On Trio Ternary Γ-Semigroups

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Abstract

In this paper the terms trio L-trio TT-ideal, La-trio TT-ideal, R-trio TT- ideal and trio TT-ideal of a TT-semi group are introduced and some examples are given. It is proved that (1) a TT-semi group T is a trio TT-semi group if and only if \( xTTT = T\overline{T}T = T\overline{T}\overline{T} \) for all \( x \in T \). (2) Every trio TT-semi group is a duo TT-semi group. (3) Every commutative TT-semi group is a trio TT-semi group and a R-trio TT-semi group. (4) Every quasi commutative TT-semi group is a trio TT-semi group.

Keywords: duo TT-semi group, idempotent, prime trio TT-ideal, semi primary TT-semi group, trio TT-semi group.

1. Introduction


2. Preliminaries

Note 2.1 : for preliminaries refer to reference [10].

3. Trio Ternary Γ-Semi group

Def 3.1: A TT-semi group Q is called a L-duo ternary Γ-semi group if every left TT-ideal of Q is a two sided TT-ideal of Q.

Def 3.2: A TT-semi group Q is called a R-duo ternary Γ-semi group if every right TT-ideal of Q is a two sided TT-ideal of Q.

Def 3.3: A TT-semi group Q is called a duo TT-semi group if it is both a left duo TTT-semi group and a right duo TTT-semi group.

Th 3.4: A TT-semi group M is a duo TT-semi group if and only if \( pFMFM = FMFMp \) for all \( p \in M \).

Def 3.5: A TT-semi group M is said to be a L-trio TT-semi group if every left TT-ideal of M is a lateral TT-ideal and right TT-ideal of M.

Def 3.6: A TT-semi group M is said to be a R-trio TT-semi group if every right TT-ideal of M is a lateral TT-ideal and left TT-ideal of M.

Def 3.7 : A TT-semi group Q is said to be a La-trio TT-semi group if every lateral TT-ideal of Q is a left TT-ideal and a right TT-ideal of Q.

Def 3.8: A TT-semi group M is said to be a trio TT-semi group if it is a L-trio TT-semi group, a La-trio TT-semi group and a R-trio TT-semi group.

Th 3.9: A TT-semi group M with identity is a trio TT-semi group iff \( xMF = MFx = Mx \) for all \( x \in M \).

Proof: Suppose that M is a trio TT-semi group and \( x \in M \). Let \( t \in xFMFM \). Then \( t = xaua, b \in G \) for some \( u, v \in M \), \( a, b \in G \). Since \( Mx \) is a left TT-ideal of M, \( xM \) is a TT-ideal of M. So \( x \in xFMFM \). Since \( xFMFM = FMFMx \) for all \( x \in M \), \( xFMFM = FMFMx \) for all \( x \in M \).

Let \( t \in xFMFM \). Then \( t = xaua, b \in G \) for some \( u, v \in M \). Since \( xFMFM = FMFMx \) for all \( x \in M \), \( xFMFM = FMFMx \) for all \( x \in M \).

Similarly, we can prove that \( MFM = xFMFM \) for all \( x \in M \). Therefore \( xFMFM = xFMFM \) for all \( x \in M \).

Hence form (1) and (2) \( xFMFM = FMFMx = FMFMx \) for all \( x \in M \).

Conversely Let \( xFMFM = FMFMx = FMFMx \) for all \( x \in M \). Let \( xFMFM = FMFMx \) for all \( x \in M \).

Therefore, \( xFMFM = FMFMx \) for all \( x \in M \).
Th 3.10: Every trio TT-semi group is a duo TT-semi group.

**Proof:** Let $Q$ be a trio TT-semi group. Then by theorem 3.9, $\exists Q'$ such that $Q = Q' \times Q' \forall x \in Q$. Therefore, $Q$ is a duo TT-semi group.

Note 3.11: The converse of the theorem 3.10, need not necessarily be true. i.e., every duo TT-semi group need not be trio TT-semi group.

Example 3.12: Consider the set $T = \{0, -s, -t, -r\}$ and $\Gamma = T$ with the following compositions:

\[
\begin{array}{cccc}
  & 0 & -s & -t & -r \\
0 & 0 & 0 & 0 & 0 \\
s & 0 & 0 & 0 & 0 \\
t & 0 & 0 & 0 & 0 \\
r & 0 & -s & -t & -r \\
\end{array}
\]

Clearly $M$ is a TT-semi group. Let $P = \{0, -t\}$ is a L-TT-ideal and R-TT-ideal of $M$, but not La-TT-ideal of $M$. Since $M$ is a duo TT-semi group but not trio TT-semi group. From theorem 2.7 and \(-a, c \in M \Rightarrow (-a)(t)(r) = \in FPM \) and \(-a, c \in M \Rightarrow (-a)(t)(r) = \in FPM \) and hence $FPM \notin P$. Therefore $P$ is not La-TT-ideal of $M$ and hence $M$ is not trio TT-semi group.

Th 3.13: Every commutative semi group is a trio TT-semi group.

**Proof:** Let $T$ be a tri commutative semi group Therefore $\forall x \in T \Rightarrow T \ast T \forall x \in T$. By theorem 3.9, $T$ is a trio TT-semi group.

Th 3.14: Every quasi commutative semi group is a trio TT-semi group.

**Proof:** Suppose that $T$ is a quasi commutative semi group. Then for each $a, b, c \in T$, there exists a $n \in N$ such that $ab = ba = ca = ac$. Suppose $\exists a \in T$ where $aab = a$. Clearly $M$ is a TT-semi group. Let $P = \{0, -t\}$ is a L-TT-ideal of $M$. Therefore, $T \ast T \forall x \in T$. By theorem 3.9, $T$ is a trio TT-semi group.

Def 3.15: An element $a$ of a TT-semi group is said to be regular if there exist $x, y \in T$ such that $ax = a$. The ternary semi group called regular TT-semi group.

Some authors may refer to the element in $T$ group if there exist an $x \in T$ and $\alpha, \beta \in T$ such that $\alpha x = \beta x$. But obviously the conditions are same.

Th 3.16: Every idempotent element in a TT-semi group is regular.

Def 3.17: An element $a$ of a TT-semi group $T$ is said to be left regular if there exist $x, y \in T$ and $\alpha, \beta \in T$ such that $a = (\alpha y)b$.

Def 3.18: An element $a$ of a TT-semi group $T$ is said to be right regular if there exist $x, y \in T$ and $\alpha, \beta \in T$ such that $a = x(\beta y)$.

Def 3.19: An element $a$ of a TT-semi group $T$ is said to be right regular if there exist $x, y \in T$ and $\alpha, \beta \in T$ such that $a = x(\beta y)$.

Def 3.20: An element $a$ of a TT-semi group $T$ is said to be intra regular if there exist $x, y, z \in T$ such that $a = x(yz)$.

Def 3.21: An element $a$ of a TT-semi group $M$ is said to be semi-simple if $q \in \langle q \rangle \Rightarrow q \in \langle q \rangle$ i.e., $\langle q \rangle \Rightarrow q \in \langle q \rangle$. Theorem 3.22: An element $a$ of a TT-semi group $M$ is said to be semi simple if $q \in \langle q \rangle$ i.e., $\langle q \rangle \Rightarrow q \in \langle q \rangle = q \Rightarrow \forall q \in M$.

Def 3.23: A TT-semi group $M$ is called simple TT-semi group provided every element in $M$ is semi simple.

Th 3.24: If $T$ is a trio TT-semi group with identity, then the following are equivalent for any element $a \in T$.

1) $a$ is regular.
2) $a$ is left regular.
3) $a$ is right regular.
4) $a$ is lateral regular.
5) $a$ is intra regular.
6) $a$ is semi simple.

**Proof:** Since $T$ is trio TT-semigroup, $\forall a \in T \Rightarrow a \ast T = T \ast a = T \ast T \ast T \Rightarrow T \ast T \ast T \Rightarrow T \ast T \Rightarrow T \ast T$.

Th 3.25: An element $a$ of a TT-semi group $T$ is said to be an idempotent element provided $a = a$.

Def 3.26: An element $a$ of a TT-semi group $T$ is said to be zero of $T$ if $a \ast bc = b \ast c = b \ast c$.

Notation 3.27: For any TT-semi group $T$, $e_T$ denotes the set of all idempotents of $T$ together with the binary relation denoted by $e \leq f$ if and only if $e = e \ast f$.

Def 3.28: A TT-ideal $Q$ of a TT-semi group $M$ is said to be semi-primary if $\forall Q$ is a prime TT-ideal of $M$.

Def 3.29: A TT-semi group $M$ is said to be a semi primary TT-semi group provided every TT-ideal of $M$ is a semi primary TT-ideal.

Th 3.30: Let $T$ be a semi primary semi TT-semi group, then the idempotents of $T$ form a chain under natural ordering.
4. Conclusion

In this paper we are introducing the concept of trio TT-ideals in TT-semigroup. Previously, many of the researchers studied about duo ideals in different algebraic structures.

References


