuels* 12, 1305-1312.

[3] Kaji R, Hishinuma Y & Nakamura Y (1985), Low temperature oxidation of coals. Effects of pore structure and coal composition. *Fuel* 64, 297-302.

[4] Quick JC & Glick DC (2000), Carbon dioxide from coal combustion: variation with rank of US coal. *Fuel* 79(7), 803–812.

[5] Makinde OD, Chinyoka T & Lebelo RS (2011), Numerical investigation into CO2 emission, O2 depletion and thermal decomposition in a reacting slab. *Mathematical Problems in Engineering* 2011, 1 – 19.

[6] Unal S (1995), A review of spontaneous combustion of coals. *Fuel Science and Technology International* 13(9), 1995, 1103-1120.

[7] Lohrer C, Krause U & Steinbach J (2005), Self-ignition of combustible bulk materials under various ambient conditions. *Process Safety and Environmental Protection* 8, 145–150.

[8] Hensel W, Krause U & Loffler U (2004), Self-ignition of materials (including dust). In: Hartwig M and Steen H (eds) Handbook of explosion prevention and protection. *Weinheim: Wiley-VCH* 2.7, 227–270.

[9] Nyamadzawo G, Gwenzi W, Kanda1 A, Kundhlande A & Masona C (2013), Understanding the Causes, Socio-economic and Environmental Impacts, and Management of Veld Fires in Tropical Zimbabwe. *Fire Science Reviews* 2(2), 1 – 13.

[10] Lebelo RS, Mahlobo RK, Adesanya SO & Muthuvalu MS, "On Heat Transfer Analysis for a Sphere of Combustible Material of Variable Thermal Conductivity" *IOP Conference Series: Materials Science and Engineering,* 269, 2017, 1 – 6, http://dx doi:10.1088/1757-899X/269/1/012062.

[11] Lebelo RS (2017), Thermal conductivity impact on thermal stability of reactive materials. *Diffusion Foundations* 11, 1-10.

[12] Makinde OD (2012), Hermite–Pade_ approach to thermal stability of reacting masses in a slab with asymmetric convective cooling. *Journal of Franklin Institute* 349, 957 – 965.

[13] Hamza BH, Massawe ES & Makinde OD (2011), Analysis of transient heating due to exothermic reaction in a stockpile of combustible material. International Journal of Physical Sciences 6(18), No 18, 4337 – 4341.

[14] Legodi AMK & Makinde OD (2011), A numerical study of steady state exothermic reaction in a slab with convective boundary conditions. *International Journal of the Physical Sciences* 6(10), 2541-2549.

[15] Lebelo RS & Makinde OD (2015), Modelling the impact of radiative heat loss on CO2 emission, O2 depletion and thermal stability in a seactive Slab. *IJST, Transactions of Mechanical Engineering* 39(M2), 351-365.