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Research paper



Indentation of rigid rough surface into polymer coating layer

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Abstract

An analysis of engineering approaches to determining the value of the introduction of spherical asperity and a rough surface into the polymer layer is given. It is shown that engineering methods of solving contact problems on the basis of simplifying hypotheses are more practical, for example, the representation of a layered body as a construction with special mechanical properties that depend on the mechanical properties of base and coating materials and the thickness of the coating. Analysis of the use of the proposed engineering solutions in calculating the value of the indentation of the sphere showed the advantages of the method based on the rigid model of a layered body. In this case, the effective modulus of elasticity and the Poisson's ratio are determined for any values of the coating thickness for an axisymmetric loading of a layered half-space. To determine the value of the indentation of a rough surface into the polymer layer, a discrete model of a rough surface is used. When contacting a rough surface through a polymer layer, it was taken into account that each asperity corresponds to a certain modulus of elasticity, which is determined by the level of the peak and the value of approach.

Keywords: *polymeric coating, layered half-space, variable modulus of elasticity, spherical asperity, indentation of sphere, rough surface.*

1. Introduction

In the sealing technology, metal-polymer seals are widely used if the temperature range permits [1-3]. In this case, it becomes necessary to solve a number of complex problems associated with ensuring minimum pressure on the sealant while maintaining a predetermined tightness of the joint, a minimum coefficient of friction of couplings operating over a wide temperature range, under conditions of increased media pressure and vacuum. One of the promising directions that increase the efficiency of seals and friction units is the application of polymer coatings on their working surfaces or the use of thin polymer films [3]. Experience in the operation of seals and friction units with such coatings shows that their performance is determined not only by the properties of the coating material, but also by its thickness.

The presence of a coating involves taking into account the change in mechanical properties as a function of the distance to the surface. Within the framework of the theory of elasticity, this means that we must consider an elastic body with varying values of the elastic modulus and Poisson's ratio [4]. Contact problems for bodies with mechanical properties that vary in depth have been examined by many researchers [3-7, etc.]. According to the author [4], research methods can be divided into three groups: analytical, numerical and numerically-analytical. However, it is not possible to apply the results obtained to solve practical problems of friction, wear and tightness. An exception is the paper [3], in which the solution of the spatial axisymmetric problem is given in a form accessible for practical use. In this regard, we should also to note [8], in which an approximate solution of the axisymmetric contact problem is given for an elastic layer of finite thickness.

Engineering methods for solving contact problems on the basis of simplifying hypotheses, for example, the representation of a layered body as a topocomposite-constructions with special mechanical properties, depending on the mechanical properties of the base and coating materials, the thickness of the coating, should be included in a separate group. In [9], it was proposed to use the Hertz theory for this purpose. On the basis of reliable results for the extreme values of the coating thickness and using the two-point Padé approximation, an expression is obtained for the dimensionless elastic geometric parameter by means of which the elastic constant of the topocomposite and all the main characteristics are determined for its axisymmetric loading.

The authors of [10-12], with the development of the method on the basis of the stiffness model of a layered body, determined the effective modulus of elasticity and the Poisson's ratio for any values of the coating thickness for an axisymmetric loading of a layered semi-space. A classical approach based on the use of potential Boussinescu functions was used to move any point along the symmetry axis into a homogeneous half-space. Using the achievements of previous works [10-12], the author of [13] proposed a simplified model of a layered body and it is an alternative and simpler engineering solution.

The purpose of this paper is to use the solution for a particular spherical asperity to determine the indentation of a rigid rough surface into the polymer layer.

2. Analysis of engineering approaches to determining the indentation of a sphere into the polymer layer

Consider a sphere of radius R with a layered half-space consisting of a base with elastic modulus E_0 and a Poisson's ratio ν_0 and a coating with a thickness δ of elastic modulus E_0 and a Poisson's

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ratio ν_1. When the sphere is indentated by value w, a contact area of radius a is formed. The contact diagram is shown in Fig. 1.

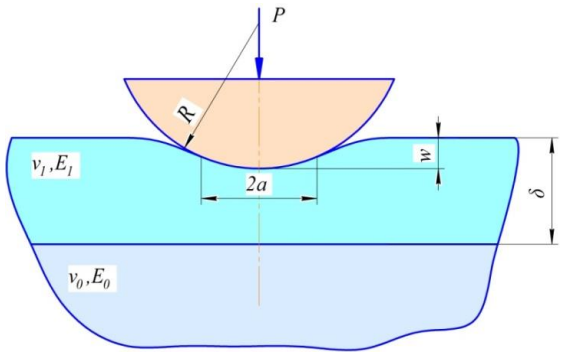


Fig. 1: Scheme of contact of a sphere with a layered half-space

According to the approximate solution of the problem of the introduction of a sphere of radius R into an elastic layer of thickness δ lying on a rigid base [8], we have

$$P=\frac{8\theta a^3}{3R}+\frac{16\theta\delta^2 a}{\pi}\cdot\frac{N_3(a/\delta)}{1-N_1(a/\delta)}; \quad (1)$$

$$w=\frac{a^2}{R}-\frac{4\delta a}{R}\cdot\frac{N_2(a/\delta)}{1-N_1(a/\delta)}, \quad (2)$$

where P is the force applied to the sphere; w is the amount of penetration; a is the radius of the contact area; $\theta=E_1\Big/[2\left(1-\nu_1^2\right)]$, E and ν are the modulus of elasticity and the Poisson's ratio of the elastic layer.

We introduce the notation: $\alpha=a/\delta$, $\bar{\delta}=\delta/R$, $\bar{w}=w/\delta$, $E^*=E/\left(1-\nu^2\right)$, $\bar{P}=P/\left(E^*R^2\right)$ and represent the expressions (1) and (2) in the form:

$$\bar{P}(\alpha,\bar{\delta})=\frac{4}{3}\alpha^3\bar{\delta}^3+\frac{8}{\pi}\alpha\bar{\delta}^3\cdot\frac{N_3(\alpha)}{1-N_1(\alpha)}, \quad (3)$$

$$\bar{w}(\alpha,\bar{\delta})=\alpha^2\bar{\delta}-\frac{4}{\pi}\alpha\bar{\delta}\cdot\frac{N_2(\alpha)}{1-N_1(\alpha)}, \quad (4)$$

$N_i(\alpha)$ is functions given in [8], are defined by the expressions:

$$N_1(\alpha)=\frac{2}{\pi}\cdot\int_0^\infty[1-L(u)]\cdot Si(\alpha u)\cdot\left(\frac{\sin\alpha u}{\alpha u}-\cos\alpha u\right)\cdot\frac{du}{u};$$

$$N_2(\alpha)=\int_0^\infty[1-L(u)]\cdot\left(\frac{\sin\alpha u}{\alpha u}-\cos\alpha u\right)\cdot\frac{du}{u^2};$$

$$N_3(\alpha)=\int_0^\infty[1-L(u)]\cdot\left(\frac{\sin\alpha u}{\alpha u}-\cos\alpha u\right)\cdot\sin\alpha u\cdot\frac{du}{u^3};$$

$$L(u)=\frac{2k\cdot sh(2u)-4u}{2k\cdot sh(2u)+1+k^2+4u^2};\quad k=3-4\nu;$$

$$Si(x)=\int_0^x\frac{\sin t}{t}dt.$$

We define analogous dependences for the indentation of a sphere into the stratified half-space, using the engineering solution from [11, 13].

The effective modulus of elasticity is determined by equations

$$E_{01}^*=E_1^*\cdot F; \quad (5)$$

$$F=F(\bar{z},I_e)=\frac{\pi/2}{\pi/2-K_1(\bar{z})+K_0(\bar{z})\cdot I_e}; \quad (6)$$

$$K_i(\bar{z})=arcctg\bar{z}+\frac{\nu_i}{1-\nu_i}\bar{z}(1-\bar{z}\cdot arcctg\bar{z}); \quad (7)$$

where $I_e=\frac{E_1^*}{E_0^*}=I\cdot\frac{1-\nu_0^2}{1-\nu_1^2}$, $I=\frac{E_1}{E_0}$; $\bar{z}=\frac{z}{a}$.

For the case of contact between a rigid sphere and a layered half-space, the approach and radius of the contact spot are determined by equations

$$w_{01}=w_1\cdot F_1^{-\frac{2}{3}},\quad a_{01}=a_1F^{-\frac{1}{3}}. \quad (8)$$

From the Hertz formula for the radius of the contact spot, we have

$$\bar{P}=\frac{P}{E^*R^2}=\frac{4}{3}\frac{a^3}{R^3}. \quad (9)$$

With taking into account the above notations and Eqs. (5), (6), (8) and (9), we have

$$\bar{P}(\alpha,\bar{\delta})=\frac{4}{3}\alpha^3\bar{\delta}^3F(\alpha,I_e),\quad \bar{w}(\alpha,\bar{\delta})=\alpha^2\bar{\delta}\cdot[F(\alpha,I_e)]^{-\frac{2}{3}} \quad (10)$$

where $\alpha=\bar{z}^{-1}=a/\delta$.

When F=1 Eq. (10) describes the contact of the sphere with a half-space of the coating material.

Let's consider other engineering approaches. Using the adopted notation, the Eq. (2.39) from [3, p. 69] can be represented as

$$F_1(\alpha,I_e)=\left(\frac{I_e^{1.33}+0.8\bar{z}(\alpha)}{\sqrt{1+(0.8\bar{z}(\alpha))^2}}\right)^{\frac{-3}{4}},\quad \bar{z}(\alpha)=\alpha^{-1}. \quad (11)$$

Similarly, from the expression (20) of [9], we have

$$F_1(\alpha,I_e)=I_e^{-1}\cdot\left[I_e^{-\frac{2}{3}}+\left(1-I_e^{-\frac{2}{3}}\right)\times\left(\frac{1+1.27\bar{z}(\alpha)^3I_e^{-1}-0.333\bar{z}(\alpha)^2I_e^{-\frac{2}{3}}-0.424\bar{z}(\alpha)\ I_e^{-\frac{1}{3}}}{1+2\bar{z}(\alpha)^4I_e^{-\frac{4}{3}}}\right)\right]^{\frac{-3}{2}} \quad (12)$$

Eqs. (11) and (12) are identical in their physical essence to Eq. (6) and determine the change in the effective modulus of elasticity, depending on the elastic properties of the coating and base materials, and also on the thickness of the coating.

In Fig. 2 shows the dependencies of relative penetration on the relative load: curve 1 corresponds to the Eq. (10); curve 2 to the Eqs. (3) and (4); curves 3 and 4 - according to the Eqs. (11) and (12); curve 5 corresponds to the introduction of a sphere into an elastic half-space of the coating material.

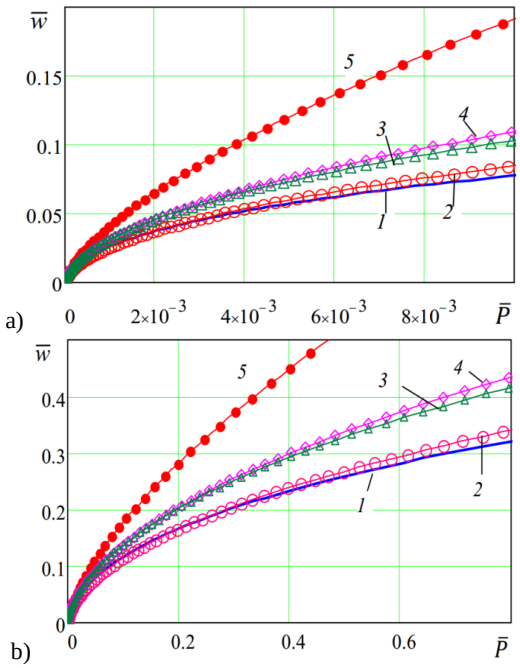


Fig. 2: Dependences of relative introduction of the relative load:
(a) $\bar{\delta}=0.2$; (b) $\bar{\delta}=1$.

3. Contact rough surfaces through a layer of polymer coating

Let us use the discrete model of a rough surface, in which microasperities are presented by identical spherical segments with the distribution of segments' peaks on height corresponds to the bearing profile curve of the real surface [14, 15]. To describe the distribution of the bearing curve is used incomplete beta function

$$\eta(\varepsilon)=\frac{B_{\varepsilon}(p,q)}{B(p,q)}, \tag{13}$$

where $B_{\varepsilon}(p,q)$ and $B(p,q)$ are incomplete and complete beta functions;

$$p=\left(\frac{R_p}{R_q}\right)^2\left(\frac{R_{\max}-R_p}{R_{\max}}\right)-\frac{R_p}{R_{\max}},\quad q=p\left(\frac{R_{\max}}{R_p}-1\right). \tag{14}$$

where R_p, R_q, $R_{\max}$ are height roughness parameters according to ISO 4281/1–1997.
In this case, the density of the asperities distribution on height function is

$$\varphi_n'(u)=\frac{u^{p-2}(1-u)^{q-2}\left[(p-1)(1-u)-(q-1)u\right]}{\varepsilon_s^{p-1}(1-\varepsilon_s)^{q-1}}, \tag{15}$$

where ε_s is determined from the condition $\varphi_n(\varepsilon_s)=1$ [14].
Geometrical parameters of a spherical segment are: the height is $\omega R_{\max}$, where $\omega=1-\varepsilon_s$; a_c is the radius of the base of the spherical segment.

$$R=\frac{a_c^2}{2\omega R_{\max}}, \tag{16}$$

where $R>>R_{\max}$.

When describing the bearing curve of the profile to the level of the midline with $\varepsilon_p=R_p/R_{\max}$ a parabola

$$\eta(\varepsilon)=t_p(\varepsilon)=b\varepsilon^{\nu}, \tag{17}$$

$$b=t_m\left(R_{\max}/R_p\right)^{\nu},\qquad \nu=2t_m\frac{R_p}{R_a}-1.$$

where $t_m=t_p(\varepsilon_p)$.
In this case, the function and density of the height distribution function of the asperities are described by expressions

$$\varphi_n(u)=\left(\frac{u}{\varepsilon_s}\right)^{\nu-1};\qquad \varphi_n'(u)=\frac{(\nu-1)u^{\nu-2}}{\varepsilon_s^{\nu-1}}. \tag{18}$$

To analyze the results obtained using different reference curves, it is important to know the relationships between the parameters p and q when describing the bearing curve of the profile with the incomplete beta function, and the parameters ν and b in the description of the bearing curve by the parabola, which are determined by the following the following expressions [14]:

$$\nu=\frac{B_{\varepsilon_p}(\alpha+1,\beta)}{\varepsilon_p B_{\varepsilon_p}(\alpha,\beta)-B_{\varepsilon_p}(\alpha+1,\beta)},\ b=\frac{B_{\varepsilon_p}(\alpha,\beta)}{B_{\varepsilon_p}(\alpha+1,\beta)}\left(\frac{\alpha+\beta}{\alpha}\right)^{\nu}, \tag{19}$$

here $\varepsilon_p=\frac{R_p}{R_{\max}}=\frac{\alpha}{\alpha+\beta}$.

The scheme of the interaction of a single asperity is shown in Fig. 1. Assuming that the distribution function of the asperities $\varphi_n(u)$ continuous, the number of asperities in the layer between the levels u and $u+du$

$$dn_r=n_c\varphi_n'(u)du \tag{20}$$

where $n_c=A_c/(\pi a_c^2)$, is the contour area.
Using the solution of the Hertz problem on the introduction of a sphere by an amount $h_i=R_{\max}(\varepsilon-u)$ in the elastic half-space, determine the necessary force for this N_i

$$N_i=\frac{4}{3}E_{01}^{*}R^{\frac{1}{2}}R_{\max}^{\frac{2}{3}}(\varepsilon-u)^{\frac{3}{2}} \tag{21}$$

Summing over all the asperities, we obtain

$$N=\frac{8}{3}n_c a_c\omega R_{\max}\int_0^{\min(\varepsilon,\varepsilon_s)}E_{01i}^{*}\left(\frac{\varepsilon-u}{2\omega}\right)^{\frac{3}{2}}\varphi_n'(u)du \tag{22}$$

Effective modulus of elasticity when contacting a particular asperity E_{01i}^{*} is defined similarly to Eqs. (5) – (7).
The amount $\bar{z}_i$ in the form:

$$\bar{z}_i=\frac{\delta}{a_i}=\frac{\delta}{a_c}\cdot\frac{a_c}{a_{ri}}=\gamma\cdot\eta_i^{-0.5} \tag{23}$$

where $\gamma=\delta/a_c$, $a_i\equiv a_{ri}$, $\eta_i=a_{ri}^2/a_c^2=\frac{\varepsilon-u}{2\omega}$.
Then

$$F_i = F_i(\gamma, \eta_i, I_e) = F_i(\gamma, \varepsilon, u, I_e) = \\ = \frac{\pi/2}{\pi/2 - K_1(\gamma, \varepsilon, u) + K_0(\gamma, \varepsilon, u) \cdot I_e} \quad (24)$$

Values $K_1(\gamma, \varepsilon, u)$ are calculated from Eq. (7) with allowance for (23). Substituting Eq. (24) into (22) and taking into account that $q_c = N/A_c$, we get

$$q_c = q_c(\gamma, \varepsilon, I_e) = \frac{8}{3\pi} \cdot \frac{\omega R_{\max}}{a_c} E_1^* \times \\ \times \int_0^{\min(\varepsilon, \varepsilon_S)} F_i(\gamma, \varepsilon, u, I_e) \cdot \left(\frac{\varepsilon - u}{2\omega} \right)^{\frac{3}{2}} \varphi_n'(u) du \ , \quad (25)$$

or in dimensionless form

$$F_{q1} = F_{q1}(\gamma, \varepsilon, I_e) = \frac{q_c a_c}{\omega R_{\max} E_1^*} = \\ = \frac{8}{3\pi} \cdot \int_0^{\min(\varepsilon, \varepsilon_S)} F_i(\gamma, \varepsilon, u, I_e) \cdot \left(\frac{\varepsilon - u}{2\omega} \right)^{\frac{3}{2}} \varphi_n'(u) du. \quad (26)$$

In Fig. 2 shows the relationship between the rough surface and the polymer layer as a function of the nominal pressure, calculated in the Mathcad in Eq. (25). The dots denote previously "digitized" experimental data given in [3, Fig. 3.14]. The following roughness parameters were used in the calculations: $R_{\max}$=8.25 *μm*, *R*=20 *μm*, *b* = 1.9, *ν* =1.59.

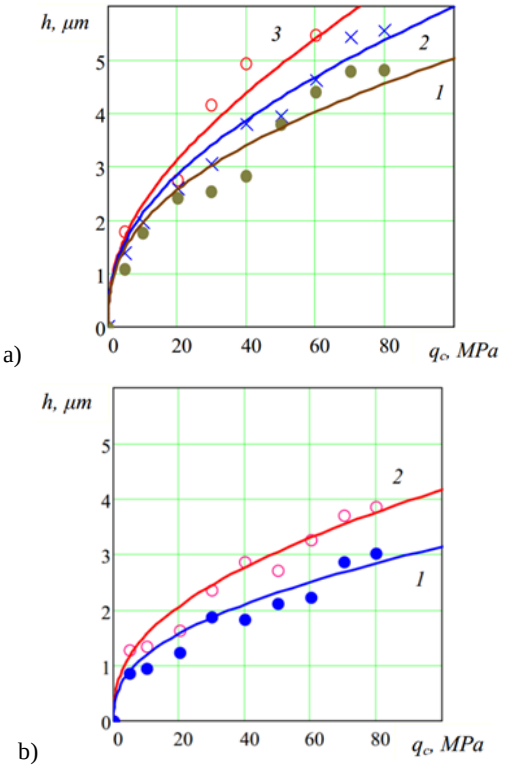


Fig. 3: Dependences of rough surface and polymer layer approach from the nominal pressure: a – fluoroplastic coating ФБФ-74Д (1 – at δ=12 μm; 2 – at δ=25 μm; 3 – bulk material); b – lavsan films δ=25 μm (curve 1) и polyamide PM-1, δ=45 μm (2).

4. The conclusion

1. Analysis of the proposed engineering solutions for the indentation of the sphere showed the advantages of the method based on the rigid model of a layered body. In this case, the effective modulus of elasticity and the Poisson's ratio are determined for any values of the coating thickness for an axisymmetric loading of a layered half-space.
2. A layered body with a polymeric coating is presented as a structure with special mechanical properties depending on the mechanical properties of the base and coating materials and the thickness of the coating.
3. When investigating the contact of a rough surface through a polymer layer, it should be taken into account that each asperity corresponds to a certain modulus of elasticity, which is determined by the level of the peak and the magnitude of the approach.
4. The proposed engineering solution for changing the effective modulus of elasticity is well combined with a discrete roughness model which allows to successfully determine any characteristics in the interaction of a rough surface with a layered body, for example, the relative contact area [16] or the gap density in the joint [17].
5. The obtained results shows high sensitivity of the proposed method to a change in the thickness of the coating and its mechanical properties. The results of calculations and experimental data on the indentation of a rough surface into polymer coatings are in satisfactory agreement.

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