Mean Square Cordial Labeling of Some Cycle Related Graphs

S Dhanalakshmi1, N.Parvathi2

1Department of Mathematics, Faculty of Engineering and Technology, SRM IST, Chennai -600089, India
2Department of Mathematics, Faculty of Engineering and Technology, SRM IST, Chennai -600089, India
*Corresponding author Email : parvathi.n@ktr.srmuniv.ac.in

Abstract

In this paper we investigate the Mean square cordial labeling for some cyclic graphs like Helm graph H_p, Closed helm graph CH_p, Gear graph G_p, Sunlet graph SL_p, Fan graph F_{1,p} and C_pOnK_2.

Keywords: Mean square cordial labeling, Closed helm graph, Gear graph, Sunlet graph.

1. Introduction

Many graph labeling [1] techniques have been discussed by different researchers and it is still getting enrichment due to its broad range of applications in various fields like electrical circuit theory, social psychology, addressing communication network Systems, finding optimal circuit layouts, channel assignment process etc.. For basics terms and notations we follow Harary[2]. Cahit initiated the Cordial labeling [3] and Ponraj and et al[4] paved the way to mean cordial labeling of a graph. Mean square cordial labeling (MSCL) was first introduced by A.Nellai murugan et al and this labeling technique is applied for few classes of graphs[5]. In addition to that they discussed that some path, tree and cycle related graph admits MSCL [6],[7]. Along with that MSCL of some acyclic graphs and upper approximations [8] were studied by Dhanalakshmi et al. In this paper we investigate the MSCL for some cyclic graphs like Helm graph H_p, Closed helm graph CH_p, Gear graph G_p, Sunlet graph SL_p, Fan graph F_{1,p} and C_pOnK_2.

2. Preliminaries

A.Nellai murugan and et al defined “A Mean Square Cordial labeling (MSCL) of a Graph G(V,E) with p vertices and q edges is a bijection from V to {0, 1} such that each edge uv is assigned the label \( \left( \left\lfloor \frac{1}{2} f(u)^2 + f(v)^2 \right\rfloor \right) \) where \( \left\lfloor x \right\rfloor \) (ceil(x)) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1”.

3. Main results

Theorem 3.1 MSCL of a Helm graph H_p when p is odd p \geq 3.

Proof: Consider a helm graph H_p be G. Let the vertex set and edge set as V(G) = {u_0, v_i, u_{i+1}, 1 \leq i \leq p}. It is very clear that the above labeling pattern proved that the...
difference between the vertices and edges labeled with 0 and 1 differ at most by 1. Hence helm graph $H_p$ admits MSCL when $m$ is odd $\forall p \geq 3$.

**Example 3.1: MSCL of a Helm graph $H_p$ is shown in the Fig 1**

![Fig 1](image1)

**Theorem: 3.2 MSCL of a closed helm graph $CH_p \ \forall p \geq 2$.**

Proof: Consider a closed helm graph $P_\theta K_p$ be $G$.

Let $V(G) = [u_0, v_0, u_i: 1 \leq i \leq p]$ and $E(G) = \{(u_0, u_i): 1 \leq i \leq p\} \cup \{(v_0, v_i): 1 \leq i \leq p\} \cup \{(u_i, u_{i+1}): 1 \leq i \leq p\} \cup \{(v_i, v_{i+1}): 1 \leq i \leq p\}$ where $u_0, u_i$ and $v_i$ be the centre vertex, vertices on a inner cycle and vertices on a outer cycle respectively.

Consider the elements of the vertex set maps either 0 or 1.

$$f(u_0) = 0$$
$$f(u_i) = 0, 1 \leq i \leq p$$
$$f(v_0) = 1$$
$$f(v_i) = 1, 1 \leq i \leq p$$

The edge labeling pattern of the above vertex labeling is as follows

$$f(u_0u_i) = 0, 1 \leq i \leq p$$
$$f(u_0u_{i+1}) = 1, 1 \leq i \leq p-1$$
$$f(v_0v_i) = 1$$
$$f(v_0v_{i+1}) = 1, 1 \leq i \leq p-1$$
$$f(u_iu_{i+1}) = 0$$

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ at most by 1.

Hence closed helm graph $CH_p$ admits MSCL when $\forall p \geq 3$.

**Example 3.2: MSCL of a closed helm graph $CH_p$ shown in the Fig 2**

![Fig 2](image2)

**Theorem: 3.3 MSCL of a Gear graph $G_p$ when $p$ is odd $\forall p \geq 3$.**

Proof: Consider a Gear graph $G_p$ be $G$

Let $V(G) = [u_0, v_0, w_i: 1 \leq i \leq p]$ and $E(G) = \{(u_0, u_i): 1 \leq i \leq p\} \cup \{(v_0, v_i): 1 \leq i \leq p\} \cup \{(w_0, w_i): 1 \leq i \leq p\}$

Consider the elements of the vertex set maps either 0 or 1.

$$f(u_0) = 0$$
$$f(u_i) = 0, 1 \leq i \leq p$$
$$f(v_0) = 1$$
$$f(v_i) = 1, 1 \leq i \leq p$$
$$f(w_0) = 0$$
$$f(w_i) = 1$$

The theorem follows

$$f(u_0u_i) = 0, 1 \leq i \leq p$$
$$f(u_0u_{i+1}) = 1, 1 \leq i \leq p-1$$
$$f(v_0v_i) = 1$$
$$f(v_0v_{i+1}) = 1, 1 \leq i \leq p-1$$
$$f(u_iu_{i+1}) = 0$$

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ at most by 1.

Hence Gear graph $G_p$ admits MSCL when $m$ is odd $\forall p \geq 3$.

**Example 3.3: MSCL of a Gear graph $G_p$ is shown in the Fig 3.**

![Fig 3](image3)

**Theorem: 3.4 MSCL of a sunlet graph $S_p \ \forall p \geq 2$.**

Proof: Consider a sunlet graph $S_p$ be $G$

Consider $V(G) = [v_0, u_i: 1 \leq i \leq p]$ and $E(G) = \{(u_0, u_i): 1 \leq i \leq p\}$
\[ \{(u_v):1 \leq i \leq p\} \cup \{(u_u):1 \leq i \leq p\} \] where \( u \) and \( v \) be the vertices on a cycle and pendent vertices respectively.

Consider the elements of the vertex set maps either 0 or 1.

\[
\begin{align*}
f(u) &= 0, \quad 1 \leq i \leq p \\
f(v) &= 1, \quad 1 \leq i \leq p \\
f(u) &= 0
\end{align*}
\]

The edge labeling pattern of the above vertex labeling is as follows

\[
\begin{align*}
f((u_u)) &= 0, \quad 1 \leq i \leq p \\
f((u_u)) &= 1, \quad 1 \leq i \leq p \\
f(u) &= 0
\end{align*}
\]

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.

Hence Fan graph \( F_{1,p} \) admits MSCL when \( p \) is even \( \forall \, p \geq 2 \).

Example 3.4 MSCL of a Sunlet graph \( SL_4 \) shown in the Fig 4

![Fig 4](image)

Theorem: 3.5 MSCL of a Fan graph \( F_{1,p} \), when \( p \) is even \( \forall \, p \geq 2 \).

Proof: Consider a Fan graph \( F_{1,n} \) be \( G \)

Let \( V(G) = \{u_0,u_1, \ldots, u_n\} \) and \( E(G) = \{(u_{2i}), \ldots, (u_{2i+n})\} \) where \( u_0 \) and \( u_1 \) be the apex vertex and the remaining vertices other than apex respectively.

Consider the elements of the vertex set maps either 0 or 1.

\[
\begin{align*}
f(u_0) &= 0 \\
f(u_i) &= \begin{cases} 
0, & 1 \leq i \leq \frac{p}{2} \\
1, & \frac{p}{2} + 1 \leq i \leq p 
\end{cases}
\end{align*}
\]

The edge labeling pattern of the above vertex labeling is as follows

\[
\begin{align*}
f((u_{2i})) &= 0, \quad 1 \leq i \leq \frac{p}{2} \\
f((u_{2i+n})) &= 1, \quad \frac{p}{2} + 1 \leq i \leq p \\
f(u_0) &= 0
\end{align*}
\]

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.

Hence Sunlet graph admits MSCL when \( \forall \, p \geq 3 \).

Example 3.5 MSCL of a Fan graph \( F_{p,s} \), shown in the Fig 5

![Fig 5](image)

Theorem: 3.6 MSCL of a graph \( C_p\cdot OnK_1 \), when \( n \) is odd \( \forall \, p \).

Proof: Consider a graph \( C_p\cdot OnK_1 \) be \( G \)

Let \( V(G) = \{u_0,u_1, \ldots, u_n\} \) and \( E(G) = \{(u_{2i}), \ldots, (u_{2i+n})\} \) where \( u_0 \) and \( u_1 \) be the vertices on a cycle and pendent vertices respectively.

Consider the elements of the vertex set maps either 0 or 1.

\[
\begin{align*}
f(u_0) &= 0, \quad 1 \leq i \leq p \\
f(u_i) &= 1, \quad 1 \leq i \leq p \\
f(u) &= 0
\end{align*}
\]

The edge labeling pattern of the above vertex labeling is as follows

\[
\begin{align*}
f((u_{2i})) &= 0, \quad 1 \leq i \leq \frac{p}{2} \\
f((u_{2i+n})) &= 1, \quad \frac{p}{2} + 1 \leq i \leq p \\
f(u_0) &= 0
\end{align*}
\]

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.

Hence Sunlet graph admits MSCL \( \forall \, p \).

Example 3.6 MSCL of a graph \( C_p\cdot OnK_1 \), shown in the Fig 6

![Fig 6](image)

5. Conclusion

Here we investigated the MSCL for different cyclic graphs. Further it is an open to all the researchers in this domain to discuss the same labeling technique for various types of graphs.

Acknowledgements

The authors are very much obliged and extend their gratitude to the reviewers for their valuable feedback and ideas for the
betterment of article.

References

[1] Gallian A Dynamic Survey of Graph Labeling Electronic