Reconstruction of RGB composite CT lung image by blind de-convolution for various blurring functions

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Abstract

A competent and flexible tool to optimize inverse problems related to image reconstruction by restoration is Alternating Direction Method of Multipliers (ADMM) with the knowledge of known blur. This method is later modified to perform Blind Image De-blurring (BID) of unknown blur on original image by using some function of regularization. But, in real world for de-blurring, the prior knowledge of blurring filter is important. In this research work, estimates of the image and blurring operator are obtained by considering significant image edges. An ADMM iteration criterion forms the base for which whiteness measurement parameter estimation which includes auto-correlation, auto-covariance. Using these parameters best ISNR is taken as input resulting from the iterations. Different degradation conditions are considered in the analysis to estimate the performance and to bring a conclusion to the degradation and restoration pair by processing composite and component images of the input RGB-CT lung image.

Keywords: Image Restoration Degradation; Blur; Convolution; De-Convolution; CT.

1. Introduction

Restored image is in general the extracted approximate of the original image at the receiver side. Enhancement is subjectively evaluated where as image restoration process is assessed objectively. Image restoration involves mathematical modeling of the degradation process which in turn based on probability. Fig 1 shows general image degradation model and restoration model. The main concept in restoration is De-convolution. The process of degradation involves blurring of a known observed image known blur function and degraded by an additive Gaussian noise. The process of degradation in matrix form is represented by convolution of original image x with blur kernel R and it is written as Rx. x is taken as a column vector obtained by stacking all columns of the image to one long column vector and 'R' is a block of Toeplitz matrix with constants along the diagonal. Mathematically the degradation model is written as y=Rx+n, where n is a Gaussian noise of variance $\sigma^2$:

$$y = Rx + n$$

Where B is the direct operator in matrix from and 'n' is noise. B represents convolution operator if deconvolution is considered. This model describes several physical phenomenons, such as relative motion between the camera and the subject resulting in motion blur, and defocusing blur called bad focusing.

Generally, image blurring can be obtained by convolving original image with impulse function. ADMM can be used to reduce blurring effect based on conjugate Gradient (CG) & Mask Decoupling (MD) algorithms in TV & FC by knowing the kernels. Observing the ISNR values of four algorithms [TV-CG, TV-MD, FC-CG, FC-MD], the FC-CG algorithm will provide better results than remaining three algorithms. All these ADMM approaches are implemented using the prior information about the kernels.

In proposed approach ADMM based method will work without prior knowledge about the kernel parameters. Hence, this process is known as Blind Image Deconvolution (BID). Fig 2 shows general deconvolution procedure. Fig 3 and Fig 4 shows difference between the images obtained by non-Blind deconvolution and Blind deconvolution. This is extended in the proposed approach by using an additional parameter called whiteness measurement, in which the iteration of ADMM will stops if ISNR value reaches to the highest when compared to previous iteration values. Five types of blur are added to the color CT lung image in two ways: directly on the RGB composite image and to the component images to validate the performance of the proposed approach. Restoration results of the two blurring procedures are validated for various BSNR (Blurred Signal-to-Noise ratio) values with ISNR (Improvement in Signal-to-Noise ratio) by estimating the best values between consecutive iterations.
2. Literature survey

Alternating Direction Method of Multipliers (ADMM) was proposed in 1970’s to solve several imaging problems [1] [2]. This method was used as a flexible and efficient tool compared to Denoising [2], Inpainting, Deblurring [4], Reconstruction [5], and Motion Segmentation [6]. All-inclusive reviews of classical problems are [7], [8], [9], [10], [11], [12], [13], [14]. Variable splitting is the concept on which the ADMM is based. This particular concept is used directly to process different regularizes/priors [15]. These are total-variation (TV) [16], [7], and insertion of several types of specifications (e.g., positivity) [3], [10]. ADMM is strongly related to other techniques, namely Bregman and split Bregman methods [10], [11], [19], [20], [21], [22], [3] and Douglas–Rachford splitting [9], [23], [13], [22] are the other related methods for inverse imaging problems similar to ADMM. These methods involve iterative procedures and are required to solve a linear system [4], [10], [3]. This fact is concurrently an advantage and a limitation. On the other hand, this iteration process is limited to the applicability, where it can be efficiently computed. Deconvolution algorithms for image inversion are competently implemented Fast Fourier transform (FFT) [24] [25] when cyclic convolution is assumed, which in turn results in diagonal in the discrete Fourier domain. But for most of the problems related to imaging circularity is an abnormal supposition. In the process of deconvolution, the pixels located in the neighborhood of the boundary of the observed picture depend on pixels of the unknown picture located outside of its domain which is unobserved. The traditional way to solve this problem is to adopt a boundary condition (BC). In Fig 5, it can be observed that the standard BCs are quite deviating because the boundaries are marked for computation benefit only and they do not relate to any type of acquisition system. It can be concluded that the assumption of periodic boundary condition has the benefit of implementation of the convolution as a sample-wise multiplication using the FFT algorithm efficiently. In most of the real imaging problems it cannot be insisted to make the external unobserved pixel to follow the periodic boundary conditions. The consequence of the of this mismatch results in the degradation of the deconvolved images. The output images may consist of ringing errors along the boundaries. Hence the errors created due to wrong assumption can be decreased by pre-processing. Pre-processing an image will extrapolate to produce a large size image in which the boundary conditions are smoothed.

3. Methodology and implementation

3.1. Methodology

1) In the proposed method RGB CT lung image slices are taken as input.
2) The image is blurred by convolving the input image with the blurring functions.
3) A spatial mask is applied to prevent the unobserved pixels which may be missed during the transmission process.
4) Tight frame approach is applied to analyze the image and perform the synthesis.
5) By using Conjugate Gradient (CG) minimizing the cost function (Mean Square Error) is computed to get best ISNR and best restored image.
6) The iteration process is the new approach introduce in the method, which is based on the whiteness measurement. The process of restoration will stop when the image gets best ISNR value compared to the previous iteration value.
7) In addition to this iteration process the filter that performs the degradation of the input image is also computed by Curvelets and CG methods, which is the BID.
8) The ISNR values and the number of iterations performed are tabulated.
9) All the previously mentioned steps are applied to the same medical image by decomposing the image in to component R, G, and B images.
10) The individual values are tabulated and the average of the component values is obtained along with the composite image from individually processed output images.
11) The validation of the approach was done by comparing the ISNR values of approach on original RGB and individual component images.

3.2. Implementation

3.2.1. BID using ADMM

Blind Image De-convolution (BID) is the process of deconvolving a blurred image without having prior knowledge of blurring kernel. This method is also applicable for real images. In proposed work, five blurs functions (out-of-focus, uniform, linear motion, random, Gaussian) are considered at BSNR (90dB). The reason for concentrating on large blur is that the effect of the boundary is apparent. Estimations are obtained by minimizing the cost (Mean Square Error) function of the image x and the blurring operator H.

3.2.1.1. Image estimation

The unconstrained formulation of estimate of image is given as

\[ f(x) = \frac{1}{2} ||y - MAx||_2^2 + \lambda \sum_{m=1}^{m} ||FmX||_2^2 \]  

(3.1)

And in constrained formulation for J= m+1, and

\[ G^{(j)} = F_{j} \text{ for } j = 1, \ldots, m \]  

(3.2)

\[ G^{(m+1)} = H \]  

(3.3)
\[ g^{(0)}(u^{(0)}) = \lambda \|u^{(0)}\|_2^2 \text{ for } j = 1, \ldots, m \] (3.4)

\[ g^{(m+1)}(u^{(m+1)}) = \frac{1}{2} \|y - Mu^{(m+1)}\|_2^2 \] (3.5)

Equation (3.4) can be written as

\[ x_{k+1} = K(pH(\tau) \left( u^{(0)}_k + d^{(1)}_k \right) + \mu \sum_{i=1}^{m} \nu_i^T (u^{(0)}_k + d^{(1)}_k) \] (3.6)

To implement algorithm Proximity Operators (PO) must be solved.

\[ \text{prox}_{g_\rho/\mu}(v) = \arg \min_{u} \frac{\rho}{2 \mu} \|u\|_2^2 + \frac{1}{2} \|v - u\|_2^2 \] (3.7)

\[ \text{prox}_{g_\rho/\mu}(v) = v - \text{shrink} \left( v, \lambda/\mu, \rho \right) \] (3.7)

\[ \text{prox}_{g_\rho/\mu}(v) = (M^*M + \rho I)^{-1} \text{prox}_m(y + \rho v) \] (3.8)

The proximity operator in equation (3.5) can be computed: \( M^* \) is the extension of \( y : \mathbb{R}^n \to \mathbb{R} \) by zero-padding, binary diagonal matrix \( M^*M \), with zeros correspond to the unobserved external boundary pixels.

### 3.2.1.2. Blur estimating by using ADMM

The estimate of blur problem of algorithm one can be written in unconstrained formula as

\[ \arg \min_{\hat{u}} \frac{1}{2} \| y - M\hat{u} \|_2^2 + l_s + (h) \] (3.9)

And in constrained form equation (2.2), with \( J=2 \), \( g(\hat{u}) = X, g(\hat{u}) = l \) and

\[ g(u^{(1)}) = \frac{1}{2} \| y - Mu^{(1)} \|_2^2, g(u^{(2)}) = l_s + \left( u^{(2)} \right) \] (3.10)

The resulting instance of Algorithm involves the inversion of the matrix \( \muX + \mu^2/2I \). FFT is used to compute in the discrete frequency domain. The PO has the same form as equation (3.5) pertaining to the two \( (J=2) \) Proximity Operator (PO). Orthogonal projection on that v-shrink is

\[ \text{prox}_{g(\rho)}(v) = \text{prox}_{g(\rho)} = p(v) \] (3.11)

### 3.3. Whiteness measures

To assess the accuracy whiteness measures of the residual are used. The stopping criterion which is based on whiteness measure parameter was tested on large resolution images. The success lead to an average decrease of ISNR by 3% compared to that obtained by stopping the algorithm at the highest ISNR.

#### 3.3.1. Rationale

The stopping rule for iterations and the criteria for selecting the regularization parameter are based on measures of the fitness of the image estimate \( \hat{x} \) and the blur estimate \( \hat{h} \) in non-blind deconvolution where \( \hat{h} \) is known. Hence \( \hat{h} = h \) is replaced in the degradation model equation (1.1). The estimate of the residual image is obtained by

\[ r = y - \hat{h} * \hat{x} \] (3.12)

#### 3.3.2. Measures of whiteness

The residual image is normalised to unit variance and with zero mean and this normalized residual is represented by \( r \),

\[ r = \frac{r - \text{mean}(r)}{\text{var}(r)} \] (3.13)

Where \( r \) and \( \text{var}(r) \) are the sample mean variance of \( r \) respectively. As the mean is zero the auto-correlation and auto-covariance of the normalized residual \( r \), at 2D lag \((m, n)\), is estimated by the following equation:

\[ R_{rr}^{(m,n)} = K \sum_{i} r(i,j)r(i - m,j - n) \] (3.14)

\[ M_{rr}^{(r)} = - \left( \sum_{(m,n) \neq (0,0)} (R_{rr}^{(m,n)})^2 \right) \] (3.15)

In the above equation the negative sign indicates is used to make \( M \) larger for whiter residuals in the image. \( L = 4 \) is used in experiments and for a typical process in which the short-range correlations are exhibited, the auto-covariance for large lags is usually smaller than for small range dependencies (small lags). Hence it can be concluded from this observation that more weight is given to the auto-covariance for smaller lags. In relation to this a weighted version of the measure is also considered and is given by

\[ M_{W}^{(r)} = - \left( \sum_{(m,n) \neq (0,0)} (R_{rr}^{(m,n)})^2 \right) \] (3.16)

\( W(m,n) \) is a weight matrix. For all practical purposes \( L=4 \) and the Gaussian function of MATLAB is used.

\[ W_{(m,n)} = \exp(-1.25(m^2 + n^2)) \] (3.17)

Let \( S_r(\omega, \nu) \) denote the power spectral density (PSD) of the residual parameter \( r \), at two-dimensional spatial frequency \((\omega, \nu)\),

\[ S_r = F(R_{rr}) \]

The magnitude of the two-dimensional DFT is given by \( F \). As the auto-correlation of a white process is a delta function, hence the signal PSD pattern is flat. The flatness of PSD \( S_r \) is measured by estimating the Shannon entropy, after the normalization process. The maximum entropy is achieved by a flat distributed PSD. The resulting measure of the entropy is given by

\[ M_H^{(r)} = - \sum \frac{S_r(\omega, \nu)}{S_r} \log S_r(\omega, \nu) \] (3.18)

Where

\[ S_r(\omega, \nu) = \frac{S_r(\omega, \nu)}{\sum_{\omega', \nu'} S_r(\omega', \nu')} \]

### 3.3.3. Local measures of whiteness

The approach mentioned previously implicitly assumes that the residual image \( r \) is a sample of a stationary and ergodic process and hence the estimate of the auto-covariance is computed by averaging over the whole image. In practical applications, the residual parameter may not be stationary. So there arises a need to consider the local versions of the previous measures of whiteness parameter, estimating the local auto-covariance by,

\[ R_{rr}^{(m,n)} = \sum_{i \in B_r} r(i,j)r(i - m,j - n) \] (3.19)

### 3.4. Blind image deconvolution (BID) on colour images

The new proposed method is also applicable for colour image and for obtaining ISNR values by applying different types of impulse responses. Again same colour image is split in to individual RGB component images and ISNR values are found by applying same...
 impulse responses which were applied on the RGB image and averaging all individual component values. Finally comparison of ISNR values for actual image and average value of individual component images are obtained.

4. Results and discussions

4.1. Lung CT slice

The performance analysis and validation of the proposed method is performed by considering CT image of lung. Fig 6 shows original RGB image of CT lung and the resultant image obtained by applying the proposed BID approach mentioned in methodology. The obtained ISNR values are tabulated in Table 1 for different blur functions.

![Fig. 6: Original (RGB) Image and Result Image.](image)

The original RGB-CT image is separated into component images as shown in Fig. 7 (row-1), the corresponding resultant images obtained by applying proposed method on individual component images are also shown in Fig 7 (row-2). The corresponding ISNR values are tabulated in Table 2. The resultant images of individual R, G and B components are combined into composite image shown in Fig 8.

![Fig. 7: Component Images and There Resultant Images.](image)

![Fig. 8: Combined Image of Individual Component Images.](image)

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<th>Table 1: ISNR Values for Lung RGB Image</th>
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<td>BSNR</td>
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![Table 2: ISNR Values For Red, Green and Blue Component Images](image)

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The individual ISNR values obtained are averaged for different distortions as shown in Table 3. Table 4 shows a comparison of ISNR values obtained by applying the proposed approach on original RGB-CT lung image and average of individually processed images of components original RGB-CT lung image for different resolutions.

![Fig. 9: Degraded Image and Initial Estimated Image for Original RGB Lung Image.](image)

Figures from 9 to 12 shows intermediate stages images of BID method applied on color image (Individual color components of original RGB-CT lung image are not shown due to more number of images). Fig 9 shows degraded image and initial estimated images for original RGB-CT lung image. Fig 10 shows estimated image and filter for RGB-CT lung image. Fig 11 is observed blurred image for original RGB-CT lung image and Fig 12 is resultant image for original RGB-CT lung image.
5. Conclusion and future scope

5.1. Conclusions

In this paper, BID ADMM iteration criteria based on whiteness measurement parameter sources like Covariance, Weighted covariance (RG), Entropy (H), Block covariance (RB), Block weighted Covariance (RGB), and Block Entropy (HB) is evaluated on RGB lung CT images. Initially RGB lung image is degraded by five blur functions such as Uniform, Out of focus, linear motion, Gaussian, and Random. The sequence of steps of BID ADMM based restoration are directly applied on individual distorted image and estimate of original image is restored. To assess the performance of proposed BID method of restoration on various distortions, ISNR values for two resolutions are computed and tabulated. Next RGB-CT lung image is decomposed in to component images (Red, Green, Blue images) and the same process of restoration is applied to each individual component image and corresponding ISNR values for two resolutions are tabulated. This restored component images ISNR values are averaged and compared with values obtained by direct restoration of original RGB CT lung image. It is observed that an increased ISNR value is obtained for full resolution than of one-third resolution for two cases. And also an improved ISNR values are obtained for different degradations for two resolutions in the case of restoring from component image than from original RGB composite image. Finally it can be concluded that transmitting a color image by decomposing into individual component images will give better ISNR values than transmitting a color composite image directly.

5.2. Future scope

The proposed approach can be further improved by varying the resolutions spatially and iterations in BID to compute the restored image. In addition compression techniques can also be applied after decomposing and before transmitting.

References


