Interval-Valued Intuitionistic Fuzzy INK-Ideals of INK-Algebra

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Abstract

Mathematical structures of Interval valued IF INK-ideal on INK-algebras are presented. We established that every IF- (INK)-ideal of an INK algebra A can be executed as an level ideal (INK) of an IF(INK) of U. As far as the idea of homomorphism, we talked about the cartesian result of i-v IF(INK)- ideal.

Keywords: INK-algebra, INK-ideal, fuzzy INK-ideal, IF INK-ideal, Interval-valued fuzzy INK-ideal, Interval-valued intuitionistic fuzzy INK-ideal

1. Introduction

Several researchers have developed algebraic structures. Imai and Iséki (1966) proposed two algebraic structure BCI and BCK-algebras. Then Hu and Li (1983) are expanding algebra is called BCH-algebra, which is a generalization of BCK- and BCI-algebras. J. Næs and H. S. Kim (1999) introduced d-algebras and studied the relationship flanked by d-algebras and BCH-algebras. Conceptual Fs and i-v Fs are then introduced by Zadeh (1982). Zadeh also used his i-v Fs to construct an near reasoning system. In addition, Attansouv (1986) introduced the concept of Intuitive Fuzzy Sets (IFs) and Interval-valued IVIFS as a generalization of ordinary FS. Attassouv and Gargov [1989] show that IFs and IVIFS are equal probability generalizations of FS. In this paper, we first introduce an i-v IF INK-ideal INK-algebra. Then we prove that each intuitionistic fuzzy INK ideal of INK algebra A can be executed as the ideal INK ideal of intuitionistic fuzzy INK- U. In relation to the concept of homomorphism, we study the Cartesian product of the interval. Pay attention to intuitionistic fuzzy INK-ideal.

2. Preliminaries

Definition 2.1: A INK-algebra $(U, \cdot, 0)$ is a nonvoid set $U$ with a non-varying value $0$ and a binary operation $\cdot$ fulfilling the additional adages

i) $a \cdot 0 = a$

ii) $(b \cdot a) \cdot (b \cdot c) = a \cdot c$ $\forall a, b, c \in U$.

In $U$ we can regulate a binary association by $a \leq b$ if and only if $a \cdot b = 0$.

The nonvoid subset $T$ of INK-algebra $(U, 0)$ is called sub-algebra of $U$, and if it is $a \cdot b \in T$, $\forall a, b \in T$.

Let $S$ be a nonvoid subset of $U$. Then $S$ is called a INK-ideal of $(U, 0)$ if (1) $0 \in A$, (2) $(c \cdot a) \cdot (c \cdot b) \in S$ and $c \in S \iff c \in S, \forall a, b, c \in U$.

Definition 2.4: Let $U$ be a universe set, $\chi$ be a Fs in $U$ is a mapping $\chi: U \rightarrow [0,1]$.

Definition 2.2: A Fs $\chi$ in a INK-algebra $X$ is called a fuzzy subalgebra of $U$ if $\chi(a \cdot b) \geq \min \{ \chi(a), \chi(b) \}, \forall a, b \in U$.

Definition 2.3: A IFs $\chi$ is in the nonvoid $U$, in the form $\{U, \chi_a, \chi_b, \chi_c\} : x \in U | \chi_a : U \rightarrow [0,1]$ and $\lambda : U \rightarrow [0,1]$ means the degree of membership of each member $x \in U$ (ie. $\chi_a(a)$ and the degree of non-membership (in particular, $\delta_a(a), 0 \leq \chi_a(a) \leq \delta_a(a) \leq 1, \forall x \in U$. We use the character $\chi = \{U, \chi_a, \delta_a\}$ for IFs $A = \{U, \chi_a, \delta_a\}$.

Definition 2.4: The IFs $\chi = \{U, \chi_a, \delta_a\}$ in a INK-algebra $U$ is called an IF INK-ideal of $U$ if

1. $\chi_a(0) \geq \chi_a(a)$ in addition $\delta_a(0) \leq \delta_a(a)$
2. $\chi_a(a) \geq \min \{ \chi_a(c \cdot a) \cdot (a \cdot b), \chi_a(b) \}$
3. $\delta_a(a) \leq \max \{ \delta_b(c \cdot a) \cdot (a \cdot b), \delta_b(b) \}, \forall a, b, c \in U$.

3. I-v IFs -ideals (INK)

Definition 3.1: An i-v IFs in INK-algebra $U$ is called an i-v IF INK-ideal of $U$ if it satisfies,

1. $\chi_a(0) \geq \chi_a(a)$, $\delta_a(0) \leq \delta_a(a)$
2. $\chi_a(a) \geq \min \{ \chi_a((c \cdot a) \cdot (c \cdot b)), \chi_a(b) \}$
3. $\delta_a(a) \leq \max \{ \delta_b((c \cdot a) \cdot (c \cdot b)), \delta_b(b) \}$

Example 3.2.

Discuss the following table of INK-Algebra: $U = \{0, p, q, r\}$.

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>p</td>
<td>p</td>
</tr>
<tr>
<td>q</td>
<td>q</td>
</tr>
<tr>
<td>r</td>
<td>r</td>
</tr>
<tr>
<td>p</td>
<td>p</td>
</tr>
</tbody>
</table>

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Let $A$ be an $i$-v IF in $\mathfrak{B}$ by
$$\begin{align*}
\psi_A(x) &= \begin{cases}
0 & \text{if } [0,0.6,0.8] 
0.4, 0.5] 
0.3, 0.4] \end{cases} 
\phi_A(x) &= \begin{cases}
0 & \text{if } [0,0.3, 0.4] 
0.5, 0.7] 
0.4, 0.6] \end{cases}
\end{align*}
$$
Then routine calculations give that $A$ is an $i$-v intuitionistic fuzzy INK ideal of $U$.

**Theorem 3.3.** Let $\tau$ be an $i$-v IF INK-ideal of $U$. If there exists a sequence $\{x_n\}$ in $\tau$ such that $\lim_{n \to \infty} x_n = [1,1]$, $\lim_{n \to \infty} \tau(x_n) = [0,0]$. Then $\overline{\tau}(0) = [1,1]$ and $\tilde{\tau}_i(0) = [0,0]$.

**Proof.** Since $\overline{\tau}(x) \leq \overline{\tau}(x)$ and $\tilde{\tau}_i(x) \leq \tilde{\tau}_i(x) \forall x \in U$. We have $\overline{\tau}(0) \geq \overline{\tau}(x) \land \tilde{\tau}_i(0) \leq \tilde{\tau}_i(x)$, $n$ is a positive integer.

Note that $[1,1] \geq [\overline{\tau}(x), \tilde{\tau}_i(x)]$. If $n \geq [\overline{\tau}(x), \tilde{\tau}_i(x)]$ is an IF of U.

Let $x, y \in U$, then
$$\begin{align*}
\overline{\tau}(x) &= \min \{[\overline{\tau}(x), \overline{\tau}(x)], [\overline{\tau}(x), \overline{\tau}(x)]\} = \min \{[\overline{\tau}(x), \overline{\tau}(x)], [\overline{\tau}(x), \overline{\tau}(x)]\}
\end{align*}
$$

And all of these $\tau$ is an $i$-v IF of $U$. TUYT8777Y9698689

Similarly, expect that $A$ is an $i$-v IF of $U$.

**Proposition 3.7.** Every $i$-v IF INK-ideal of a INK-ideal algebra $U$ is $i$-v IF sub algebra of $U$.

**4. Product of $i$-v intuitionistic fuzzy INK-ideal**

**Definition 4.1.** An IF, $\varnothing$ on any set is a $i$-v IF subset $A$ with a membership function $HWHWVHWHVHJVHJ: \mathbb{U} \times [0,1]$ and a non-membership function $XHCHFTS6988=5654 \forall x, y \in \mathbb{U}$, where $\varnothing \{U \times V \}$.

**Definition 4.2.** Let $\hat{u} = \{\hat{u}_i, \hat{u}_i\}$ and $\hat{w} = \{\hat{w}_i, \hat{w}_i\}$ be two $i$-v IFs in a set $U$. The product of $\hat{u} \times \hat{w}$ is defined by $GTU990$.

**Theorem 3.4.** An $i$-v IF IF $\tau = \{[\overline{\tau}(x), \tilde{\tau}_i(x)]\}$ is an $i$-v intuitionistic fuzzy INK ideal of $U$ if and only if $\overline{\tau}(x)$ and $\tilde{\tau}_i(x)$ are IF of $U$.

**Proof.** Since $\overline{\tau}(x) = \min \{[\overline{\tau}(x), \overline{\tau}(x)], [\overline{\tau}(x), \overline{\tau}(x)]\}$ and $\tilde{\tau}_i(x) = \max \{[\tilde{\tau}_i(x), \tilde{\tau}_i(x)], [\tilde{\tau}_i(x), \tilde{\tau}_i(x)]\}$.

Along these lines, $\tau$ is an $i$-v IF of $U$. TUYT8777Y9698689

**Proposition 3.5.** Every $i$-v IF INK-ideal of a INK-algebra $U$ is an $i$-v IF.

**Definition 3.6.** An IF $\tau$ in $U$ is called an interval-valued intuitionistic fuzzy INK-sub algebra of $U$ if
i) $\overline{\tau}(x, y) \geq \max \{\overline{\tau}(x), \overline{\tau}(y)\}$ and
ii) $\tilde{\tau}_i(x, y) \leq \max \{\tilde{\tau}_i(x), \tilde{\tau}_i(y)\}$, $\forall x, y \in U$.
Definition 4.4. Let $\overrightarrow{\omega}$ and $\overleftarrow{\omega}$ respectively be an i-v membership and non-membership function of every element $x \in U$ to the set $\omega$. Then strongest i-v IFS relation on $U$, that is a membership function relation $\overrightarrow{\omega}$ and $\overleftarrow{\omega}$ whose i-v membership and non-membership function of every element $(x, y) \in U \times U$ and defined by

i) $\overleftarrow{\omega}(x, y) = \min\{|\overleftarrow{\omega}(x), \overleftarrow{\omega}(y)|$ and

ii) $\overrightarrow{\omega}(x, y) = \max\{|\overrightarrow{\omega}(x), \overrightarrow{\omega}(y)|$

Definition 4.5. Let $\phi = \{(\overrightarrow{\phi}, \overleftarrow{\phi}) : (u, v) \in U \times U\}$ be an i-v subset in a set $U$, then the strongest i-v IFS relation on $U$ that is a i-v A on $\omega$ is $\phi$ and defined by

$\delta_{\phi} = \{(\mu_{\phi}(x, y), \mu_{\phi}(x, y)) : (u, v) \in U \times U\}$

Theorem 4.6. Let $\phi = \{(\overrightarrow{\phi}, \overleftarrow{\phi}) : (u, v) \in U \times U\}$ be an i-v subset in a set $U$ and be the strongest i-v IFS relation on $U$, at that point $\phi$ is an i-v INK-ideal of $U$ if and only if $\phi$ is an i-v INK-ideal of UxU.

Proof. Let $B$ be an i-v IF INK-ideal of U,

$\overrightarrow{\omega}(0, 0) = \min\{|\overrightarrow{\omega}(0), \overrightarrow{\omega}(0)|$

Furthermore, $\max\{|\overrightarrow{\omega}(0), \overrightarrow{\omega}(0)| = \overrightarrow{\omega}(0, 0)$

and

$\delta_{\phi}^*(0, 0) = \max\{|\delta_{\phi}(0), \delta_{\phi}(0)|$

On the other hand,

$\overrightarrow{\omega}(x, y) = \min\{|\overrightarrow{\omega}(x, y), \overrightarrow{\omega}(y)|$

$r \min\{|\overrightarrow{\omega}(x, y), \overrightarrow{\omega}(y)|$

Also,

$\overleftarrow{\omega}(x, y) = \max\{|\overleftarrow{\omega}(x, y), \overleftarrow{\omega}(y)|$

$r \min\{|\overleftarrow{\omega}(x, y), \overleftarrow{\omega}(y)|$

Equally, let $\delta_{\phi}$ be an i-v IF-ideal (INK) of UXU.

$x, y \in U \times U$. Then

$r \min\{|\delta_{\phi}(x, y), \delta_{\phi}(y)|$

and

$\delta_{\phi}(x, y) \leq \delta_{\phi}(y)$

$r \min\{|\delta_{\phi}(x, y), \delta_{\phi}(y)|$

$r \min\{|\delta_{\phi}(x, y), \delta_{\phi}(y)|$

Hence complete the proof.
Theorem 4.9. If \( \tilde{X}_\phi \) is also an i-v IF INK-ideal of \( U \), then \( \tilde{X}_{\phi\omega} \) is also an IF INK-ideal of \( U \).

Proof. For all \( x, y, z \in U \).

\[
\begin{align*}
\tilde{X}_{\phi\omega}(0) & \geq \tilde{X}_\phi(x) \\
\min \{ \tilde{X}_\phi(x), \tilde{X}_\omega(y) \} & \geq \min \{ \tilde{X}_{\phi\omega}(x), \tilde{X}_{\phi\omega}(y) \}
\end{align*}
\]

Also,

\[
\begin{align*}
\tilde{d}(x) & \leq \tilde{d}(x) \\
\tilde{d}(x) & \leq \tilde{d}(x) \\
\tilde{d}(x) & \leq \tilde{d}(x) \\
\tilde{d}(x) & = \tilde{d}(x)
\end{align*}
\]

5. Homomorphism of i-v Intuitionistic fuzzy INK-Algebra

Definition 5.1. Let \((U, \cdot, 0)\) and \((V, \cdot, 0)\) be INK-algebras. A function \( \phi: U \to V \) is called a homomorphism if

\[
\phi(x \cdot y) = \phi(x) \cdot \phi(y), \quad \forall x, y \in U.
\]

For any interval-valued IF \( X = (V, \tilde{X}_\phi, \tilde{d}_\phi) \) in \( V \) we define a new i-v IF \( \hat{X} = (U, \hat{X}_\phi, \hat{d}_\phi) \) in \( U \) by

\[
\hat{X}_\phi = \tilde{X}_\phi(\phi(x)) \quad \text{and} \quad \hat{d}_\phi = \tilde{d}_\phi(\phi(x)).
\]

Theorem 5.2. Let \((U, \cdot, 0)\) and \((V, \cdot, 0)\) be INK-algebras. An onto homomorphic image of an i-v IF INK-ideal of \( U \) is also an i-v IF INK-ideal of \( V \).

Proof. Let \( \psi: U \to V \) be an onto homomorphism of INK-algebras. Suppose \( \chi = (V, \tilde{X}_\phi, \tilde{d}_\phi) \) is the image of an i-v IF INK-ideal \( \rho = (U, \tilde{X}_\phi, \tilde{d}_\phi) \) of \( U \). We have to prove that \( \rho = (V, \tilde{X}_\phi, \tilde{d}_\phi) \) is an i-v IF INK-ideal of \( V \). Since \( h: U \to V \) is onto, then \( x, y, z \in V \) there exist \( x, y, z \in U \) such that \( \phi(x) = x \) and \( \phi(y) = y \) also we study \( \phi(z) = 0 \).

Then,

\[
\begin{align*}
\tilde{X}_\phi(0) & = \tilde{X}_\phi(0) \\
\tilde{d}_\phi(0) & = \tilde{d}_\phi(0) \\
\tilde{d}_\phi(y) & \geq \tilde{d}_\phi(x)
\end{align*}
\]

Also,

\[
\begin{align*}
\tilde{d}_\phi(x) & = \tilde{d}_\phi(x) \\
\tilde{d}_\phi(y) & = \tilde{d}_\phi(y)
\end{align*}
\]

Hence \( A = (V, \tilde{X}_\phi, \tilde{d}_\phi) \) is an i-v IF INK-ideal of \( U \).

Theorem 5.3. Let \((X, \cdot, 0)\) and \((Y, \cdot, 0)\) be INK-algebras. An onto homomorphic inverse image of an i-v IF INK-ideal of \( V \) is also an i-v IF INK-ideal of \( U \).

Proof. Let \( \psi: U \to V \) be an onto homomorphism of INK-algebras. Suppose \( \psi = (X, \psi \tilde{X}_\phi, \psi \tilde{d}_\phi) \) is the inverse image of an i-v IF INK-ideal \( \chi = (V, \tilde{X}_\phi, \tilde{d}_\phi) \) of \( V \).

For any \( x, y, z \in V \) there exist \( x, y, z \in U \) such that \( \psi(x) = x \) and \( \psi(y) = y \) and \( \psi(z) = 0 \).

\[
\begin{align*}
\tilde{X}_\phi(0) & = \tilde{X}_\phi(0) \\
\tilde{d}_\phi(x) & = \tilde{d}_\phi(x)
\end{align*}
\]

implies that \( \psi \tilde{X}_\phi(0) = \psi \tilde{X}_\phi(x) \) and \( \psi \tilde{d}_\phi(x) = \tilde{d}_\phi(x) \).
\[ \leq \bar{\sigma}(x) \]
\[ = \bar{\sigma}(\psi(x)) \]

Implies that \( \psi \bar{\sigma}(0) = \psi \bar{\sigma}(x) \).
\[ \psi \bar{\sigma}(x) = \bar{\sigma}(\psi(x)) \]
\[ = \bar{\sigma}(x) \]

\[ \psi \bar{\sigma}(x) = \min \{ \bar{\sigma}(z \cdot x) \cdot (z \cdot y), \bar{\sigma}(y) \} \]
\[ = \min \{ \bar{\sigma}(\psi(z \cdot x) \cdot (z \cdot y)), \bar{\sigma}(\psi(y)) \} \]
\[ = \min \{ \bar{\sigma}(\psi((z \cdot x) \cdot (z \cdot y))), \bar{\sigma}(\psi(y)) \} \]
\[ \geq \min \{ \psi \bar{\sigma}(z \cdot x) \cdot (z \cdot y), \psi \bar{\sigma}(y) \} \]
and
\[ \psi \bar{\sigma}(x) = \bar{\sigma}(\psi(x)) \]
\[ = \bar{\sigma}(x) \]
\[ \psi \bar{\sigma}(x) = \max \{ \bar{\sigma}(z \cdot x) \cdot (z \cdot y), \bar{\sigma}(y) \} \]
\[ = \max \{ \bar{\sigma}(\psi(z \cdot x) \cdot (z \cdot y)), \bar{\sigma}(\psi(y)) \} \]
\[ = \max \{ \bar{\sigma}(\psi((z \cdot x) \cdot (z \cdot y))), \bar{\sigma}(\psi(y)) \} \]
\[ \psi \bar{\sigma}(x) \leq \max \{ \psi \bar{\sigma}(z \cdot x) \cdot (z \cdot y), \psi \bar{\sigma}(y) \} \]

Therefore, complete the proof.

6. Conclusions

In this paper, we INK-algebra between the current value of the member and non-member functions of IF-INK classical with introducing the concept, and to study their properties.

References