Bipolar Fuzzy Soft Hyperideals and Homomorphism of Gamma-Hypersemeigroups

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Abstract

In this paper, we introduce the concept of bipolar fuzzy soft gamma hyperideals in gamma hyper semigroups. We define bipolar fuzzy soft hyper ideals, bi-ideals and interior ideals of gamma hyper semigroups and discuss some properties.

Keywords: Soft set, Γ- hyper semigroups, bipolar valued fuzzy set, hyper ideal, homomorphism.

1. Introduction

Zadeh [18] introduced the concept of fuzzy sets in 1965. Algebraic hyper structures represent a natural extension of classical algebraic structures, and they were originally proposed in 1934 by Marty [7]. One of the main reasons which attract researchers towards hyperstructures is its unique property that in hyperstructures composition of two elements is a set, while in classical algebraic structures the composition of two elements is an element. Zhang [19] introduced the notion of bipolar fuzzy sets. Lee [4] used the term bipolar fuzzy sets as applied to algebraic structures. Bipolar fuzzy Γ-hyperideals in Γ-hyper semigroups was studied by Naveed Yaqoob et al [14]. Soft set theory was introduced by Molodtsov [8] in 1999, and its a new mathematical model for dealing with uncertainty from a parameterization point of view. Maji et al [6] studied the some new operations on fuzzy soft sets. Aygunoglu and Aygun [3] introduced the notion of a fuzzy soft group. The concept of bipolar fuzzy soft sets has been introduced by Naz et al [12]. Aslam et al [2] worked on bipolar fuzzy soft sets and their special union and intersection. Bipolar fuzzy soft Γ-semigroups was introduced by Muhammad Akram et al [10]. Γ-semigroups was introduced by Sen and Saha [16]. In this paper, we define a new notion of bipolar fuzzy soft Γ- hyper semigroups and investigate some of its properties with examples.

2. Preliminaries

In this section, we list some basic definitions.

Definition 2.1 [16]
Let S = {a, b, c, …} and {α, β, γ, …} be two non-empty sets. Then S is called a Γ-semigroup if it satisfies the conditions
(i) αβδ ∈ S,
(ii) (αβ)γ = α(βγ) ∀ α, β, γ ∈ Γ.

Definition 2.2
A map o : H × H → P*(H) is called a hyper operation or join operation on the set H, where H is a non-empty set and P*(H) = P(H)\{ϕ} denotes the set of all non-empty subset of H. A hypergroupoid is a set H together with a (binary) hyperoperation.

Definition 2.3
A hypergroupoid (H, o), which is associative, that is x o (y o z) = (x o y) o z for all x, y, z ∈ H, is called a hyper semigroup. Let A and B be two non-empty subsets of H. Then we define

A o B = \{a o b | a ∈ A, b ∈ B\}

Definition 2.4 [1]
Let S and Γ be two non-empty sets. S is called a Γ-hypersemigroup if every γ ∈ Γ is a hyperoperation on S that is xγy ⊆ S for every x, y ∈ S, and for every α, β ∈ Γ and x, y, z ∈ H we have xγ(yβz) = (xγy)βz. If every γ ∈ Γ is a hyper operation, then S is a Γ-semigroup. If (S, γ) is a hypergroup for every γ ∈ Γ, then S is called a Γ-hypergroup. Let A and B be two non-empty subsets of S and γ ∈ Γ. We define AγB = U{ayb|a ∈ A, b ∈ B}.

Also AΓΓB = U{ayb|a ∈ A, b ∈ B and γb} = U AΓB . Let S be a Γ-hypersemigroup and let γ ∈ Γ. A non-empty subset A of S is called a Γ-hypersemigroup of S if aγa2 ⊆ A for every aγa2 ∈ A. A Γ-semihypergroup S is called commutative if for all x, y ∈ S and γ ∈ Γ we have xyγ = yxγ.

Definition 2.5 [8] Let U be an universe set and E be the set of parameters. P(U) denote the power set of U. Let A be a non empty subset of E then the pair (F, A) is called a soft set over U, where F is a mapping given by F : A → P(U).

Definition 2.6
[18] Let X be a non-empty set. A fuzzy subset μ of X is a function from X into the closed unit interval [0,1]. The set of all fuzzy subset of X is called the fuzzy power set of X and is denoted by FP(X).
Definition 2.7 [4]
A bipolar fuzzy set $A$ in a universe $U$ is an object having the form
\[ A = \{ (x, \mu^{+}_{A}(x), \mu^{-}_{A}(x)) : x \in U \}, \]
where $\mu^{+}_{A}: X \rightarrow [0,1]$ and $\mu^{-}_{A}: X \rightarrow [-1,0]$. Here $\mu^{+}_{A}(x)$ represents the degree of satisfaction of an element $x$ to the property and $A = \{ (x, \mu^{+}_{A}(x), \mu^{-}_{A}(x)) : x \in X \}$ and $\mu^{-}_{A}(x)$ represents the degree of satisfaction of $x$ to some implicit counter property of $A$. For simplicity the symbol $(\mu^{+}_{A}, \mu^{-}_{A})$ is used for the bipolar fuzzy set $A = \{ (x, \mu^{+}_{A}(x), \mu^{-}_{A}(x)) : x \in X \}$.

Definition 2.8 [2]
Let $U$ be the universe set and $E$ be the set of parameter. Let $A \subseteq E$ and $B_{E}^{U}$ denotes the set of all bipolar fuzzy subsets of $U$. Then a pair $(F, A)$ is called a bipolar fuzzy soft sets over $U$, where $F$ is a mapping given by $F: A \rightarrow B_{E}^{U}$. It is defined as $(F, A) = (\{ (x, \mu^{+}_{A}(x), \mu^{-}_{A}(x)) : x \in U \land a \in A \})$ for any

\[ a \in A, F(a) = \{ (x, \mu^{+}_{F(a)}(x), \mu^{-}_{F(a)}(x)) : x \in U \} \equiv (\mu^{+}_{F(a)}(x), \mu^{-}_{F(a)}(x)) \] for $a \in A \land x \in U$.

Definition 2.9 [2]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$, then $(F, A)$ AND $(G, B)$ denoted by $(F, A) \Lambda (G, B)$ is defined as $(F, A) \Lambda (G, B) = (C, H)$ where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in A \times B$.

Definition 2.10 [2]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$, then $(F, A)$ OR $(G, B)$ denoted by $(F, A) \vee (G, B)$ is defined as $(F, A) \vee (G, B) = (C, H)$ where $C = A \times B$ and $H(a, b) = F(a) \cup G(b)$, for all $(a, b) \in A \times B$.

Definition 2.11 [2]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$ then their extended union is a bipolar fuzzy soft set over $U$ denoted by $(F, A) \cup_{u} (G, B)$ and is defined by $(F, A) \cup_{u} (G, B) = (C, H)$ where $C = A \cup B, H : C \rightarrow B_{E}^{U}$ and

\[ F(c) \ if \ c \in A \land \neg B \]
\[ H(c) = G(c) \ if \ c \in B \land \neg A \]
\[ F(c) \cap G(c) \ if \ c \in A \land B \]

Definition 2.12 [2]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$ then their extended intersection is a bipolar fuzzy soft set over $U$ denoted by $(F, A) \cap_{u} (G, B)$ and is defined by $(F, A) \cap_{u} (G, B) = (C, H)$ where $C = A \cap B, H : C \rightarrow B_{E}^{U}$ and

\[ F(c) \ if \ c \in A \land \neg B \]
\[ H(c) = G(c) \ if \ c \in B \land \neg A \]
\[ F(c) \cap G(c) \ if \ c \in A \land B \]

Definition 2.13 [13]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$ such that $A \land B \neq \emptyset$. The restricted union of $(F, A)$ and $(G, B)$ is defined to be a bipolar fuzzy soft set $(H, C)$ over $U$ where $C = A \cup B$ and $H(c) = F(c) \cup G(c)$, for all $c \in C$. This is denoted by $(H, C) = (F, A) \cup_{r} (G, B)$.

Definition 2.14 [11]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$ such that $A \land B \neq \emptyset$. The restricted intersection of $(F, A)$ and $(G, B)$ is defined to be a bipolar fuzzy soft set $(H, C)$ over $U$ where $C = A \cap B$ and $H(c) = F(c) \cap G(c)$, for all $c \in C$. This is denoted by $(H, C) = (F, A) \cap_{r} (G, B)$.

Definition 2.15 [9]
Let $(F, A)$ be a bipolar fuzzy soft set over $U$ for each $t \in [0,1]$ and $s \in [-1,0]$ the set $(F, A)^{(t,s)} = (F^{(t,s)}, A)$ where $(F^{(t,s)}, A)_{a} = (x \in U | \mu^{+}_{F(a)}(x) \geq t, \mu^{-}_{F(a)}(x) \leq s)$ for all $a \in A$.

Definition 2.16 [17]
Let $\phi: H_{1} \rightarrow H_{2}$ and $h: E_{1} \rightarrow E_{2}$ be two maps, $A \subseteq E_{1}$ and $B \subseteq E_{2}$, where $E_{1}$ and $E_{2}$ are sets of parameters viewed on $H_{1}$ and $H_{2}$, respectively. The pair $(\phi, h)$ is called a fuzzy soft map from $H_{1}$ to $H_{2}$. If $\phi$ is a hypergroup homomorphism, then $(\phi, h)$ is called a fuzzy soft homomorphism from $H_{1}$ to $H_{2}$.

Definition 2.17 [3]
Let $(A, (G, B))$ be two fuzzy soft sets over $H_{1}$ and $H_{2}$, respectively, and $(\phi, h)$ be a fuzzy soft function from $H_{1}$ to $H_{2}$

(i) The image of $(\phi, h)$ under the soft function $(\phi, h)$ denoted by $(\phi, h)(A, (G, B))$, is a fuzzy soft set over $H_{2}$ defined by $(\phi, h)(A, (G, B)) = (\phi(h), h(A))$, where for all $b \in h(A)$ and for all $y \in H_{2}$, then

\[ \phi(h)(y) = \begin{cases} \vee y & \text{if } y \in \phi^{-1}(y) \\ 0 & \text{otherwise} \end{cases} \]

(ii) The inverse image of $(\phi, h)$ under the fuzzy soft function $(\phi, h)$ denoted by $(\phi, h)^{-1}(b, (G, B))$, is a fuzzy soft set over $B$ defined by $(\phi, h)^{-1}(b, (G, B)) = (\phi^{-1}(y), h^{-1}(A))$, where for all $\phi \in h^{-1}(A)$ and for all $x \in H_{1}$, $\phi^{-1}(y)(x) = g_{h(a)}(\phi(x))$. If $\phi$ and $h$ is injective (surjective), then $(\phi, h)$ is said to be injective (surjective).

3. Bipolar Fuzzy Soft $\Gamma$-Hyper Ideals

In this section, we introduce the notion of bipolar fuzzy soft gamma hyperideals in gamma semigroups and discuss some of its properties $S$ denotes the $\Gamma$-hyper semigroup.

Definition 3.1
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$-hypersemigroups $S$ is called a bipolar fuzzy soft $\Gamma$-subhypersemigroup over $S$ if

(i) $\inf_{xy} \mu_{F(a)}(x) \geq \min(\mu_{F(a)}(y), \mu_{F(a)}(z))$

(ii) $\sup_{xy} \mu_{F(a)}(x) \leq \max(\mu_{F(a)}(y), \mu_{F(a)}(z))$ for all $x, y, z \in S$, $x, y, z \in S$, $y \in \Gamma$ and $a \in A$.

Definition 3.2
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$-hypersemigroups $S$ is called a bipolar fuzzy soft left $\Gamma$-hyperideal over $S$ if

(i) $\inf_{xy} \mu_{F(a)}(x) \geq \mu_{F(a)}(z)$

(ii) $\sup_{xy} \mu_{F(a)}(x) \leq \mu_{F(a)}(z)$ for all $x, y, z \in S$, $y \in \Gamma$ and $a \in A$.

Definition 3.3
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$-hypersemigroups $S$ is called a bipolar fuzzy soft right $\Gamma$-hyperideal over $S$ if

(i) $\inf_{xy} \mu_{F(a)}(x) \geq \mu_{F(a)}(y)$

(ii) $\sup_{xy} \mu_{F(a)}(x) \leq \mu_{F(a)}(y)$ for all $x, y, z \in S$, $y \in \Gamma$ and $a \in A$.

Definition 3.4
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$-hypersemigroups $S$ is called a bipolar fuzzy soft $\Gamma$-hyperideal of $S$ if

(i) $\inf_{xy} \mu_{F(a)}(x) \geq \max(\mu_{F(a)}(y), \mu_{F(a)}(z))$
(ii) $\sup_{z \in xy} \mu_{F}(a)(x) \leq \min\{\mu_{F}(a)(y), \mu_{F}(a)(z)\}$ for all $x, y, z \in S$,

$\gamma \in \Gamma$ and $a \in A$.

**Definition 3.5**

A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$-hypersemigroups $S$ is called a bipolar fuzzy soft $\Gamma$-hyperbi-ideal over $S$ if

(i) $\inf_{p \in xy \in \mathbb{B}} \mu_{F}(a)(p) \geq \min\{\mu_{F}(a)(x), \mu_{F}(a)(z)\}$

(ii) $\sup_{p \in xy \in \mathbb{B}} \mu_{F}(a)(p) \leq \max\{\mu_{F}(a)(x), \mu_{F}(a)(z)\}$ for all $x, y, z \in S$,

$\alpha, \beta \in \Gamma$ and $a \in A$.

**Definition 3.6**

A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$-hypersemigroups $S$ is called a bipolar fuzzy soft $\Gamma$-hyperinterior ideal over $S$ if

(i) $\inf_{p \in xy \in \mathbb{B}} \mu_{F}(a)(p) \geq \min\{\mu_{F}(a)(x), \mu_{F}(a)(y)\}$

(ii) $\sup_{p \in xy \in \mathbb{B}} \mu_{F}(a)(p) \leq \max\{\mu_{F}(a)(x), \mu_{F}(a)(y)\}$ for all $x, y, z \in S$,

$\alpha, \beta \in \Gamma$ and $a \in A$.

**Theorem 3.7**

Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hypersemigroups over $S$, then $(F, A) \vee (G, B)$ and $(F, A) \wedge (G, B)$ are bipolar fuzzy soft $\Gamma$-hypersemigroup of $S$.

**Proof.** Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hypersemigroups over $S$ defined as $(F, A) \wedge (G, B)$ where $\mathbb{C} = A \times B$. Let $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in \mathbb{C} = A \times B$, $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\inf_{z \in xy} \mu_{H}^+(a)(z) = \inf_{z \in xy} \mu_{F}^+(a)(z), \mu_{G}^-(b)(z))$$

$$\sup_{z \in xy} \mu_{H}^+(a)(z) = \sup_{z \in xy} \mu_{F}^+(a)(z), \mu_{G}^-(b)(z))$$

Hence $(F, A) \wedge (G, B)$ is a bipolar fuzzy soft $\Gamma$-hyper-bi-ideal over $S$.

**Theorem 3.8**

Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hyper-semigroups over $S$ defined as $(F, A) \vee (G, B)$ where $\mathbb{C} = A \times B$. Let $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in \mathbb{C} = A \times B$, $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\inf_{z \in xy} \mu_{H}^-(a)(z) = \inf_{z \in xy} \mu_{F}^-(a)(z), \mu_{G}^-(b)(z))$$

$$\sup_{z \in xy} \mu_{H}^-(a)(z) = \sup_{z \in xy} \mu_{F}^-(a)(z), \mu_{G}^-(b)(z))$$

Hence $(F, A) \vee (G, B)$ is a bipolar fuzzy soft $\Gamma$-hyper-ideal over $S$. It can be similarly proved that $(F, A) \wedge (G, B)$ is a bipolar fuzzy soft $\Gamma$-hyper-ideal over $S$.

**Theorem 3.9**

Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hyper-bi-ideals over $S$, then $(F, A) \wedge (G, B)$ and $(F, A) \vee (G, B)$ are bipolar fuzzy soft $\Gamma$-hyper-bi-ideals over $S$.

**Proof.** Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hyper-bi-ideals over $S$ defined as $(F, A) \wedge (G, B)$ where $\mathbb{C} = A \times B$. Let $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in \mathbb{C} = A \times B$, $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\inf_{z \in xy} \mu_{H}^+(a)(z) = \inf_{z \in xy} \mu_{F}^+(a)(z), \mu_{G}^+(b)(z))$$

$$\sup_{z \in xy} \mu_{H}^+(a)(z) = \sup_{z \in xy} \mu_{F}^+(a)(z), \mu_{G}^+(b)(z))$$

Hence $(F, A) \wedge (G, B)$ is a bipolar fuzzy soft $\Gamma$-hyper-bi-ideal over $S$.

**Theorem 3.10**

Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hyper-bi-ideals over $S$, then $(F, A) \wedge (G, B)$ and $(F, A) \vee (G, B)$ are bipolar fuzzy soft $\Gamma$-hyper-bi-ideals over $S$.

**Proof.** Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hyper-bi-ideals over $S$ defined as $(F, A) \wedge (G, B)$ where $\mathbb{C} = A \times B$. Let $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in \mathbb{C} = A \times B$, $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\inf_{z \in xy} \mu_{H}^+(a)(z) = \inf_{z \in xy} \mu_{F}^+(a)(z), \mu_{G}^+(b)(z))$$

$$\sup_{z \in xy} \mu_{H}^+(a)(z) = \sup_{z \in xy} \mu_{F}^+(a)(z), \mu_{G}^+(b)(z))$$

Hence $(F, A) \wedge (G, B)$ is a bipolar fuzzy soft $\Gamma$-hyper-bi-ideal over $S$.
\[ \inf_{x \in xy} [\mu^*_{H/G}(x)] = \inf_{x \in xy} [\mu^*_{H/G}(y)] \]
\[ \geq \min [\mu^*_{H/G}(x), \mu^*_{H/G}(y)] = \min [\mu^*_{H/G}(x), \mu^*_{H/G}(y)] \]
and
\[ \sup_{x \in xy} [\mu^*_{H/G}(x)] = \sup_{x \in xy} [\mu^*_{H/G}(y)] \]
\[ \leq \max [\mu^*_{H/G}(x), \mu^*_{H/G}(y)] = \max [\mu^*_{H/G}(x), \mu^*_{H/G}(y)] \]
Case (iii) \( C \in A \cap B \) and \( \gamma \in \Gamma \) then \( H(\gamma) = F(\gamma) \cap G(\gamma) \) then by theorem 3.7.
\[ \inf_{x \in xy} [\mu^*_{H/G}(x)] \geq \inf_{x \in xy} [\mu^*_{H/G}(x), \mu^*_{H/G}(y)] = \min [\mu^*_{H/G}(x), \mu^*_{H/G}(y)] \]
and
\[ \sup_{x \in xy} [\mu^*_{H/G}(x)] \leq \sup_{x \in xy} [\mu^*_{H/G}(x), \mu^*_{H/G}(y)] = \max [\mu^*_{H/G}(x), \mu^*_{H/G}(y)] \]
Hence \((F, A) \cap (G, B)\) is a bipolar fuzzy soft \( \Gamma \)-hypersubsemigroup over \( S \).

**Theorem 3.11**

Let \((F, A)\) and \((G, B)\) be two bipolar fuzzy soft \( \Gamma \)-hypersubsemigroup over \( S \), then \((F, A) \cup (G, B)\) is a bipolar fuzzy soft \( \Gamma \)-hypersubsemigroup of \( S \).

Proof. Straight forward.

**Theorem 3.12**

Let \((F, A)\) and \((G, B)\) be two bipolar fuzzy soft \( \Gamma \)-hyperideal over \( S \), then \((F, A) \cup (G, B)\) is a bipolar fuzzy soft \( \Gamma \)-hyperideal over \( S \).

Proof. Straight forward.

**Theorem 3.13**

Let \((F, A)\) and \((G, B)\) be two bipolar fuzzy soft \( \Gamma \)-hyperideal over \( S \), then \((F, A) \cap (G, B)\) is a bipolar fuzzy soft \( \Gamma \)-hyperideal over \( S \).

Proof. Straight forward.

**Theorem 3.14**

Let \((F, A)\) and \((G, B)\) be two bipolar fuzzy soft \( \Gamma \)-hypersubsemigroup over \( S \), then \((F, A) \cap (G, B)\) is a bipolar fuzzy soft \( \Gamma \)-hypersubsemigroup of \( S \).

Proof. Straight forward.

**Theorem 3.15**

Let \((F, A)\) and \((G, B)\) be two bipolar fuzzy soft \( \Gamma \)-hypersubsemigroup over \( S \), then \((F, A) \cup (G, B)\) is a bipolar fuzzy soft \( \Gamma \)-hypersubsemigroup of \( S \).

Proof. Straight forward.

**Theorem 3.16**

Let \((F, A)\) and \((G, B)\) be two bipolar fuzzy soft \( \Gamma \)-hyperideal over \( S \), then \((F, A) \cap (G, B)\) is a bipolar fuzzy soft \( \Gamma \)-hyperideal over \( S \).

Proof. Straight forward.

**Example 3.18**

Every bipolar fuzzy soft \( \Gamma \)-hyperideal is bipolar valued fuzzy soft \( \Gamma \)-hyperideal but converse is not true.

Let \( S = \{a, b, c, d, e\} \) and \( \Gamma = \{\gamma\} \) then \( S \) is a semihypergroup.

Let \( E = \{u, v, w, x, y\} \) and \( A = \{u, v, y\} \). Define the bipolar fuzzy soft set \((F, A)\) as \((F, A) = (F(u), F(v), F(y))\), where

\[ F(u) = \{(a, 0.6, -0.5), (b, 0.7, -0.6), (c, 0.4, -0.2), (d, 0.3, -0.1), (e, 0.9, -0.8)\} \]

\[ F(v) = \{(a, 0.8, -0.4), (b, 0.9, -0.7), (c, 0.6, -0.3), (d, 0.2, -0.1), (e, 1, -0.9)\} \]

\[ F(y) = \{(a, 0.7, -0.8), (b, 0.8, -0.9), (c, 0.5, -0.4), (d, 0.2, -0.3), (e, 1, -0.9)\} \]

Hence \((F, A)\) is a bipolar fuzzy soft sub \( \Gamma \)-hyperideal over \( S \), but not bipolar valued fuzzy hyperideal. Since

\[ \inf_{x \in xy} [\mu^*_{F(A)}(x)] \geq \max [\mu^*_{F(A)}(x), \mu^*_{F(A)}(y)] \]

\[ = 0.4 \not\geq 0.6 \]

**Example 3.19**

Every bipolar fuzzy soft \( \Gamma \)-hyperideal is bipolar valued fuzzy soft \( \Gamma \)-hyperideal but converse is not true.

Let \( S = \{a, b, c, d, e\} \) and \( \Gamma = \{\alpha, \beta\} \) then \( S \) is a semihypergroup.

Let \( E = \{u, v, w, x, y\} \) and \( A = \{w, x, y\} \). Define the bipolar fuzzy soft set \((F, A)\) as \((F, A) = (F(w), F(x), F(y))\), where

\[ F(w) = \{(a, 0.2, -0.1), (b, 0.4, -0.3), (c, 1, -0.9), (d, 0.6, -0.7), (e, 0.7, -0.8)\} \]

\[ F(x) = \{(a, 0.1, -0.2), (b, 0.2, -0.3), (c, 0.7, -0.8), (d, 0.4, -0.5), (e, 0.5, -0.6)\} \]
Hence is a bipolar fuzzy soft hyperbi-ideal but not bipolar valued fuzzy hyper ideal, since $\inf \mu^{+}_{F(a)}(a) \geq \max \{ \mu^{+}_{F(a)}(d), \mu^{+}_{F(a)}(e) \} = 0.6 \neq 0.7$.

**Example 3.20**

Every bipolar fuzzy soft hyperideal is bipolar valued fuzzy soft $\Gamma$-hyperideal but converse is not true.

Proof. Assume that $\{ F, A \}$ is a bipolar soft hyperideal over $S$ for each $t \in [0,1]$ and $s \in [-1,0]$. For each $x, y \in S$ and $a \in A$, let $t = \min(\mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(y)) \leq s = \max(\mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(y))$, then $\alpha, \beta \in \mu^{+}_{F(a)}$. Since $\mu^{+}_{F(a)}$ is a $\Gamma$-hyperideal of $S$, then $x \in \mu^{+}_{F(a)}(x)$ and $\forall (x, y) \in \mu^{+}_{F(a)}(x, y)$. That is $\mu^{+}_{F(a)}(x, y) \subset \mu^{+}_{F(a)}(x) \times \mu^{+}_{F(a)}(y)$, which shows that $\mu^{+}_{F(a)}(x, y) \subset \mu^{+}_{F(a)}(x) \times \mu^{+}_{F(a)}(y)$. This shows that $\mu^{+}_{F(a)}$ is bipolar fuzzy $\Gamma$-hyperideal of $S$ over $A$. Thus $A$ is a bipolar fuzzy soft $\Gamma$-hyperideal over $S$. Conversely, assume that, for each $a \in A$, $t \in [0,1]$ and $s \in [-1,0]$ and $x, y \in S$, we have $\mu^{+}_{F(a)}(x) \supseteq t, \mu^{+}_{F(a)}(y) \supseteq t$ and $\mu^{+}_{F(a)}(x) \cap \mu^{+}_{F(a)}(y) \subseteq s$. Therefore $\mu^{+}_{F(a)}$ is a bipolar fuzzy $\Gamma$-hyperideal of $S$. Thus $\gamma \in \Gamma$ there exists $x \in S_{x,y}$ such that $\inf \{ z \} \geq \max \{ \mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(y) \} \supseteq \gamma$ and $\sup \{ z \} \leq \mu^{+}_{F(a)}(x) \times \mu^{+}_{F(a)}(y) \subseteq s$. Therefore for all $x \in S_{x,y}$ we have $z \in \mu^{+}_{F(a)}$. This implies that $x \in \mu^{+}_{F(a)}$, for each $x \in S$. Hence $\mu^{+}_{F(a)}(x, y) \subset \mu^{+}_{F(a)}(x) \times \mu^{+}_{F(a)}(y)$, which shows that $\mu^{+}_{F(a)}$ is bipolar fuzzy $\Gamma$-hyperideal of $S$. By definition 3.2, for each $a \in A$, $t \in [0,1]$ and $s \in [-1,0]$ and $x, y \in \mu^{+}_{F(a)}$ we have $\mu^{+}_{F(a)}(x) \supseteq t, \mu^{+}_{F(a)}(y) \supseteq t$ and $\mu^{+}_{F(a)}(x) \cap \mu^{+}_{F(a)}(y) \subseteq s$. Therefore $\mu^{+}_{F(a)}(x, y) \subset \mu^{+}_{F(a)}(x) \times \mu^{+}_{F(a)}(y)$.

**Theorem 3.21**

Let $\{ F, A \}$ be a bipolar fuzzy soft set over $S$. $\{ F, A \}$ is a bipolar fuzzy soft $\Gamma$-hyperideal if and only if $\{ F, A \}$ is a soft $\Gamma$-hyperideal of $S$ for each $t \in [0,1]$ and $s \in [-1,0]$. Proof. The proof follows from theorem 3.22.

**Theorem 3.22**

Let $\{ F, A \}$ be a bipolar fuzzy soft set over $S$. $\{ F, A \}$ is a bipolar fuzzy soft $\Gamma$-hyperideal if and only if $\{ F, A \}$ is a soft $\Gamma$-hyperideal of $S$ for each $t \in [0,1]$ and $s \in [-1,0]$. Proof. Let $\{ F, A \}$ be a bipolar fuzzy soft set over $S$. $\{ F, A \}$ is a bipolar fuzzy soft $\Gamma$-hyperideal if and only if $\{ F, A \}$ is a soft $\Gamma$-hyperideal of $S$ for each $t \in [0,1]$ and $s \in [-1,0]$. Proof. Suppose that $\{ F, A \}$ is a bipolar soft hyperideal of $S$ for each $t \in [0,1]$ and $s \in [-1,0]$ and $a \in A, \gamma \in \Gamma$. For each $t$, let $t = \min(\mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(y)) \leq s = \max(\mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(y))$, then $\alpha, \beta \in \mu^{+}_{F(a)}$. Since $\mu^{+}_{F(a)}$ is a $\Gamma$-hyperideal of $S$, then $\forall (x, y) \in \mu^{+}_{F(a)}(x, y) \subseteq s = \max(\mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(y))$. This shows that $\mu^{+}_{F(a)}$ is bipolar fuzzy $\Gamma$-hyperideal of $S$. By definition 3.2, $\{ F, A \}$ is a bipolar fuzzy soft $\Gamma$-hyperideal of $S$. Conversely, assume that $\{ F, A \}$ is a bipolar fuzzy soft $\Gamma$-hyperideal of $S$. For each $a \in A, t \in [0,1]$ and $s \in [-1,0]$ and $x, y \in \mu^{+}_{F(a)}$, we have $\mu^{+}_{F(a)}(x) \supseteq t, \mu^{+}_{F(a)}(y) \supseteq t$ and $\mu^{+}_{F(a)}(x) \cap \mu^{+}_{F(a)}(y) \subseteq s$. Thus for $\gamma \in \Gamma$ there exists $z \in x_{y} \subset S$ such that $\inf \{ z \} \geq \max \{ \mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(y) \} \supseteq \gamma$ and $\sup \{ z \} \leq \mu^{+}_{F(a)}(x) \times \mu^{+}_{F(a)}(y) \subseteq s$. Therefore $\mu^{+}_{F(a)}(x, y) \subset \mu^{+}_{F(a)}(x) \times \mu^{+}_{F(a)}(y)$.

**Definition 4.1**

[9] Let $\eta_{1} : H_{1} \rightarrow H_{2}$ and $\psi : A \rightarrow B$ be two functions, $A$ and $B$ be two parametric sets from the crisp sets $H_{1}$ and $H_{2}$ respectively. Then the pair $(\eta, \psi)$ is called a bipolar fuzzy soft function from $H_{1}$ to $H_{2}$.

**Definition 4.2**

Let $\{ F, A \}$ and $\{ G, B \}$ be two bipolar fuzzy soft sets over the sets $H_{1}$ and $H_{2}$, respectively, and $(\eta, \psi)$ be a bipolar fuzzy soft map from $H_{1}$ to $H_{2}$.

(i) The image of $(F, A)$ under $(\eta, \psi)$ denoted by $(\eta, \psi)(F, A)$, is a bipolar fuzzy soft set over $H_{2}$ defined by $(\eta, \psi)(F, A) = (\eta(F), \psi(A))$, where for all $b \in \psi(A)$ and for all $y \in H_{2}$, $\mu^{+}_{\eta(\psi)(A)}(y) = \sup \{ \sup \{ \mu^{+}_{\eta(F)}(x), \mu^{+}_{\psi(A)}(y) \} \}$. Otherwise, $\mu^{+}_{\eta(\psi)(A)}(y) = 0$.

(ii) The inverse image of $(G, B)$ under $(\eta, \psi)$ denoted by $(\eta, \psi)^{-1}(G, B)$, is a bipolar fuzzy soft set over $H_{1}$ defined by $(\eta, \psi)^{-1}(G, B) = (\eta^{-1}(G), \psi^{-1}(B))$, where for all $a \in \psi^{-1}(B)$ and for all $x \in H_{1}$, $\mu^{-}_{\eta(\psi)(A)}(y) = \mu^{-}_{\eta(\psi)(A)}(x)$. Therefore $(\eta, \psi)^{-1}(G, B)$ is a bipolar fuzzy soft $\Gamma$-hyperideal of $H_{1}$.

**Theorem 4.3**

Let $\eta_{1} : H_{1} \rightarrow H_{2}$ be a homomorphism of $S$. If $(G, B)$ is a bipolar fuzzy soft $\Gamma$-hyperideal of $H_{2}$, then $(\eta, \psi)^{-1}(G, B)$ is a bipolar fuzzy soft $\Gamma$-hyperideal of $H_{1}$.

Proof. Let $(G, B)$ be a bipolar fuzzy soft $\Gamma$-hyperideal of $H_{2}$. Then for all $x, y \in H_{2}$, $\gamma \in \Gamma$, we have $\mu^{-}_{\eta(\psi)(A)}(y) = \inf \{ \mu^{-}_{\eta(\psi)(A)}(x), \mu^{-}_{\eta(\psi)(A)}(y) \}$. Therefore $\mu^{-}_{\eta(\psi)(A)}(y) = \inf \{ \mu^{-}_{\eta(\psi)(A)}(x), \mu^{-}_{\eta(\psi)(A)}(y) \} = \inf \{ \mu^{-}_{\eta(\psi)(A)}(x), \mu^{-}_{\eta(\psi)(A)}(y) \}$. Therefore for all $x, y \in H_{2}$, $\gamma \in \Gamma$, we have $\mu^{-}_{\eta(\psi)(A)}(y) = \inf \{ \mu^{-}_{\eta(\psi)(A)}(x), \mu^{-}_{\eta(\psi)(A)}(y) \}$. Therefore for all $x, y \in H_{2}$, $\gamma \in \Gamma$, we have $\mu^{-}_{\eta(\psi)(A)}(y) = \inf \{ \mu^{-}_{\eta(\psi)(A)}(x), \mu^{-}_{\eta(\psi)(A)}(y) \}$. Therefore for all $x, y \in H_{2}$, $\gamma \in \Gamma$, we have $\mu^{-}_{\eta(\psi)(A)}(y) = \inf \{ \mu^{-}_{\eta(\psi)(A)}(x), \mu^{-}_{\eta(\psi)(A)}(y) \}$.
Therefore $(\eta, \psi)^{-1}(G, B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_1$.

**Theorem 4.4**
Let $\eta: H_1 \rightarrow H_2$ be a homomorphism of $S$. If $(G, B)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right, bi-ideal, interior) of $H_2$, then $(\eta, \psi)^{-1}(G, B)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right, bi-ideal, interior)ideal of $H_1$.

Proof. Straightforward.

**Theorem 4.5**
Let $\eta: H_1 \rightarrow H_2$ be a homomorphism of $S$. If $(G, A)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_1$, then $(\eta, \psi)(G, A)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_2$.

Proof. Let $(G, A)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_1$. Let $\eta(x) \in H_2$, $y \in H_2$ then we have

\[
\inf_{x, y, y'} \{\inf_{x, y, y'} \left\{ \sup_{\eta(x)} \sup_{\eta(y)} (G, A) \right\} = \inf_{x, y, y'} \left\{ \sup_{\eta(x)} \sup_{\eta(y)} (G, A) \right\} \}
\]

and

\[
\sup_{x, y, y'} \{\inf_{x, y, y'} \left\{ \sup_{\eta(x)} \sup_{\eta(y)} (G, A) \right\} = \sup_{x, y, y'} \left\{ \sup_{\eta(x)} \sup_{\eta(y)} (G, A) \right\} \}
\]

Therefore $(\eta, \psi)(G, A)$ is a bipolar fuzzy soft $\Gamma$-hyper subsemigroup of $H_2$.

**Theorem 4.6**
Let $\eta: H_1 \rightarrow H_2$ be a homomorphism of $S$. If $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right, bi-ideal, interior)ideal of $H_1$, then $(\eta, \psi)(F, A)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right, bi-ideal, interior)ideal of $H_2$.

Proof. Straightforward.

**References**


