Creation of Model and Calculation of Thermal Processes in the Ring Graphite Electrode of the Plasmatron

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Abstract

This paper presents the theoretical and experimental material obtained in the study of the erosion and thermal state of the ring graphite electrode for a plasmatron. Thermal processes in graphite electrodes of plasmatrons are quite complex and multifaceted. A mathematical model of thermal processes that occur at the ring electrodes of plasmatrons has been developed. The mathematical model is based on the differential heat conduction equation for a ring electrode in cylindrical coordinates. With the use of this mathematical model, the inverse problem of heat conduction is solved: determination of the regularities of the heat exchange process by the temperatures of individual points on a solid surface. An experimental study of the temperature distribution at the end of the electrode and along the length of the electrode was carried out. Experiments have shown that the temperature on the side surfaces drops sharply towards the cold end of the electrode. When reducing the length of the electrode, the maximum temperature at the end decreases, and the temperature on the inner and outer edges of the electrode increases slightly. The most significant factors determining the temperature field at the end of the ring electrode are the power and size of the heat source. Comparison of the results of experimental studies and mathematical modeling showed a match with an acceptable degree of accuracy.

Keywords: graphite electrode, ring electrode, thermal processes, thermal conductivity, electric arc, plasmatron, thermal characteristics.

1. Introduction

If the dependence on heat flux or on temperature is known at the boundary of a solid, then the temperature distribution in the whole body can be determined. This is the so-called direct problem. In many cases, when analyzing dynamic heat transfer processes, the law of change in heat flux or surface temperature must be determined from the data of temperature measurements at one or several internal points of the solid. This is the inverse problem. In particular, during the last decades, the specific problem of estimating the thermal boundary condition from the results of measurements of the internal temperature is called the inverse problem of heat conduction.

One of the first papers on the inverse heat conduction problem was published by Stolz back in 1960. It described the method for calculating heat fluxes with different cooling of bodies having simple geometric shapes and finite sizes. For the semi-infinite body, the same Murseppassian approach was used, and both numerical and graphical methods were used for it. Practically at the same time, in the field of fundamental and applied mathematics a new direction was being formed concerning the methods for solving ill-posed problems. At the same time, academician A.N. Tikhonov for the first time formed his own and now generally accepted and widely used regularization method. The inverse heat conduction problem is one of many problems that are ill-posed in the mathematical sense. These are, as a rule, inverse problems for which the solution is extremely sensitive to measurement errors. Currently, an extensive bibliography on methods for solving incorrect inverse problems of heat conduction has more than a thousand papers. Most of the papers consider methods for solving ill-posed problems, their algorithmic implementation, and the results of calculations using model examples. And only a small part of the research is devoted to solving specific problems with specific experimental data. So, for example, in the work of M.P. Kuzmichev the temperature of the hot medium, the heat transfer coefficient and the heat flux density during asymmetric heating are determined from the results of measuring the wall temperature at three points at a distance from a hot surface. Also of some practical interest is the work of S.L. Balakovsky, the author investigated the dependence of the error in recovering the heat flux density at the boundary on the accuracy of setting the coordinates of the temperature sensor located inside the object.

There are extremely few works on inverse heat conduction problems in relation to ceramic materials. In the works of N.V. Pashatsky and V.F. Obesnyuk the method is proposed for identifying the temperature dependence of the thermal conductivity of ceramic materials based on solving the inverse thermal conductivity problem. The installation for conducting experiments is described. The temperature dependence of the thermal conductivity in the ceramic materials was obtained, and the hyperbolic temperature asymptotics was found when the silicate surface was heated by a moving and extended heat source. The results of the analytical assessment of the maximum temperature at different depths are consistent with the measurement data obtained when a plate was processed by a plasma generator. The authors stipulate the range of applicability of the obtained asymptotics (0.5 ≤ z ≤ 150 mm, where z is the heating depth) and further show how with its help, provided that the diameter of the plasmatron head is known, the inverse problem can be partially solved: to estimate the power of the surface heat source.
Taking into account the above analysis of literary material in the study of heat transfer processes during electric arc processing of materials, numerical simulation and creation of engineering methods for calculating temperature fields in graphite electrodes of a plasma generator are the most relevant.

2. Methods

Thermal processes in graphite electrodes of the plasmatron are quite complex and multifaceted. When building a model and numerical calculation of the temperature fields in graphite electrodes, it is necessary to investigate the influence of such factors as the geometry of the electrodes, the dependence of the thermal conductivity of the material on temperature, heat loss on radiation and evaporation of material from the outer side and end surfaces, convective heat removal by plasma gas, Joule heat generation, and also to take into account the final size of the surface heat source. From this point of view, the calculation of the ring electrode temperature is of the greatest theoretical interest. Figure 1 shows a diagram with the design of the plasmatron head, consisting of ring and rod graphite electrodes between which plasma-forming gas is fed through an annular gap.

![Figure 1: Schematic drawing of the plasmatron head.](image)

The following assumptions were made for calculations: the size changes of the electrodes due to erosion are insignificant; the heat flux is delivered only through the reference spots of the electric arcs on the surface of the electrode end. Joule heat dissipation inside the electrode is neglected, since, according to [1], it amounts to less than 3% of the total heat flux into the electrode. Since heat sources are surface, their action is taken into account in the boundary conditions by specifying the heat flux density q. It is further assumed that the sources are uniformly smeared out along a ring of width δ at the end of the electrode. Such a transition from spots to a ring (arc track) is justified by the fact that, as experiments show, arc spots are in continuous motion near the middle circle on the end surface. The diameter of the spots coincides with the width of the arc path and is equal to

\[ d_s = \delta = (2 \pm 3) \times 10^{-3} \text{ m} \]  

(1)

The differential equation of thermal conductivity of a ring electrode in cylindrical coordinates is as follows:

\[
\frac{\partial T}{\partial r} = \frac{\partial}{\partial r} \left[ a(T) \frac{\partial T}{\partial r} \right] + a(T) \frac{1}{r} \frac{\partial}{\partial r} \left[ a(T) \frac{\partial T}{\partial r} \right]
\]  

(2)

Equation (2) is solved under the following boundary conditions:

\[
\frac{\partial T}{\partial r} \bigg|_{r=r_1} = hT
\]  

(3)

\[
\frac{\partial T}{\partial r} \bigg|_{r=r_2} = \frac{1}{\rho c a(T)} \left[ bT^4 + v(T)p(cT + Q_{cor}) \right]
\]  

(4)

\[
\frac{\partial T}{\partial z} \bigg|_{z=0} = \frac{1}{\rho c a(T)} \left[ bT^4 + v(T)p(cT + Q_{cor}) - q \right]
\]  

(4)

\[
T(r, l, \tau) = T_0
\]  

(6)

Initial conditions:

\[
T(r, \tau, 0) = T_0
\]  

(7)

and the following relations are satisfied:

\[
q = \frac{\rho}{\pi \delta (r_1 + r_2)}
\]

\[
r_1 \leq r \leq \frac{r_1 + r_2 - \delta}{2} u \bigg( \frac{r_1 + r_2 + \delta}{2} \bigg) < r \leq r_2
\]

The value of h in (3) determines only the convective heat removal from the inner surface of the ring electrode. The value of h * for the swirling flux is selected based on the data [2].

The temperature of the external (end) electrode surfaces is decisively influenced by thermal radiation, therefore, we neglect the convection of heat for these surfaces.

Under conditions (4) and (5), the first term in square brackets describes the process of radiation, and the second describes the process of electrode material evaporation.

The product of the graphite density by its specific heat capacity \( \rho \cdot c \) is assumed to be constant and independent of temperature [3].

To calculate the thermal conductivity of graphite, the formula given in [4] is used:

\[
a(T) = 3.64 \cdot 10^{-5} \left[ \exp \left( \frac{575}{T} \right) - 1 \right]
\]  

(8)

The dependence of the evaporation rate \( v(T) \) of graphite on temperature is described by the formula (1).

3. Results and discussion

The values of the geometric and physical parameters in the calculation were as follows: \( r_1 = 0.03 \), \( r_2 = 0.05 \), \( l = 0.2 \), \( \delta = 2.5 \cdot 10^{-3} \text{ m} \), \( \rho = 1700 \text{ kg/m}^3 \), \( c = 2078 \text{ J/kg K} \), \( Q = 2.14 \cdot 10^{-7} \text{ J/kg (for c)} \), \( b = 5.67 \cdot 10^{-8} \text{ W/(m}^2 \text{K}^4) \), \( T_0 = 293 \text{ K} \). The power \( P \) of the warm flow from the arcs into the electrode is \( 2 \cdot 10^4 \text{ W} \), as in [6].

For numerical calculations using this model, the method of variable directions was used using an implicit finite difference scheme of the first order of accuracy in time and second order of accuracy in spatial variables \( Q(h_1, h_2, h_z, z) \) [7]. The calculation was carried out with the use of a \( 8 \times 40 \) grid \( (r, x, z) \) with a time step \( h_x = 0.25 \text{ s} \), the counting error is \( \pm 20 \text{ K} \). The stationary mode along the electrode, as was shown in [6], is achieved in about 5 minutes. Due to the fact that from a technological point of view, the temperature field on the working surface of the end face is of greatest interest, and here the steady state is achieved in almost 1–2 minutes, all calculations were carried out for \( \tau \leq 60 \text{ s} \). From figure 2 it can be seen that the state close to the stationary state is set at approximately 60 s at the working end of the electrode, as expected. The maximum temperature is 3040 K.
The temperature on the side surfaces drops sharply towards the cold end of the electrode: it decreases by almost half at $z = (0.1 \div 0.2) \cdot l$ (Figure 4). Despite the different boundary conditions on the outer and inner surfaces of the electrode, the temperatures of these surfaces differ slightly from each other ($\Delta T \approx 100 \text{ K}$ at $z = 0$) and this difference disappears at a distance $z \approx 0.1 \cdot l$ from the working end.

When reducing the length of the electrode, the maximum temperature at the end decreases, and the temperature on the inner and outer edges of the electrode slightly increases (curve 1 in Figure 5). The same picture is observed if in the course of calculations we take the graphite thermal conductivity (in the temperature range $293 \div 3000 \text{ K}$) constant and equal to $12.5 \cdot 10^6 \text{ m}^2/\text{s}$. Finally, if to neglect the evaporative member under conditions (4) and (5), the temperature increases markedly with approaching to the center of the source, remaining almost the same as in the main model on the edges of the electrode.

Since the size of the source (the average diameter of the reference spot) and its power depend on the mode of operation of the plasma head, it is also of interest to investigate the dependence of the electrode temperature on these factors. With increasing the width $\delta$ of the arc path (from $2.5 \cdot 10^{-3}$ to $7.5 \cdot 10^{-3}$ m), which in practice may correspond to the transition to a stationary thermal state (heating) of electrodes, the temperature at the point of maximum at the end decreases sharply (Figure 6). The change in power of the heat source (within $\pm 25\%$) also has a significant effect on temperature, especially in the center of the electrode’s end.

**4. Summary**

Thus, the numerical experiment showed that the most significant factors determining the temperature field at the end of the ring electrode are the power and size of the heat source. The dependence of the thermal conductivity on temperature and the evaporation of the material have less influence on the temperature.
of the electrode. The conservative calculated temperature value which is close to stationary, on the inner and outer edges of the end face compared with the experimental value [6] can be explained by the neglect in the model of radial movements of the arc spots, which equalize the temperature at the end of the electrode.

5. Conclusions

The developed mathematical model made it possible to carry out a theoretical study of the thermal processes occurring on circular graphite electrodes of plasmatrones. Experimental studies have confirmed the adequacy of the developed model. This made it possible to identify patterns in thermal processes at the electrodes and to determine the factors that have the greatest influence on the heat transfer processes.

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References