Novelty Ranking Approach with Z-Score and Fuzzy Multi-Attribute Decision Making Combination

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Abstract

This study aims to recommend a new approach in the ranking system by analyzing the combination of the Z-Score method and the Fuzzy Multi-Attribute Decision Making (FMADM) method. This fusion is based on the merging of the advantages of Z-Score and FMADM as a superiority method in statistical rank data processing with weighting data distribution. The lack of Z-Score in processing multi-attributes weighted data can be improved by the FMADM method. In this study, the integration of the Analytical Hierarchy Process (AHP) and Weighted Product (WP) methods was used as the FMADM method with the Z-Score statistical technique. The results of the analysis in the case study show that the integration of the Z-Score and AHP-Weighted Product (Z-WeP) methods can provide maximum results with similarities to the Z-Score results by 86%. Analysis of criterion values on alternatives also shows that Z-WeP can work better than some other of FMADM approaches.

Keywords: Z-Score, FMADM, Z-WeP, Ranking

1. Introduction

The ranking technique in the field of statistical science consists of many methods. The use of this method is intended to obtain sequencing of data as an evaluation based on a criterion [1]. For example in the modern education system, the ranking is often used as an evaluation tool in measuring the quality of the implementation of the education system [2]. Various statistical techniques that are often used in a ranking include Simple Rank, Percentile Rank [3], Deviation Standard, and Z-Score [4]. Some of these techniques ranking will be better if using the Standard Score (Z-Score) method because it can compare the quality of achievement of a datum to the average distribution of data in groups based on the standard deviation value [5]. The functionality of the Z-Score technique in the classification of data is outstanding as the studies have been carried out, but Z-Score technique will not work well if implemented in a rating system with weighted data distribution or having a degree of importance. This condition occurs because the Z-Score technique only compares the difference between the datum and the distribution average for the standard deviation on each type of criteria locally then summed so that the output of the final value produced only looks at each criterion of the data distribution. Another approach that can be done to improve this by a combination of weight multiplication. Weighting techniques in a ranking certainly need to adjusting the data preferences to produce the ranking system, because each criterion positively weights interest in a data distribution [6]. Of course, a fair ranking must adjust the optimal alternatives for each data sequence from some alternatives. Fuzzy Multi-Attribute Decision Making (FMADM) is one of the alternative solutions in the weighting system to find optimal alternatives from some alternatives with specific criteria according to the level of importance of the data [7]. In this study, rank system testing is conducted using the Analytical Hierarchy Process (AHP) and Weighted Product (WP) methods. Weighted Product (WP) method selection is based on its ability to provide optimal solutions in the ranking system based on normalized weight values and can be adjusted to the needs of ranking weights [8]. Whereas the AHP as a method to obtain weight values is based on its ability to provide consistent weighting system recommendations. AHP method as in several studies such as by [9], used as a weighting system so that the determination of multicriteria decisions will be easily analyzed, and obtain alternatives for decisions. The application of AHP as a multicriteria weight analysis technique as done by [10], where AHP is applied as a multicriteria data ranking analysis model. The combination of Z-Score and Fuzzy Multi-Attribute Decision Making (FMADM) methods, especially in Weighted Products (WP), has been done even though only to classify groups of image data based on biometric feature classification. The results of classification with Weighted Product (WP) and normalization methods such as Min-Max, Median-MAD are not so optimal. In that study the maximum results obtained when combining the Z-Score method with Weighted Product (WP) with the achievement of image recognition accuracy up to 97.5% [11], meanwhile, the combination of WP and other methods has also been conducted by Prasetyo, H. A. 2017 which combines the WP method with AHP as a Decision Support System to determine the best producer [12]. The advantages of the Z-Score method and the FMADM method are combined. The lack of Z-Score on the distribution of weighted data will be able to be refined by the use of the WP method. The combination of Z-Score and Weighted Product (WP) techniques abbreviated as Z-WeP will be carried out as a new approach in the system ranking on the distribution of weighted data with varying levels of importance with AHP as the initial weighting technique.
2. Statistical Ranking System

The ranking is the position of a datum object in a data set where it is sorted according to specific criteria. The sorting can be in the order of the largest or smallest or based on the criteria of weight, percentage and so on [4]. In ranking process or sequence of the datum to groups of data can be done in several ways, including Simple Rank, Percentile Rank, Ranking Standard Deviation, and Standard Score (Z-Score).

Among the several statistical ranking techniques, the Standard Score (Z-Score) has a much better ranking. Z-Score is a ranking technique that shows a comparison of the score/data differs from the Mean value with the Standard Deviation. The new alternative values to a data rank will be compared to a common standard, so the use of this method is fair enough to provide ranking recommendations [5]. To determine of the Z-Score, the average score (X) of the group (M) and the standard deviation of the score (SD) must be known. The formula used is as follows:

\[ Z = \frac{X - M}{SD} \]  

1) The implementation of Z-Score is widely used in ranking a data if it has the same the sum of values because the results of calculations with Z-Score are not affected by the amount of values

3. Fuzzy Multi-Attribute Decision Making (Fuzzy MADM)

Fuzzy Multi-Attribute Decision Making (FUZZY MADM) is a method used to find optimal alternatives from some alternatives with specific criteria. The essence of FADM is to determine the weight values for each attribute, then proceed with a ranking process that will select the alternatives. There are three approaches to finding attribute weight values, namely subjective approaches, objective approaches and integration approaches between subjective & objective [13].

Each approach has strengths and weaknesses. In the subjective approach, the weight value is determined based on the subjectivity of the decision makers, so that several factors in the alternative ranking process can be determined freely. Whereas in the objective approach, the weight value is calculated so that it ignores the subjectivity of the decision maker [14].

Several methods can be used to solve FMADM problems, such as ELECTRE’s Simple Additive Weighting Method (SAW), Weighted Product (WP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and Analytic Hierarchy Process (AHP).

The combination and development of various methods of FMADM have been widely implemented in research that looks for classification of data based on ranking, as done by [15] who developed the TOPSIS method by combining triangular fuzzy membership function in the uncertain attribute. While the combination implementation of FMADM which implements MADM fuzzy hybrid technique on a classification of uncertain information on mobile-based commercial applications [16].

The combination of FMADM was done to improve the performance of multi-attribute analysis so that in this study the Analytic Hierarchy Process (AHP) technique was used as the initial method of determining consistent weight values as implemented [9]. AHP method is widely applied as a Decision Support System tool [17], so basically this algorithm can also be utilized as a ranking support system [18]. Gradually the AHP method starts from defining the problem and determining the desired solution, screening the problem, formulating the issue in a hierarchical structure, forming a pairwise comparison matrix, consistency test [19]. As shown in the following Flowchart.

![AHP Process Flowchart](image)

The weighted product (WP) in this study is used as a weighting normalization technique. As with the FMADM method, WP is a finite set of alternative decisions which are explained regarding several decision criteria. The Weighted Product method uses multiplication to connect the attribute rating, where the rating of each attribute must be raised first with the weight of the attribute in question. This process is the same as the normalization process The level of importance can also be referred to as the weight of interest which can be determined according to needs according to the use of Fuzzy Logic in general. For example, a preference for data consists of Very Unimportant, Unimportant, Important enough, Important, Very important.

Then, the data is converted and normalized by weight (W) with the formula:

\[ W_j = \frac{w_j}{\Sigma w_j} \]  

\( W_j \) is rank positive for profit attribute, and its value is negative for cost attribute. The normalization process as follows the formula:

\[ S_i = \prod_{j=1}^{n} X_{ij}^W \]  

Where, \( S_i \) is the result of normalizing decisions on alternatives of, \( X_{ij} \) is an alternative rating per attribute, \( i \) is alternative index, \( j \) is attribute index, and \( \prod_{j=1}^{n} X_{ij} \) is multiplying alternative ratings per attribute from \( j=1 \) to \( n \), where for every alternative, \( \Sigma W_j = 1 \).

The relative preference of each alternative \( (V) \), as the following formula:

\[ V_i = \frac{\prod_{j=1}^{n} X_{ij}^W}{\prod_{j=1}^{n} X_{ij} \cdot W_j} \]  

Where, \( V_i \) is preference result for alternative-\( i \) and \( \prod_{j=1}^{n} X_{ij} \cdot W_j \) is the sum of multiplication of alternative ratings per attribute.
4. Methodology

This study was conducted starting from literature search, observation, data collection, model development and comparative analysis as compared to previous research comparative analysis techniques [20]. Visually as illustrated in Figure 1.

![Figure 2: Stage of study](Image)

The stages of the study were carried out with literature studies, and research analyzes related to the Z-Score method and FMADM method analysis by the Z-Score combination. At this stage, the results of the state of the art study are obtained, and according to the needs and analysis of the system, the FMADM is applied using AHP and WP. Observation and collection of data are conducted at institutions or fields as well as expert systems to prepare data targets by the themes and topics of these study.

The ranking model development with a combination of Z-Score and FMADM is done by comparing the results of the ranking of each method analyzed. At this stage, the formulation of each technique is developed based on data references. At this stage separately will test the performance of the rating application made using Z Score with the combination of AHP and WP. The purpose of this stage is to produce a formula and develop a rating system.

The combined stages of the method are conducted through the algorithm stages as explained in sub-section 4.2. The results of the ranking development with a combination of Z-Score and AHP-WP (Z-WeP) were then tested using data that had been collected previously. Data testing is done by using 30 alternative controlled data with variate data based on sample level of importance.

4.1 Flowchart Implementation of Z-WeP

From the results of system analysis, according to the flow of research conducted, the implementation of the algorithm in applying the combination of the Z-WeP method as shown in the system flowchart is shown in Figure 2 below.

![Figure 3: Flowchart Implementation of Z-WeP](Image)

Based on the flowchart, it can figure out that the system flow entity consists of Website Administration, which functions as an application interface in inputting data, processing, and output. The initial stage of the Z-WeP algorithm is normalization of weights by calculating consistent weights using the AHP model, then the coded data will be normalized using WP and will be adjusted to the weight preference, according to the level of importance of the data to be processed. Data from the results of WP processing then goes into the processing stage using Z-Score, which will produce data on the Z-Score sum of each criterion. The final stages of this process will produce the order of the largest and smallest of the process Preferences and WP Vectorization of each criterion.

5. Results and Discussion

5.1 Weighting with AHP

In this study, the weighting system was carried out by the AHP method to obtain a weighting value that was consistent with each criterion. The initial stages as presented in previous studies [21]. The initial step was done by determining the priority of elements with a paired comparison matrix, namely comparing elements in pairs according to the criteria given. In this case, four criteria are used (C1-C4) with the importance of simulation values as shown in Table 1.

| Table 1: Pair-wise matrix comparison |
|-----------------------------|----------------|----------------|----------------|----------------|
| Criterion                | C1  | C2  | C3  | C4 |
| C1                        | 1.0 | 3.0 | 4.0 | 5.0 |
| C2                        | 0.333 | 1.0 | 2.0 | 2.0 |
| C3                        | 0.25 | 0.5 | 1.0 | 3.0 |
| C4                        | 0.2  | 0.5 | 0.333 | 1.0 |

As with the AHP process shown in Figure 1, after the value of the paired comparison matrix is obtained, then add the values of each column matrix from table 1, so that C1 (1.703), C2 (5), C3 (7.333) and C4 (11) are obtained. The added value was then divided into each value of each column so that the matrix value was obtained as Table 2.

| Table 2: Number of comparison matrix |
|-----------------------------|----------------|----------------|----------------|----------------|
| Criterion                | C1  | C2  | C3  | C4 |
| C1                        | 0.560 | 0.6 | 0.545 | 0.454 |
| C2                        | 0.186 | 0.2 | 0.272 | 0.181 |
| C3                        | 0.140 | 0.1 | 0.136 | 0.272 |
| C4                        | 0.112 | 0.1 | 0.045 | 0.090 |

Furthermore, the sum of the values of each row is based on table 2, then the amount of the C1 (2.160), C2 (0.841), C3 (0.649) and C4 (0.384) values were obtained. Calculation of Eigen Value (EV) as AHP weight value was done by dividing the sum of the results of each row with a number of criteria (n = 4), so that we get the weight values of each criterion C1 (0.540), C2 (0.210), C3 (0.162) and C4 (0.087).

The final process of using the AHP method measured the consistency of values to ensure that the AHP produce the right weight. Based on Figure 1 in the consistency calculation process the value reached 1.6, with the Consistency Index (CI) by 0.054, and Consistency Ratio (CR) by 0.06. The CR value obtained was 0.06, so the weighting of each criterion can be said to be consistent because of CR <0.1. In this process, if the CR value is more than 0.1, the assessment of the data in the AHP process must be corrected or repeated.

5.2 Normalization with AHP-WP

The weight values obtained in the AHP process, at C1 (0.540), C2 (0.210), C3 (0.162) and C4 (0.087), are used as weights in the WP process as well as the WP process presented in the process Figure 2. Steps did as done in previous studies [22], the results are as shown in table 3.
5.3 Combination of Z-Score and AHP-WP

Processing data using criteria data in the AHP-WP process is then combined with the Z-Score (Z-WeP) technique, where each standardized criterion uses the average value and standard deviation of each criterion. The results of processing criteria data in each alternative using the Z-WeP technique, as shown in Table 4.

Table 4: Results of Z-WeP process

<table>
<thead>
<tr>
<th>Alt.</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>Σ-Z-WeP</th>
<th>Rank Z-WeP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>-0.42</td>
<td>0.61</td>
<td>-0.61</td>
<td>2.45</td>
<td>2.04</td>
<td>5</td>
</tr>
<tr>
<td>A2</td>
<td>2.21</td>
<td>1.01</td>
<td>1.42</td>
<td>0.98</td>
<td>5.62</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>2.72</td>
<td>-1.90</td>
<td>-0.61</td>
<td>0.95</td>
<td>4.17</td>
<td>30</td>
</tr>
<tr>
<td>A4</td>
<td>1.35</td>
<td>0.20</td>
<td>0.57</td>
<td>2.45</td>
<td>4.57</td>
<td>2</td>
</tr>
<tr>
<td>A5</td>
<td>0.18</td>
<td>-1.04</td>
<td>1.14</td>
<td>0.60</td>
<td>0.87</td>
<td>9</td>
</tr>
<tr>
<td>A6</td>
<td>-0.42</td>
<td>0.20</td>
<td>0.85</td>
<td>0.22</td>
<td>0.86</td>
<td>10</td>
</tr>
<tr>
<td>A7</td>
<td>-1.02</td>
<td>-1.04</td>
<td>-1.51</td>
<td>-0.56</td>
<td>-4.14</td>
<td>29</td>
</tr>
<tr>
<td>A8</td>
<td>-1.02</td>
<td>-1.47</td>
<td>-0.61</td>
<td>-0.17</td>
<td>-3.26</td>
<td>27</td>
</tr>
<tr>
<td>A9</td>
<td>1.64</td>
<td>-0.21</td>
<td>0.57</td>
<td>-0.95</td>
<td>1.04</td>
<td>8</td>
</tr>
<tr>
<td>A10</td>
<td>1.35</td>
<td>-0.62</td>
<td>-0.61</td>
<td>-0.17</td>
<td>-0.05</td>
<td>18</td>
</tr>
<tr>
<td>A11</td>
<td>0.77</td>
<td>-0.62</td>
<td>-0.01</td>
<td>0.60</td>
<td>0.73</td>
<td>11</td>
</tr>
<tr>
<td>A12</td>
<td>1.64</td>
<td>-0.62</td>
<td>-0.01</td>
<td>-0.56</td>
<td>0.44</td>
<td>15</td>
</tr>
<tr>
<td>A13</td>
<td>0.77</td>
<td>-0.21</td>
<td>-0.61</td>
<td>0.22</td>
<td>0.17</td>
<td>17</td>
</tr>
<tr>
<td>A14</td>
<td>0.42</td>
<td>1.41</td>
<td>-1.51</td>
<td>-0.95</td>
<td>-1.47</td>
<td>22</td>
</tr>
<tr>
<td>A15</td>
<td>-1.32</td>
<td>-0.62</td>
<td>-0.91</td>
<td>-0.56</td>
<td>-3.41</td>
<td>28</td>
</tr>
<tr>
<td>A16</td>
<td>0.47</td>
<td>1.41</td>
<td>1.42</td>
<td>0.22</td>
<td>3.52</td>
<td>3</td>
</tr>
<tr>
<td>A17</td>
<td>-1.02</td>
<td>-0.21</td>
<td>-2.45</td>
<td>-2.45</td>
<td>-1.22</td>
<td>21</td>
</tr>
<tr>
<td>A18</td>
<td>-0.12</td>
<td>-0.62</td>
<td>1.42</td>
<td>-0.17</td>
<td>0.51</td>
<td>14</td>
</tr>
<tr>
<td>A19</td>
<td>-0.12</td>
<td>1.81</td>
<td>-0.91</td>
<td>-0.56</td>
<td>0.23</td>
<td>16</td>
</tr>
<tr>
<td>A20</td>
<td>0.47</td>
<td>1.01</td>
<td>0.28</td>
<td>-0.56</td>
<td>1.21</td>
<td>7</td>
</tr>
<tr>
<td>A21</td>
<td>-0.72</td>
<td>-0.21</td>
<td>0.85</td>
<td>0.60</td>
<td>0.53</td>
<td>13</td>
</tr>
<tr>
<td>A22</td>
<td>0.47</td>
<td>0.20</td>
<td>0.57</td>
<td>-0.56</td>
<td>0.68</td>
<td>12</td>
</tr>
<tr>
<td>A23</td>
<td>0.42</td>
<td>-0.62</td>
<td>0.85</td>
<td>-1.05</td>
<td>1.75</td>
<td>22</td>
</tr>
<tr>
<td>A24</td>
<td>-0.42</td>
<td>1.41</td>
<td>1.42</td>
<td>-0.17</td>
<td>1.97</td>
<td>6</td>
</tr>
<tr>
<td>A25</td>
<td>-0.12</td>
<td>-1.47</td>
<td>0.28</td>
<td>-0.56</td>
<td>-1.87</td>
<td>24</td>
</tr>
<tr>
<td>A26</td>
<td>-1.32</td>
<td>0.20</td>
<td>-0.91</td>
<td>-0.56</td>
<td>-2.58</td>
<td>26</td>
</tr>
<tr>
<td>A27</td>
<td>-1.32</td>
<td>1.01</td>
<td>-0.61</td>
<td>-0.56</td>
<td>-1.47</td>
<td>23</td>
</tr>
</tbody>
</table>

The ranking comparison was conducted by comparing several types of techniques starting with the ranking technique with (a) the multiplication of WP criteria and weight values, (b) multiplying the criteria value and Z-Score and AHP-WP (Z-WeP). The several types of approaches obtained a ranking pattern as shown in Figure 3.

Figure 4: Model of the comparison method

Based on Figure 4, it can be seen that the ranking with the three techniques has a similar pattern, so that it can be said that the data used can represent the data testing performed. Ranking with a combination of Z-Score and AHP-WP (Z-WeP) techniques produces a ranking sequence that tends to be very similar to the Z-Score technique. This shows that the implementation of the Z-Score combination on the results of data analysis on the WP method can be done. Meanwhile, if compared to the ranking difference, the pattern as shown in Figure 5.

Figure 5: Comparison of ranking differences

Figure 5 showed that the ranking gap between Z-Score and Z-WeP. It has the same ranking tendency, and different things are shown in the comparison of Z-Score and WP rankings, where the ranking gap has the same tendency between Z-WeP and WP. Data analysis by comparing differences in classification and ranking shows that the differences that occur in the ranking after combining the Z-Score technique on AHP-WP reached 86.67% of the ranking similarity. The difference by 13.33% occurs in alternative data A1, A18, A21, and A29.

Data analysis to measure the performance between the Z-WeP and Z-Score approaches can be seen in alternatives A1 and A29, in A1 Z-WeP classifies the 4th rank while the Z-Score classifies 4th place, and vice versa on A1 Z-WeP ranks 4th while Z-Score classifies 5th. Comparative analysis of different alternatives as shown in Table 5.

Table 5: Comparison of criteria values

<table>
<thead>
<tr>
<th>Alternativ e</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>Σ</th>
<th>Rank Z-WeP</th>
<th>Rank.Z-Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>78</td>
<td>81</td>
<td>91</td>
<td>90</td>
<td>34</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>A28</td>
<td>-0.72</td>
<td>1.41</td>
<td>-0.91</td>
<td>-0.17</td>
<td>-0.38</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>A29</td>
<td>0.77</td>
<td>0.61</td>
<td>0.85</td>
<td>-0.17</td>
<td>2.06</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>A30</td>
<td>-1.32</td>
<td>-1.04</td>
<td>0.57</td>
<td>-0.17</td>
<td>-1.97</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>
Table 5 shows that A29 has better criterion values for larger values, although the highest value on A1 is in C3 and C4, the high value is in the lowest weighting criteria. Likewise, if analyzed from the sum of criteria, values on the A29 is better than A1. Other comparisons are seen in A18 and A21, wherein A18 Z-WeP classifies 14th place while Z-Score classifies 13th, and vice versa on A21 Z-WeP classifies 13th place while Z-Score classifies ranked 14th. Although in terms of the number in Alternative A18 it has a higher value and different number of A21, the ranking using Z-WeP gives a better rating in A2, groups of values influence this at C2 and C4 which have more deviation values small compared to C1 and C3, where A18 has a higher value.

6. Conclusion

The results of the data analysis and discussion show that the combination of methods is a breakthrough so that it can be a rating recommendation system, even with further study and implementation can be used as a classification technique. The results of testing and report on the data used can be concluded that by using a combination of Z-Score and AHP-WP methods (Z-WeP) can provide ranking recommendations that are different from conventional statistical models, even with the stand-alone FMADM method. The combination of Z-Score statistical techniques with the MADM method was able to provide a better ranking analysis and according to the weighting needs. Also, the test results show that the ranking with the Z-WeP technique is very thorough in giving a rating because it can trace the results of the classification based on the weighting of criteria, the sum of alternatives, to the averages and deviations of values contained in each criteria. Of course, the use of this method is only suitable for a ranking and classification of data with weighted criteria. For further development can be used on more criteria data, besides that the use of the Z-WeP method is one of the FMADM combination techniques for statistical techniques for ranking and classification so that for further development other FMADM models can be developed.

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