On R#-Closed and R#-Open Maps in Topological Spaces

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Abstract

Defining and investigating properties of R\(^#\)-closed maps, R\(^#\)-open maps, R\(^#\ Here denotes the function or, and R\(^#\)-open maps in topological spaces. 2010 Mathematics Classification: 54A05, 54A10

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1. Introduction


2. Preliminaries

In the paper M or (M,\(\tau\)) and N or (N,\(\sigma\)) denote topological spaces and separation axioms are not assumed. For A\subseteq M, cl(A), int(A), M-A, represent closure of A, interior of A and complement of A in M respectively. \(CD\) denotes the closed, \(h\) denotes the function or map of the function.

Definition 2.1: \(BC\subseteq M\) is said to be R\(^#\)-C(M) [2] if \(\text{gcd}(B)\subseteq W\) whenever \(B\subseteq W\) and \(W\) is R\(^#\)-O(M).

Definition 2.2: \(h\) is said to be R\(^#\)-continuous [5] if \(h^{-1}(V)\) is R\(^#\)-C(M) for every CD \(V\) of Y.

Definition 2.3: \(h\) is said to be R\(^#\)-Irresolute [1] if \(h^{-1}(V)\) is R\(^#\)-C(M) for every R\(^#\)-subset \(V\) of Y.

Results 2.4: [1]
i. Every CD (respectively r-CD, g-CD, p-CD, \(\tilde{g}\)-CD) set is R\(^#\)-C(M).
ii. Every R\(^#\)-C(M) is \(\text{gcd}(\text{respectively gpr-CD, rwg-CD, gspr-CD, r}\tilde{g}\text{-CD, rg}\tilde{g}\text{-CD})\) set in M.

Results 2.5: [1] Assume \(BC\subseteq M\) then
i. If B is r-o(M) and rg-C(M), then B is R\(^#\)-C(M) in M.
ii. If B is g-o(M) and rg-C(M) then B is R\(^#\)-C(M).
iii. If B is r-o(M) and rwg-C(M) then B is R\(^#\)-C(M).
iv. If B is r-o(M) and gpr-C(M) then B is R\(^#\)-C(M).
v. If B is r-o(M) and r\(\tilde{g}\)-g(C(M) then B is R\(^#\)-C(M).
vi. If B is r-o(M) and \(\beta\tilde{g}\) g\(^#\)-C(M) then B is R\(^#\)-C(M).

Definition 2.6: \(BG\subseteq X\) is said to be T\(^#\)-space [1] if every R\(^#\)-C(M) is CD

Results 2.7: [3]
1. If G and H \(\subseteq M\) then
   i. R\(^#\)-cl(G)=G
   ii. If H is any R\(^#\)-O(X) set contained in G then H=\(\text{R}\tilde{g}\text{-int}(G)\)
2. If N\(\subseteq M\) is R\(^#\)-O(M) then N is a R\(^#\)-nbd of each of its points

3. R\(^#\)-closed maps and R\(^#\)-open maps

Definition 3.1: Assume \(g\) is known as R\(^#\)-C(M) (resp-R\(^#\)-O(M)), if the image of every CD (resp-open) in X is R\(^#\)-C(M) in N.

Theorem 3.2: If g is CD map then it is R\(^#\)-CD map, converse is false.

Proof: Assume g be CD map, \(\forall\subseteq C(M)\). Then g(A)\subseteq C(N), hence \(f(M)\) is R\(^#\)-C(N) set in Y, g is R\(^#\)-C(M).

Example 3.3: Accept M=\{1, 2, 3\}, \(\tau=\{\emptyset, M, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}\}\), \(\sigma=\{\emptyset, N, \{1\}, \{2\}, \{1, 2\}\}\). Let g be an identical map then g is R\(^#\)-C(N) but not a CD map, G=\{2\} in M is g\(^#\)-C(Y).

Theorem 3.4: If g is g-CD map then it is R\(^#\)-CD map, converse is false.
Assume $M=\mathbb{N} \subseteq [1, 2, 3, 4]$, $\tau=\{\emptyset, M, \{1\}, \{2\}, \{1, 2\}\}$, and $\sigma=\{\emptyset, N, \{1\}, \{2\}, \{1, 2, 3\}\}$, $g$ be map $g(1)=1$, $g(2)=2$, $g(3)=3$, $g(4)=4$. $g$ is $R^2$-CD but not a $g$-$CD$ map because closed set $\{4\}$ in $M$ is $\{1, 2, 3\}$ which is not $g$-$C(N)$.

**Theorem 3.6:** If $g$ is w-$CD$ map (resp. $\hat{g}$-$CD$ map, r-$CD$ map) then it is $R^2$-$CD$ map converse is false.

Proof: The proof is obvious every w-$CD$ set is $R^2$-$CD$.

**Example 3.7:** Assume $M=\mathbb{N} \subseteq [1, 2, 3, 4]$, $\tau=\{\emptyset, M, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$, $g$ be an identical map then it is $R^2$-$CD$, not a $g$-$CD$ map because closed $CD$ $\{1, 2, 3\}$ in $M$ is $\{1, 2, 3\} \notin w$-$C(Y)$.

**Theorem 3.8:** If $g$ be $R^2$-$CD$ map then it is $rg$-$CD$ converse is false.

Proof: Accept $g$ be $R^2$-$CD$ map and $\text{AEC}(M)$. Then $g(A)$ is $R^2$-$C(N)$ and hence $g(A)$ is $rg$-$C(N)$. Hence $g$ is $rg$-$CD$.

**Example 3.9:** In example 3.7 $g(1)=1$, $g(2)=2$, $g(3)=3$, $g(4)=4$, $g(2)=2$ is $rg$-$CD$ but not a $R^2$-$CD$ map, $G=\{4\}$, $g(1)=1 \notin R^2$-$C(N)$.

**Theorem 3.10:** Each $R^2$-$CD$ map is $g$-$spr$-$CD$ (resp. $g$-$pr$, $rg$, $rgf$, $r^g$, $r^g_w$, $wgpr$-closed) converses is false.

Proof: The proof is obvious every $R^2$-$CD$ map is $g$-$spr$-$C(M)$.

**Example 3.11:** In example 3.7, $g(1)=1$, $g(2)=2$, $g(3)=3$, $g(4)=4$ is $g$-$spr$-$CD$ but not a $R^2$-$CD$ map as $g(1)=1 \notin \{1, 2\}$ in $Y \notin R^2$-$C(N)$.

**Remark 3.12:** $R^2$-$CD$ map is independent with some existing CD maps in topological spaces as below.

**Example 3.13:** In example 3.11. Assume $f$ be a map defined by $g(1)=1$, $g(2)=2$, $g(3)=3$, $g(4)=4$ is $R^2$-$CD$ map but not a $rg$-$CD$ map.

**Example 3.14:** Assume $M=\mathbb{N} \subseteq [1, 2, 3, 4]$, $\tau=\{\emptyset, M, \{1\}, \{2\}, \{1, 2\}\}$, $g$ be a map defined by $g(1)=1$, $g(2)=2$, $g(3)=3$, $g(4)=4$ is pre. Semi, sp, $b$, $swg$, $g\alpha$, $sgb$, $rg\alpha$, $b, gw\alpha$, $\alpha$, $rgb\alpha$, $\alpha$, $sgb\alpha$, $rg\alpha$, $\alpha$, $sgb\alpha$, $rg\alpha$, $\alpha$, $sgb\alpha$ and $\#rg$-$CD$ set in $M$ but not $R^2$-$C(N)$.

**Remark 3.15:** From the discussion and facts, the relation between $R^2$-$CD$ map and some existing CD maps in topological space is shown as follows.

**Theorem 3.16:** If $g$ is contra r-$CD$ and r$\omega$-$CD$ map then $g$ is $R^2$-$CD$ map.

Proof: Accept $V \subseteq C(X)$. Then $g(A)$ is r-O(N) and r$\omega$-$C(N)$. By results $2.5$, $f(A)$ is $R^2$-$C(N)$. Therefore $f$ is $R^2$-$CD$ map.

**Theorem 3.17:** If $g$ is contra r-$CD$ and $r$-$wg$-$CD$ map then $g$ is $R^2$-$CD$ map.

Proof: Accept $V \subseteq C(M)$. Then $g(A)$ is r-O(N) and r$\omega$-$C(N)$. By results $2.5$, $g(A)$ is $R^2$-$C(N)$. Therefore $f$ is $R^2$-$CD$ map.

**Theorem 3.18:** If $g$ is contra r-$CD$ and $g$-$pr$-$CD$ map then $g$ is $R^2$-$CD$ map.

**Remark 3.25:** The composition of two $R^2$-$CD$ maps need not be $R^2$-$CD$.

**Example 3.26:** Assume $M=\mathbb{N} \subseteq [1, 2, 3, 4]$, $\tau=\{\emptyset, M, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$ and $\eta=\{\emptyset, Z, \{1\}, \{2\}, \{1, 2, 3\}\}$. The identical map of $f$ and $g$ are $R^2$-$CD$ maps. $g$ be another map defined by $g(1)=1$, $g(2)=2$, $g(3)=3$, $g(4)=4$. Then the composition $g$-$C(X)$-$Z$ is not a $R^2$-$CD$ map as the image of the closed set $\{s\}$ in $M$ is $\{s\}$ which is not a $R^2$-$CD$ set in $Z_n$. That is $g(1)=3=g(3)=1, 2, 3$.

**Theorem 3.27:** If $g$ is CD map and $h$ is $R^2$-$CD$ map then the composition $gh$ is $R^2$-$CD$ map.
Proof: Accept $F$-$C(M)$. Since $g$ is CD then $g(F)$-$C(N)$. As $g$ is $R^2$-CD map then $g(f(F))$ is $R^2$-$C(Z)$, therefore $gof$ is $R^2$-$CD$ map.

**Remark 3.28:** If $g$ is $R^2$-CD map and $h$ is CD map then the composition homog: $M \rightarrow Z$ need not be a $R^2$-CD map.

**Example 3.29:** Assume $M=\mathbb{N}=\{1, 2, 3, 4\}$, $\tau=\{\emptyset, N, \{1, 2\}, \{1, 3\}\}$, $\sigma=\{\emptyset, Z, \{1\}\}$ and $\eta=\{\emptyset, Z, \{1, 2\}\}$. Let a map $f$ defined as $f(1)=4, f(2)=2, f(3)=4, f(4)=2$ and $g$ as $g(1)=1, g(2)=1, g(3)=4, g(4)=1$. Then $f$ and $g$ are $R^2$-CD maps and their composition $gof$ is not a $R^2$-CD map as the image of the CD set $\{3, 4\} \in X$ is $\{1, 2\} \in R^2$-$C(Z)$ That is $(gof)(4)=g(f(4))=g(1, 2, 3)\neq\{1, 2\}$.

**Theorem 3.30:** Assume $g$ and $h$ are $R^2$-CD maps and $N$ is $T^*_g$ space then $hog$ is $R^2$-CD map. If $N$ is $T\omega$ space, then $hog$ is $R^2$-CD map.

Proof: Let $H\in\mathcal{C}(M)$. Since $g$ is $R^2$-CD then $g(H)$ is $R^2$-$C(N)$. Since $Y$ is $T^*_g$ space $h(h)$ is CD. Since $g$ is $R^2$-C, $h(g(H))$ is $R^2$-$C(Z)$. Therefore $gof$ is $R^2$-CD map.

**Theorem 3.31:** If $f$ is $g$-CD map, $g$ is $R^2$-CD map and $N$ is $T\omega$ space then $frog$ is $R^2$-closed map.

Proof: Let $H\in\mathcal{C}(M)$. As $f$ is $g$-CD then $f(H)$ is $g$-$C(N)$. By hypothesis, $f\in\mathcal{C}(M)$. Given $g$ is $R^2$-CD, then $g(f(A))$ is $R^2$-$C(Z)$. Therefore $gof$ is $R^2$-CD map.

**Definition 3.32:** $h$ is known as $R^2$-open map if $g(A)$ is $R^2$-$O(Y) \forall O(M)$.

By the known facts we have

**Theorem 3.33:**

1. Each open set (respectively r,g,w, g-open) sets in $M$ is $R^2$-open map, converse is false.
2. Each $R^2$-O map is rg (respectively gpr,rgw, gspr,r*g,rgβ-open) converse is not prove.

**Theorem 3.34:** For every bijection map $g$, the following results are identical

1. $g^{-1}: Y\rightarrow X$ is $R^2$-continuous
2. $g$ is $R^2$-$O$ map
3. $g$ is $R^2$-CD map

**Theorem 3.35:** Assume a map $g$ is $R^2$-$O$ then $g(int(H)) \subseteq R^2$-$O(int(g(U))) \forall A \subseteq X$.

Proof: For a open map $g$, let $H\subseteq M$ and $int(H)$ be $O(M)$. Then $g(int(H))$ is $R^2$-$O(N)$. We know that $g(int(H)) \subseteq g(H)$. By results 2.7, $g(int(H)) \subseteq R^2$-$O(int(g(H))$space. For the $R^2$-closed set $\{q\}$ with $\eta \{q\}$, $\exists$ no disjoint open sets $U, V \subseteq \{q\} \subseteq U$ and $r \in V$.

**Theorem 3.36:** If a map $g$ is $R^2$-O map then for each neighbourhood $U$ of $n$ in $M$, $\exists$ a $R^2$-nbd $W$ of $f(x)$ in $N$ such that $W \subseteq f(U)$.

Proof: Given a $R^2$-open map $g$. Let $x\in M$ and $U$ be a nbd of $x$ in $M$. By the definition of $R^2$-nbd $\exists R^2$-$O(G(M)$ such that $x \subseteq G(U)$. Consider $g(x) \subseteq g(U) \subseteq g(U)$. Since $g$ is $R^2$-$O$, then $g(G)$ is $R^2$-$O(N)$. By results 2.7, $F(G)$ is $R^2$-nbd of each of its points. Take $g(G)=W$, $W$ is $R^2$-nbd of $g(x)$ in $N$ such that $W \subseteq f(U)$.

4. $R^2^g$-closed maps and $R^2^g$-open maps

**Definition 4.1:** A map $g$ is said to be a $R^2^g$-closed map if the image $g(A)$ is $R^2^g$-$C(N) \forall R^2^g$-$C(A)$ in $M$.

**Theorem 4.2:** Every $R^2^g$-closed map is a $R^2$-CD converse is false.

Proof: The proof follows is obvious that every CD set is $R^2$-CD.

**Example 4.3:** Assume $M=\mathbb{N}=\{1, 2, 3, 4\}, r=\{\emptyset, N, \{1, 2\}, \{1, 3\}\}$ and $\eta=\{\emptyset, Z, \{1\}\}$. Let $f$ be a map, $f(1)=4, f(2)=2, f(3)=2, f(4)=4$ is $R^2$-CD map but not a $R^2^g$-CD map as the image of the $R^2^g$-$C\{1, 2\}$ in $M$ is $\{3\}$ which is not a $R^2^g$-$C(N)$.

**Theorem 4.4:** If the maps $g$ and $h$ are $R^2^g$-C maps then the composition homog: $M \rightarrow Z$ is $R^2^g$-$C$.

Proof: Accept $F$ by any $R^2^g$-$C(M)$. Since $g$ is $R^2^g$-$C$ then $g(F)$ is $R^2^g$-$C(N)$. Since $h$ is $R^2^g$-$C$ then $h(g(F))$ is $R^2^g$-$C(Z)$. Therefore $hog$ is $R^2^g$-$C$ map.

**Definition 4.5:** A map $g$ is known as $R^2^g$-open map if $g(H)$ is $R^2^g$-$O(N) \forall R^2^g$-$O(H(M)$.

**Theorem 4.6:** Each $R^2$-$O$ map is $R^2^g$-$O$ map converses is false.

Proof: The proof is obvious.

**Example 4.7:** Assume $M=\mathbb{N}=[1, 2, 3], r=\{\emptyset, N, \{1\}, \{1, 2\}\}$ and $\eta=\{\emptyset, Z, \{1\}\}$. Let $f$ be a map defined by $f(1)=3, f(2)=1, f(3)=2$ is a $R^2$-$O$ map but not a $R^2^g$-$O$ map as the image of the $R^2$-$O$ set $\{1, 2\}$ in $M$ is $\{2, 3\}$ in $N$ which is not a $R^2^g$-$O(N)$.

**Theorem 4.8:** Assume the maps $g$ and $h$ are $R^2^g$-$O$ map then their composition homogg is also $R^2^g$-$open$.

Proof: The proof is as theorem 4.6.

5. Conclusion

In this paper we defined and studied $R^2$-closed maps, $R^2$-open maps, $R^2^g$-closed maps and $R^2^g$-open maps.

References


