# Making Unit Cost in Production Process More Accurate - the Role of Queueing 

Viktor Molnar *, Abel Tumik<br>University of Miskolc<br>*Corresponding author E-mail: szvmv@uni-miskolc.hu


#### Abstract

Queueing is a general phenomenon in the life of almost every organization. Not only people but also processes connecting to manufacturing, machine maintenance, food delivery etc. can be modeled by queueing theory. Queueing always contains waiting waste and the latest management approaches endeavor to eliminate all wastes from the system. This paper introduces and demonstrates a solution based on Activity-Based Costing that aids in the more accurate identification of wastes and therefore in more accurate costing. An experiment was conducted in which queueing of products in a warehouse was analyzed. The queueing as waiting time was built in the ABC costing model. The paper highlights that the model supports thorough business-as-usual decision-making.


Keywords: Activity-based costing; Lean management; Six sigma; Queueing

## 1. Introduction

General types of waste and their financial effects are difficult to analyze in practice because not only defective products or scrap but many activities that do not create value for customers can be considered as waste. One of the widely analyzed types of waste is 'waiting' of products. There are cost items connected to waiting: either a product is stored or it is waiting for the next phase of processing after one piece has finished. In addition, inventory costs and further types of alternative costs also appear. If, for example, no physical transformation (machining, painting, packaging, etc.) is made on a product within a given part of lead time, waiting time could be substituted with value creating time by applying more exact production scheduling. It has to be noted that more precise scheduling requires more working hours in planning activities, which is also a unit-cost-increasing extra item connected to the analyzed product (Molnar \& Tumik 2017).
Scheduling is a significant problem in industry and also in services; therefore several approaches for reduction or elimination of waiting time have been developed in the last decades. Lean management, whose main objective is the continuous reduction of waste, has been a cutting-edge management system for two decades. Although the intensity of application is not quite stable, the approach has started to appear in many industrial branches, from the automotive industry to healthcare. Molnar \& Kerchner (2016) give a comprehensive analysis of application areas and types of lean thinking and toolsets. Application of lean solutions has intensified also in public utility organizations recently.
Another direction which obviously started to merge with lean solutions is the approximately 30 -year-old six-sigma management system. The major focus in this approach is the reduction of the defect rate through well-structured and applied process management solutions.
Although both the lean and six-sigma systems use precise financial calculations for quantifying the efficiency of processes or their improvement, some connecting areas have also emerged
through the years to help in more exact calculations. One of them is the queueing theory, in which the expected value of waiting time is determined on the basis of probability theory models.
Costing of units is highly complicated due to the several types of waste, the problem that in several cases waiting can only be modeled with the use of probability variables, and the fact that production enterprises, servicers or public utility institutions produce or offer many different. Activity-Based Costing (ABC) is a solution for supporting exact unit cost calculations because it is easy to build as many items as needed into the calculations and it provides clear information on the efficiency of processes (Musinszki, 2016).
An ABC-based model is introduced in this paper. The waiting time of raw material as an unnecessary cost element that could have been avoided by Just in Time (JIT) shipment was considered in the model. The model is supplemented by a solution which deals with the stochastic nature of a buffer defined by inventory whose objective is preparing for fluctuation in the customer's demand. Shipment is considered as arbitrary, i.e. as a Poissondistribution probability variable.

## 2. Literature review

### 2.1. Types of waste

According to the lean approach, waste has three typical forms (widely referred to as the 3 MU model): muda - an activity or process without added value; mura - inconsistence or unevenness of workload; muri - overburden or unnecessary stress.
The main task in lean management and process improvement is the elimination of non-value adding operations and the other two types of waste. In order to remove mura, for example JIT production, levelling (Heijunka) or Kanban techniques are recommended so that production or resource use can be smooth in time. Muri is not only the overburden of resources but intensive stress of employees. Pienkowski (2014) summarizes the major
types of waste from lean's point of view, giving the logical connections between the types and introducing useful metrics for measuring these types exactly. Another widely used model is the so-called 7 wastes of lean or TIMWOOD from the initial letters of the types. This collection summarizes muda-type wastes (Meran et al. 2013).
Many publications give useful practical advices for reduction of wastes (e.g. Tamas, 2017; Konyha \& Banyai, 2017). In the last decade several special organizations started to use the lean or six sigma approaches, e.g. hospitals (Cima et al. 2011; Dickson et al. 2009).

### 2.2. Queueing models

Wastes are not only detectable failures; many hidden parts of processes can be considered as waste. Once a batch of product is finished at an assembly plant, it moves to the material handling phases for storage until delivery. If there is not enough staff in the warehouse, only a relatively few workers can deal with such activities and a bottleneck can form. Queueing models cannot only be applied to describe or simulate production or service processes in warehouses but in many areas where processes can be defined.
Queueing theory has an enormous literature base. Kerbache \& Smith (2004) modeled the supply chain as a network of queues and analyzed congestion problems. Another direction of supply chain analysis is the numerical explanation of performance metrics (Zhou et al. 2014). A similarly difficult problem is the analysis of flexible manufacturing systems (Jain et al. 2008). Nuyens et al. (1996) modeled mechanisms of flexible manufacturing systems as a dynamic queueing problem.
Queueing is widely used in modeling assembly lines in general (Manitz 2008), or connected to real applications. Zhuang et al. (1998) modeled and simulated a complex assembly system which consisted of many resources and processes. Other important research topics are the synchronization behavior of different types of manufacturing systems (Schipper et al. 2016) or modeling breakdown while a servicer is in operation (Gray et al. 2000). A typical area of queueing theory in manufacturing is the situation in which waiting is modeled as a probability variable that describes the period when a certain number of products has piled up (Gray \& Scott 1986).
Lynes \& Miltenburg (1994) describes the connections between the inventory, throughput, cycle time, and cost in modeling an open queueing network. Kochel (1996) modeled a multi-location inventory problem. Other useful inventory modeling can be found in Boxma \& Perry (2001) and Chang \& Lu (2010). Considering servicers, Creemers \& Lambrecht (2009) modeled appointmentdriven service systems. A mean overland flow-rate is modeled in Nuyens et al. (1996) and the model can be adopted to estimate the waiting time of customers.
In the literature, several optimization problems associated with queueing networks can be summarized as follows (Kerbache \& Smith 2004:255):

- "Optimal Topological Problem (OTOP): Deals with strategic planning and focuses on the design of the network that determines the number of nodes and arcs as well as their locations and their topological interconnections.
- Optimal Routing Problem (OROP): Deals with tactical planning and is concerned with the routing of customers, given a predefined network topology, and the consequent results of the routing configuration on the resources and congestion created by the routing decision.
- Optimal Resource Allocation Problem (ORAP): Deals with operational planning and focuses on the optimal allocation of resources given that the OTOP and OROP have already been solved."


## 3. Algebraic formulation of the Activity-Based Costing method

Activity-based costing has proved to be a useful model for considering waiting as waste in the unit cost of products. The well-known calculation procedure is given in a formalized manner by the following formulation (Table 1) and equations.

Table 1: Algebraic formulation of ABC procedure

| Vector of activity pools: |
| :---: |
| $\mathbf{a}=\left(a_{i}\right) ; 1 \leq i \leq n$ |
| Vector of products: |
| $\mathbf{p}=\left(p_{j}\right) ; 1 \leq j \leq m$ |
| Matrix of cost drivers: |
| $\mathbf{C}=\left[c_{i j}\right]$ |
| Vector of cost drivers (sum for all products): |
| $\mathbf{c}=\mathbf{C i}=\left(c_{i}\right)$ |
| Matrix of unit cost components: |
| $\mathbf{U}=\left[u_{i j}\right] ; u_{i j}=\frac{a_{i}}{c_{i}} \cdot \frac{c_{i j}}{p_{j}}$ |
| Matrix of direct costs: |
| $\mathbf{D}_{\Sigma}=\left[d_{\text {Eki }}\right] ; 1 \leq k \leq l$ |
| Matrix of direct unit costs: |
| $\mathbf{D}=\left[d_{k j}\right] ; d_{k j}=\frac{d_{\Sigma k j}}{p_{j}}$ |
| Matrix of unit cost components (direct cost elements added): |
| $\mathbf{D}^{*}=\left[d_{q j}^{*}\right]=\left[\begin{array}{l}d_{k j} \\ \ddot{u}_{i j}\end{array}\right] ; 1 \leq q \leq r ; r=n+l$ |
| Vector of unit costs: |
| $\mathbf{d}^{*}=\mathbf{i}^{\mathbf{T}} \mathbf{D}^{*}=\left(d_{j}^{*}\right)$ |

Activity pools are activity groups to which costs are connected. Cost drivers are variables (mainly activities) that cause costs, and connect mostly to indirect costs. Here cost drivers are timeconsuming activities (waiting). Unit cost refers to the cost of one component or one product. To calculate the profit of a product, the price vector has to be defined as:
$\mathbf{v}=\left[\nu_{j}\right]$
Therefore the profit of each product can be collected in the profit vector:
$\boldsymbol{\pi}=\mathbf{v}^{\mathbf{T}} \mathbf{p}-\mathbf{d}^{* T} \mathbf{p}=\left(\mathbf{v}^{\mathrm{T}}-\mathbf{d}^{* T}\right) \mathbf{p}$
Since this paper focuses on one cost driver, i.e. waiting time, a sensitivity analysis can give useful information about how the unit cost of a product changes by one percent in the value of the cost driver. Let the waiting times of products be collected in vector $\mathbf{e}$, therefore the weight vector is calculated as in Eq. (3). This vector is identical to the $x$ th row of $\mathbf{C}$ matrix.
$\mathbf{w}=\left[w_{j}\right] ; w_{j}=\frac{e_{j}}{\Sigma e_{j}} \equiv c_{x j}$
$u_{i j}=\frac{a_{i}}{p_{j}} \cdot \frac{1.01 e_{j}}{\Sigma e_{j}+0.01 e_{j}}$

Modification of one element of this vector by one percent results in a change in the $u_{i j}$ element of the matrix of unit cost elements. Since $c_{x j}$ are weights, the $c_{x}$ element of the $\mathbf{c}$ vector of cost drivers equals 1. Therefore, the $u_{i j}$ elements can be calculated as in Eq.(4).

## 4. Experiment and results

Waiting resulting from queueing is typical in material handling problems. To calculate the unit cost that includes the cost of waiting time, three products (I, II and III) of the plant were analyzed. After manufacturing they were delivered to the warehouse for packaging. The rate of waiting time is calculated for all the products analyzed in the considered period ( 1 month). In Table 2 the parameters of queueing and the average waiting times $\left(T_{q}\right)$ of the products are summarized. The estimated direct costs and the cost drivers for each product are given in Table 3.

Table 2: Waiting times [ h ] of products

|  | Products |  |  |
| :---: | :---: | :---: | :---: |
|  | I | II | III |
| Average service rate ( $\mu$ ) | 0.2 | 0.167 | 0.5 |
| Deviation of service time distribution ( $\sigma_{S}$ ) | 0.97 | 2.55 | 1.4 |
| Average arrival rate ( $\lambda$ ) | 0.167 | 0.125 | 0.25 |
| Deviation of arrival time distribution $\left(\sigma_{A}\right)$ | 2.1 | 4.9 | 2.86 |
| Average waiting time ( $T_{q}$ ) | 2 | 5 | 1 |

Table 3: Direct costs and cost drivers

|  | Products |  |  |
| :---: | :---: | :---: | :---: |
|  | I | II | III |
| Direct material cost (thousand HUF) | 500 | 700 | 300 |
| Direct wages (thousand HUF) | 150 | 150 | 300 |
| Lead Time of batch (day) | 6 | 12 | 12 |
| Number of changeovers | 2 | 1 | 2 |
| Rate of waiting time in warehouse | 0.250 | 0.625 | 0.125 |

The characterization of queueing according to Kendall's classification is $\mathrm{G} / \mathrm{G} / 1 / \mathrm{GD} / \infty / \infty$. The average waiting time is calculated by Kingman's formula (Eq.(5)). This formula includes the $\rho$ parameter, which is the rate of average arrival rate and service rate. The parameters $C_{A}$ and $C_{S}$ are the relative deviations of arrival time and service time, respectively.
$T_{q}=\frac{1}{\mu}\left[1+\frac{\rho}{1-\rho}\left(\frac{C_{\mathrm{A}}^{2}+C_{\mathrm{S}}^{2}}{2}\right)\right]$
Activity pools and the connecting cost drivers are given when the cost driver of logistics is only the direct material cost (version 1, Table 4) and when the rate of W/T in the warehouse is included as part of the logistics overhead (version 2, Table 5).

Table 4: Activity pools and cost drivers - version 1

| Activity pool | Cost (thousand HUF) | Cost driver |
| :---: | :---: | :---: |
| Logistics | 300 | Direct material cost |
| HR administration | 100 | Direct wages |
| Production mgmt. | 50 | Lead time of batch |
| Scheduling | 50 | Number of <br> changeover |

The structure of unit costs is summarized in Tables 6 and 7 for the original costing and the modified one, respectively. Due to the more exact allocation the unit cost of Product II increased by 47.5 HUF and that of products I and III decreased by 25 and 11.25 HUF, respectively.
In order to improve the process one possibility is to decrease the average waiting time of the preceding material handling and packaging in the warehouse. To get a clearer picture about the connections between this waiting time and unit cost, a sensitivity analysis is recommended to perform so that determination of goal values can be more exact.

Table 5: Activity pools and cost drivers - version 2

| Activity pool | Cost (thousand HUF) | Cost driver |
| :---: | :---: | :---: |
| Logistics | 150 | Direct material cost |
| HR administration | 100 | Direct wages |
| Production mgmt. | 50 | Lead time of batch |
| Scheduling | 50 | Number of <br> changeover |
| Material handling / <br> packaging | 150 | Rate of waiting time in <br> warehouse |

Table 6: Structure of unit costs (thousand HUFs) without considering waiting time in warehouse

| Activity pool | Products |  |  |
| :---: | :---: | :---: | :---: |
|  | I | II | III |
| Direct material cost | 1 | 1.4 | 0.3 |
| Direct wages | 0.3 | 0.3 | 0.3 |
| Logistics | 0.2 | 0.28 | 0.06 |
| HR administration | 0.05 | 0.05 | 0.05 |
| Production management | 0.02 | 0.04 | 0.02 |
| Scheduling | 0.04 | 0.02 | 0.02 |
| Total | 1.61 | 2.09 | 0.75 |

Table 7: Structure of unit costs (thousand HUF) considering waiting time in warehouse

| Activity pool | Products |  |  |
| :---: | :---: | :---: | :---: |
|  | I | II | III |
| Direct material cost | 1 | 1.4 | 0.3 |
| Direct wages | 0.3 | 0.3 | 0.3 |
| Logistics | 0.1 | 0.14 | 0.03 |
| HR administration | 0.05 | 0.05 | 0.05 |
| Production management | 0.02 | 0.04 | 0.02 |
| Scheduling | 0.04 | 0.02 | 0.02 |
| Material handling/packaging | 0.075 | 0.19 | 0.019 |
| Total | 1.585 | 2.15 | 0.74 |

The result of the sensitivity analysis is demonstrated in Figs. 1-3. The data show that the changes in the different waiting rates influence the unit costs of the other products to variable extents, and the difference between them is significant.


Fig. 1: Sensitivity analysis, basis: Product I
For example a 2 percent increase in the average waiting time of Product I causes a 0.09 HUF decrease in the unit cost of Product III (Fig.1). Similarly a 2 percent increase in the average waiting time of Product III causes a 0.19 HUF decrease in the unit cost of Product I (Fig.3)


Fig. 2: Sensitivity analysis, basis: Product II


Fig. 3: Sensitivity analysis, basis: Product III

## 5. Conclusion and further research

The introduced model is based on the well-known concept that processes always include a certain rate of waste and there are costs connected to them. The classic activity-based calculation was used for showing the significance of an alternative cost element waiting time. Consideration of this type of waste is essential in enterprises where lean production is the focus, because the more precise the costing is the more detailed information is available for decision makers about the efficiency and effectiveness of the operation of production or service processes.
Application of the model was illustrated by an experimental example highlighting the fact that significant changes in unit costs emerged with consideration of additional important elements such as alternative costs.
Further research possibilities are testing the model in a complex business environment and supplementing the calculation with additional alternative cost elements such as movement, overprocessing or defect rate. Moreover, in production processes or services there are several types of queueing. This model considered only the $\mathrm{G} / \mathrm{G} / 1 / \mathrm{GD} / \infty / \infty$ type but different processes relate to different queueing models that alter the cost results in different manners. Extension of the model to these types could lead to generalization of the model.

## References

[1] Boxma OJ \& Perry D, "A Queueing Model with Dependence between Service and Interarrival Times", European Journal of Operational Research, No.128, (2001), pp.611-624.
[2] Cima RR, Brown MJ, Hebl JR, Moore R, Rogers JC ,Kollengode A, Amstutz GJ, Weisbrod CA, Narr BJ \& Deschamps C, "Use of Lean and Six Sigma Methodology to Improve Operating Room Efficiency in a High-Volume Tertiary-Care Academic Medical

Center", Journal of American College of Surgeons, No.213, (2011), pp.83-94.
[3] Chang K-H \& Lu Y-S, "Queueing Analysis on a Single-Station Make-to-Stock/Make-to-Order Inventory-Production System", Applied Mathematical Modelling, No.34, (2010), pp.978-991.
[4] Creemers S \& Lambrecht M, "An Advanced Queueing Model to Analyze Appointment-Driven Service Systems", Computers \& Operations Research, No.36, (2009), pp.2773-2785.
[5] Dickson EW, Singh S, Cheung DS, Wyatt CC \& Nugent AS, "Application of Lean Manufacturing Techniques in the Emergency Department", The Journal of Emergency Medicine, Vol.37, No.2, (2009), pp.177-182.
[6] Gray WJ \& Scott M, "A Queueing Model with Bonus Service for Certain Customers", Applied Mathematical Modelling, Vol 10, (1986), pp.241-245.
[7] Gray WJ, Wang PP \& Scott M, "A Vacation Queueing Model with Service Breakdowns", Applied Mathematical Modelling, No.24, (2000), pp.391-400.
[8] Jain M, Maheshwari S \& Baghel KPS, "Queueing Network Modelling of Flexible Manufacturing System Using Mean Value Analysis", Applied Mathematical Modelling, No.32, (2008), pp.700-711.
[9] Kerbache L \& Smith JM, "Queueing Networks and the Topological Design of Supply Chain Systems", International Journal of Production Economics, No.91, (2004), pp.251-272.
[10] Kochel P, "On Queueing Models for some Multi-Location Problems", International Journal of Production Economics, No.45, (1996), pp.429-433.
[11] Konyha J \& Banyai T, Sensor networks for smart manufacturing processes", Solid State Phenomena, No.261, (2017), pp.456-462.
[12] Meran R, John A, Roenpage O, Staudter C, Six Sigma + Lean Toolset, Springer, (2013), pp.193-194.
[13] Lynes K \& Miltenburg J, "The Application of an Open Queueing Network to the Analysis of Cycle Time, Variability, Throughput, Inventory and Cost in the Batch Production System of a Microelectronics Manufacturer", International Journal of Production Economics, No.37, (1994), pp.189-203.
[14] Manitz M, "Queueing-Model Based Analysis of Assembly Lines with Finite Buffers and General Service Times", Computers \& Operations Research, No.35, (2008), pp.2520-2536.
[15] Molnar V \& Kerchner A, "A lean menedzsment alkalmazási lehetőségei a közszférában (Application potentials of lean management in the public sector)". Proceeding of the Conference "Műszaki Tudomány az Észak-Kelet Magyarországi Régióban 2016", (2016), pp.425-432.
[16] Molnar V \& Tumik A, "Várakozási veszteségből adódó költségek Lean Six Sigma megközelítésben: egy ABC-alapú döntési modell (Waiting Loss Costs of Non-Production Processes in Lean Six Sigma Approach: an ABC-Based Decision Support Model)", Controller Info, Vol.5, No.1, (2017), pp.35-40.
[17] Musinszki Z, "Innovations and cost systems trends and ways in the cost accounting", Organizational and economic mechanisms of development of the financial system: Collective monograph, ISMA University, (2016), pp.209-219.
[18] Nuyens RPA, Van Dijk NM, Van Wassenhove LN \& Yiicesan E, "An Experimental Analysis of Steady State Convergence in Simple Queueing Systems: Implications for Flexible Manufacturing System Models", Simulation Practice and Theory, No.4, (1996), pp.1-29.
[19] Pienkowski M, "Waste Measurement Techniques for Lean Companies", International Journal of Lean Thinking, Vol.5, No.1, (2014), pp.9-24.
[20] Schipper MA, Chankov SM \& Bendul J, "Synchronization Emergence and its Effect on Performance in Queueing Systems", Procedia CIRP, No.52, (2016), pp.90-95.
[21] Tamas P, "Application of a Simulation Investigational Method for Efficiency Improvement of SMED Method", Academic Journal of Manufacturing Engineering, Vol.15, No.2, (2017), pp.23-30.
[22] Zhou W, Huang W \& Zhang R, "A Two-Stage Queueing Network on Form Postponement Supply Chain with Correlated Demands", Applied Mathematical Modelling, No.38, (2014), pp.2734-2743.
[23] Zhuang L, Wong YS, Fuh JYH \& Yee CY, "On the Role of a Queueing Network Model in the Design of a Complex Assembly System", Robotics and Computer-Integrated Manufacturing, No.14, (1998), pp.153-161.

