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Research paper

A performance analysis of fractional order based MARC controller over optimal fractional order PID controller on inverted pendulum

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Abstract

This paper presents a new way to design MIT rule as an advanced technique of MARC (Model Adaptive Reference Controller) for an integer order inverted pendulum system. Here, our work aims to study the performance characteristics of fractional order MIT rule of MARC controller followed by optimal fractional order PID controller in MATLAB SIMULINK environment with respect to time domain specifications. Here, to design fractional order MIT rule Grunwald-Letnikov fractional derivative calculus method has been considered and based on Grunwald-Letnikov fractional calculus rule fractional MIT rule has been designed in SIMULINK. The proposed method aims finally to analyze overall desired closed loop dynamic performance on inverted pendulum with different performance criteria and to show the desired nature of an unstable system over optimal fractional order PID controller.

Keywords*: Inverted pendulum, optimal fractional order PID controller, MARC controller, fractional order, MIT rule.*

1. Introduction

In this article a simple case study has been shown to design and implement model adaptive reference control [1] of inverted pendulum system [2] which is one of the most real time problems of control engineering, using normal MIT rule [3] followed by fractional order MIT rule [4] and to compare their control performances. Several researchers proposed various scheme to analyze the performance of inverted pendulum using different control techniques. Mohammad Ali[5] proposed control of inverted pendulum cart system by use of PID controller. Moghaddas[6] proposed design of PID controller for inverted pendulum using genetic algorithm. Adrian-Duka [7] proposed MARC using lyapunov theory and fuzzy model reference control for inverted pendulum. Adaptive controller has been chosen as it is more effective than fixed gain PID controller to handle difficult situations inspite of changing the environmental condition frequently.

In this paper, design of model adaptive reference control for inverted pendulum has been suggested by normal MIT rule based on adaptation gain which is altered to adjust the minimal error between the reference model and the system output despite the variations in the plant parameters but this rule by itself does not gurantee stability and adaptive controller designed using MIT rule is very sensitive to the amplitudes of the signals. So the adaptive gain is generally kept small to make stable. Recently fractional order provides an innovative concept on mathematical calculus with one extra degree of freedom through which any kind of system can be controlled better. So, MIT rule has been modified as fractional order MIT rule with extra degree of freedom which has been varied with fixed adaptation gain to test the characteristics of error between the reference model and plant output and to analyze the nature of output of fractional order over integer order MIT rule using MATLAB SIMULINK.

2. Inverted pendulum

The inverted pendulum [8] is among one of the challenging tasks to control in the field of control engineering and has been suggested as a benchmark for testing control strategies. A free body diagram of the inverted pendulum is shown in Figure 1 in below consisting of a cart which is maintained to balance a pendulum. Inverted pendulum is a nonlinear system including a stable equilibrium point when pendulum is at pending position and upright position. The main task has been considered that pendulum has to be swinged up the from stable to unstable equilibrium point and then it is balanced at the upright position by moving the cart again from right to left.

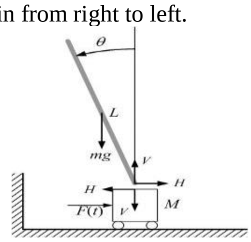

Figure 1: Free body diagram of inverted pendulum

In the above figure m= mass of pendulum, M= mass of cart, theta = angular position of pendulum, g = acceleration and L=length of pendulum. In our work, rotational single arm pendulum has been assumed which is shown in Figure 2 in below.

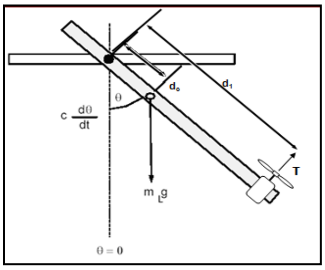

Figure 2: Rotational single arm inverted pendulum

In the above Figure the pendulum is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its resting, equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position at θ=0. In the above figure, d_1= length of pendulum, c=frictional constant, m= mass of pendulum, g= acceleration, d_0=half length at centre and T= tension.

The equation of motion for rotational single arm pendulum from free body diagram is shown in below.

$$J\frac{d^2\theta}{dt^2}+c\frac{d\theta}{dt}-mgd_0\cos\theta=(d_1)t \tag{1}$$

Taking the laplace transform ,

$$\frac{\theta(s)}{T(s)}=\frac{d_1}{Js^2+cs-mgd_0} \tag{2}$$

The parameters are given in below:

a. J (Inertia) = 1 N/S
b. C (Frictional Constant) = 0.0389
c. M (Mass) = 1 Kg
d. g (Acceleration) = 9.81
e. d_1 (Length of Pendulum) = 1.89 cm
f. d_0 (Half length at center) = 0.945 cm

Substituting the values of parameters for a real time process the overall system transfer function is shown in (3).

$$\frac{\theta(s)}{T(s)}=\frac{1.89}{s^2+0.0389s-10.17} \tag{3}$$

3. MARC controller

Model adaptive reference control [9] has been chosen to design the adaptive controller which is dependent on adaptive gain by varying it output of plant tracks reference model having same reference input. The basic block diagram of model adaptive reference control is shown in Figure 3 in below.

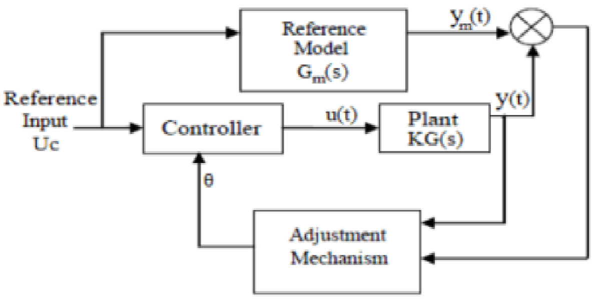

Figure 3: Block diagram of MARC controller

Generally, a reference model is used to define performance characteristics of the process being controlled and the adaptation law uses the error between the process and the model output. These parameters are varied so as to minimize the error between the process and the reference model. The transfer function of plant and model has been taken as shown in below.

Transfer function of the model :

$$\frac{1.02}{s^2+8.27s+1.02} \tag{4}$$

Transfer function of the plant:

$$\frac{1.89}{s^2+0.0389s-10.17} \tag{5}$$

Normal MIT (massachusetts institute of technology) Rule

Mainly, the adaptation law attempts [10] to find a set of parameters that minimize the error between the plant and the model outputs. To do this, the parameters of the controller are incrementally adjusted until the error has reduced to zero.

To design rule, Cost function $J(\theta)=\frac{e^2}{2}$ (6)

Where e is errror between the outputs of plant and model and θ is the adjustable parameter.

$$\text{Error(e)}= y(t)-y_m(t) \tag{7}$$

Where y denote system output and y_m denote reference output.

To make minimize the cost function to zero θ has been kept in the direction of negative gradient of J.

$$\frac{d\theta}{dt}=-\gamma\frac{dJ}{d\theta} \tag{8}$$

Where γ is adaptive gain of the controller.

From Eq(7)

$$\frac{d\theta}{dt}=-\gamma e\frac{de}{d\theta} \tag{9}$$

$\frac{de}{d\theta}$ is the sensitivity derivative of the system , can be evaluated under the assumption that θ is constant.

Now from Eq.(6) taking the laplace transform,

$$E(s)=KG(s)U(s)-K_0G(s)U_c(s) \tag{10}$$

Where $KG(s)$ is the transfer function for plant and $K_0G(s)$ is the transfer function for model.According to the control law it has been defined as,

$$u(t)=\theta*u_c \tag{11}$$

Taking partial differentiation from Eq(10) & Eq(11)

$$\frac{dE(s)}{d\theta}=KG(s)u_c(s)=\frac{K}{K_0}y_m(s) \tag{12}$$

Now from Eq(9) & Eq(12),

$$\boldsymbol{\frac{d\theta}{dt}=-\gamma e y_m} \tag{13}$$

Fractional order MIT rule

The controller is very sensitive to the changes in the amplitude of the reference input inspite of giving satifactory result using MIT rule and the system may become unstable for large values of reference input. Now to overcome this problem a new variant of MIT rule[11] has been introduced for parameter adjustment by using fractional order calculas to develop the control law. In our work, G-L fractional derivaive has been approached to design fractional MIT rule following Eq.(9) and the new equation beomes as shown in below.

$$\frac{d\theta}{dt} = -\gamma e \frac{d^{\alpha} e}{d\theta^{\alpha}} \tag{14}$$

Where rate of change of parameter θ depneds on both adaptation gain and the derivative order alpha.So,G-L fractional derivative [12][13] has been defined as,

$$D^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{n} (-1)^{k} \binom{n}{k} f(t - kh) \tag{15}$$

Where h is defined as step size .

Now assuming $D^{\alpha}f(t) \sim D_h^{\alpha}f(t)$, it has been obtained as

$$D_h^{\alpha} f(t) = h^{-\alpha} \sum_{j=0}^{k} (-1)^{j} \binom{\alpha}{j} f(kh - jh) \tag{16}$$

Now,$\binom{\alpha}{j}$ can be approximated as

$$\frac{\alpha!}{j!(\alpha - j)!} = \frac{\Gamma(\alpha + 1)}{\Gamma(j+1)\Gamma(\alpha - j + 1)} \tag{17}$$

Where Γ is defined as euler function. Now, this euler function has been applied on error signal and it has been obtained following the normal MIT rule [14] as shown in below.

$$\frac{d\theta}{dt} = -\gamma \left(\frac{d^{\alpha}}{d\theta^{\alpha}} e \right) y_m \tag{18}$$

4. Result & analysis

Optimal FOPID controller

Under FOPID controller unlike PID controller two extra degrees of freedom are used to analyze the performance of pendulum. The general transfer function of fractional PID controller [15] is given as shown in Eq.(19).

$$G(s) = K_P + K_I / s^{\alpha} + K_D s^{\beta} \tag{19}$$

Where two additional fractional parameters alpha and beta are used as two extra degrees of freedom. The optimal fractional order PID controller has been designed using FOMCON toolbox. FOMCON [16] is basically fractional order calculus based toolbox for system modeling and control design. The core of the toolbox is derived from an existing mini toolbox FOTF(fractional order transfer function). To design optimal fractional order PID controller NELDER-MEAD optimization technique has been approached. Nelder-Mead optimization method [17] is more heuristic search method than fmincon (interior point or active set) optimization method that can converge to non-stationary points. The method compares function values at the three vertices of triangle which are considered as $f(x_1)$, $f(x_2)$ and $f(x_3)$. Here x_1, x_2 and x_3 are considered as good, better and worst point of triangle**.** Simplex method has been used to continue a process of reflection (R), expansion (E), contraction(C), reduction(S) until the function is minimized after iteration process. After optimization five parameters of fractional order PID controller have been obtained as K_P with 26.08, K_I with 18.45, K_D with 6.74, Alpha with 0.854 and Beta with 0.504 respectively. The performance of inverted pendulum using optimal fractional PID controller is shown in below in Figure 4.

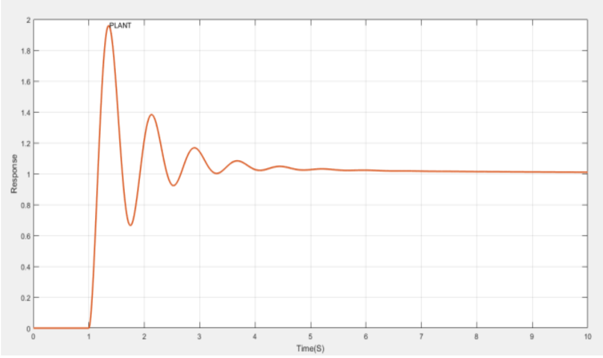

Figure 4: Performance of inverted pendulum using optimal FOPID

Table 1: Parameters of Optimal FOPID Controller

ISE	IAE	ITSE	ITAE	T_r	T_s	M_p
22.83	36.69	60.20	96.75	1.15	2.63	1.95

From the above Table I it has been observed the performance of inverted pendulum with more overshoot which reaches at the peak value of 1.956. To overcome this maximum overshoot of optimal FOPID controller fractional order MIT rule has been approached on the same plant.

Fractional MIT rule

Fractional order MIT rule as modified MIT rule has been approached to make compare with optimal FOPID and it has been studied the nature of response of fractional order MARC [18] with one extra degree of freedom alpha which has been varied with fractional order less than one keeping fixed adaptive gain value. The SIMULINK model using fractional order MIT rule is shown in Figure 5 in below.

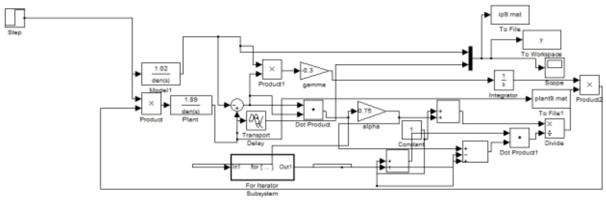

Figure 5: Simulink model of FMIT of MARC controller

The performance indices with nature of pendulum using fractional order MIT rule is shown in below in Figure 6, Figure 7, Figure 8 and Figure 9.

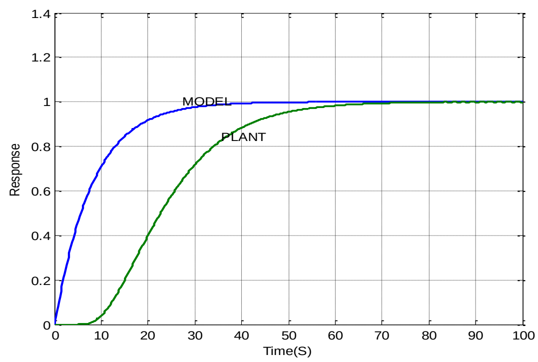

Figure 6: Alpha=0.5 and Gamma=0.6

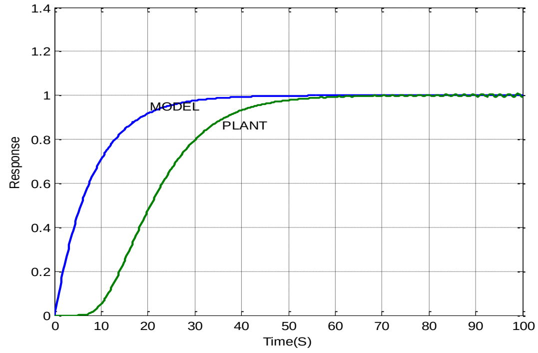

Figure 7: Alpha=0.5 and Gamma=0.8

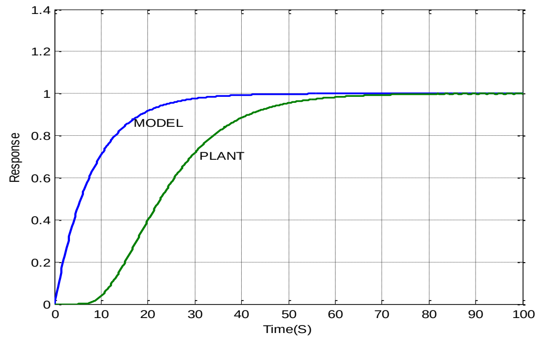

Figure 8: Alpha=0.75 and Gamma=0.6

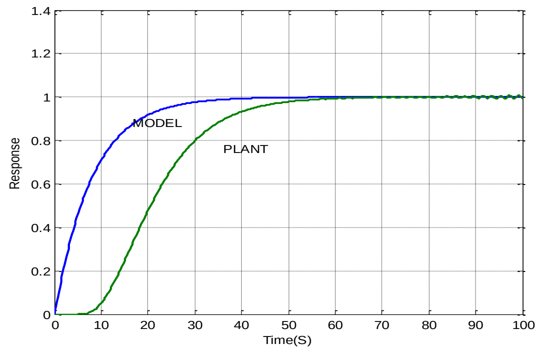

Figure 9: Alpha=0.75 and Gamma=0.8

Table 2: Parameters for Reference Model of MARC

ISE	IAE	ITSE	ITAE	T_r	T_s	M_p
3.89	7.40	121.5	231.0	18.60	31.20	0.999

Table 3: Parameters for Plant of MARC

Alpha	Gamma	ISE	IAE	ITSE	ITAE	T_r	T_s	M_p
0.5	0.6	12.9	19.0	691.5	1015.4	36.8	53.2	1.0
0.5	0.8	11.4	16.6	522.9	760	31.6	45.6	1.01
0.75	0.6	13.0	19.2	692.8	1014.4	36.6	53.2	1.0
0.75	0.8	11.2	15.6	523.4	759.7	31.6	45.6	1.01

Now it has been observed the desired performance of inverted pendulum that tracks the reference model with less overshoot keeping fixed adaptive gain as 0.6 followed by 0.8 and varying extra degree of freedom as 0.5 followed by 0.75. It has been also analyzed that keeping the fixed adaptive gain as 0.6 or 0.8 and varying the extra parameter as 0.5 followed by 0.75 the values of rise time, settling time and overshoot are more or less same and the desired value of performance metric error has been achieved with alpha as 0.75 and gamma as 0.8.

5. Conclusion

An unstable nature of inverted pendulum has been improved by optimal fractional order PID controller and MARC controller using fractional order MIT rule. It has been analyzed the stability of inverted pendulum with different performance metric values under optimal fractional order PID controller and MARC controller using fractional order MIT rule. I t has been studied that using fractional order MIT rule of MARC controller the response of plant achieves the set point with very less overshoot where optimal FOPID provides oscillation but the values of rise time and settling time of optimal FOPID are less than fractional order MIT rule. Except these parameters the values of ISE and IAE under fractional order based MIT rule are very less than optimal FOPID controller. So, the behavior of pendulum has been stabled using both of controllers but fractional order MIT rule shows the stability of inverted pendulum with no oscillation.

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