Durability Analysis for Coil Spring Suspension Based on Strain Signal Characterisation

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Abstract

This paper aims to predict the durability of an automobile coil spring by characterising the captured strain data. The load histories collected at coil spring are often presented in time domain but time domain cannot provide sufficient information for fatigue life prediction. The objective of this study was to characterise the strain signal in time domain, frequency domain and time-frequency domain for fatigue life prediction. The signal obtained in time domain was used to predict the fatigue life of the coil spring through Rainflow cycle counting technique and models of strain-life relationships. In frequency domain, fast Fourier transform revealed that the frequency components in the strain signal ranged between 0-5 Hz. The frequencies can be further categorised into two ranges: 0-0.3 Hz and 1-2 Hz. Power spectral density confirmed that the frequencies with high energy content were 0-5 Hz and the total energy content in the signal is 4.0872x10^9 μ². Short time Fourier transform can identify the local time and frequency properties of the signal but it has a limitation in time-frequency resolutions. Wavelet transform can provide a better time-frequency resolutions and it confirmed that the transients in the time domain had frequency range of 1-2 Hz. In summary, this study revealed different possible approaches of signal processing in fatigue life assessment of automotive components as guidance for the selection of suitable approach based on the type of information needed for the analysis.

Keywords: Time domain; Frequency domain; Time-frequency domain; Fatigue life; Durability

1. Introduction

As a vehicle is moving on a road, the uneven road surface profile will give excitation to the tyres and causes the tyres to move up and down, perpendicular to the road surface. Potholes or bumps on the road also cause sudden vertical movement as the wheel strike into them. The vibrations caused by the road noise, bumps and potholes can significantly affect the movement of the wheel and the comfort of the users. Hence, suspension system is needed to absorb the vibrations in order to provide safety, driving pleasure and ride comfort to the users. The continuous exposure of suspension components to the random load gives rise to the complications of fatigue failure in the components. Since metal fatigue is a cumulative phenomenon which does not recover while a component is under stationary state, fatigue life assessment becomes important to prevent catastrophic failure in suspension components [1].

The well-established practice of fatigue life assessment includes the determination of the loading history, identification of the damaging cycles and calculation of the total damage. This is a time domain approach as the input takes the form of loading history and the function in the time of function. From the loading history, the number of cycle can be identified by the means of cycle counting methods, for example, the Rainflow counting algorithm [2]. The combination between Rainflow counting and Palmgren-Miner rule is widely used in many studies [3, 4] and generally accepted as one the best time domain methods for fatigue life estimation [5]. Apart from time domain approach, signals can also be observed in frequency domain. The most commonly used algorithm for time to frequency domain transformation is the Fourier transform (FT) [6]. Through FT, all frequency components that cannot be seen from time domain can be revealed. It is convenient to convert back and forth between time and frequency domain using fast Fourier transform (FFT) and inverse Fourier transform (IFT), respectively [1]. Power spectral density (PSD) which represents the spread of mean square amplitude over frequency range is a statistical approach to describe a random process in frequency domain. Palmieri et al. [3] studied the effects of Gaussianity of a random loading on the fatigue life using PSD. The random load excited from the road surface can be viewed as a realization of random Gaussian process [5]. Hence, PSD can be an appropriate approach to analyse the random loading of automotive suspension.

Despite the popularity of FT, it has certain theoretical drawbacks in signal processing. It has difficulty to analyse any local property of time domain signals in frequency domain using FT [6]. To overcome this limitation, a sliding window function is introduced to FT and the resultant transformation is named as short time Fourier transform (STFT). STFT is widely used for time-frequency analysis before wavelet transform. However, STFT also has its own limitation. According to the Heisenberg uncertainty principle, STFT has a resolution problem in which high resolution in time and frequency cannot be attained simultaneously. Furthermore, STFT is not a suitable approach to analyse a non-stationary signal because of its constant time-frequency resolution [6]. Wavelet transform (WT) was developed to meet the demand for an adaptive time-frequency analysis. The continuous wavelet transform converts a time domain signal into a function called wavelet coef-
ficients, represented by scale and time [7]. WT is often presented in wavelet scalogram, a two dimensional contour plot in which the one axis represents the time (translation) while another axis represents the scale or frequency. Abdullah et al. [8] used wavelet transform to identify and extract the damaging segments in a strain signal. Oh [4] used discrete wavelet transform to decompose and denoise a fatigue history.

The fatigue life assessments are often performed using load histories in time domain. However, time domain approach is not suitable in applications that involve random vibrations due to the limitation of time interval. The time signal of random loading has to be sufficiently long to approximate the statistics. Finite Element simulations used in durability analysis are not capable to process long time signal in combination with huge vibrating structures. Hence, it is necessary to analyse the signals in other domains such as frequency and time-frequency domain to extract important characteristics of the signal for durability analysis. This study aims to characterise a strain signal in time domain, frequency domain and time-frequency domain for fatigue life prediction. In this study, various approaches in time domain, frequency domain and time-frequency domain will be demonstrated and discussed with their advantages and limitations. The findings in this study are important as a guideline for future studies in the selection of a suitable approach to extract desired information from a strain signal for durability analysis.

2. Theoretical Background

2.1. Fatigue Life Assessment

Strain signals are widely used to predict fatigue life of coil spring with strain loadings as the input. There are three approaches in fatigue life prediction, namely, stress life approach, strain life approach and fatigue crack growth [9]. Stress life approach is widely applied in automotive applications because it applies on ductile materials since the low cycle fatigue of small components are within 10^3 cycles. Strain life approach is based on stress life approach introduced by O.H. Basquin in 1910, as reported in [10]. The stress is presented by the following equation:

\[ \Delta \sigma = \frac{E \cdot \varepsilon_s}{2} = \sigma_y \cdot (2N)^n \]  

(1)

where \( \varepsilon_s \) is the strain in elastic range, \( N \) is the number of cycles to failure, \( 2N \) is the number of load reversals to failure, \( \sigma_y \) is the fatigue strength coefficient, and \( b \) is the fatigue strength exponent (\( b \) has a negative sign).

Coffin-Manson relationship describes the plastic strain range as following:

\[ \frac{\varepsilon_p}{2} = \varepsilon_y \cdot (2N)^c \]  

(2)

where \( \varepsilon_y \) is the fatigue ductility coefficient and \( c \) is the fatigue ductility exponent (the sign of \( c \) is negative).

The total strain amplitude can be considered as a superposition of the elastic and plastic strain. By combining equations (1) and (2), the total strain amplitude can be expressed as follows:

\[ \varepsilon_{total} = \varepsilon_s + \varepsilon_p = \frac{\sigma_y}{E} \cdot (2N)^n + \varepsilon_y \cdot (2N)^c \]  

(3)

Morrow proposed to replace \( \sigma_y \) with \( \sigma_y - \sigma_u \) as a consideration to the mean stress effect. Morrow’s relationship is as shown in Equation (4):

\[ \Delta \sigma = \frac{E \cdot \varepsilon_s}{2} = (\sigma_y - \sigma_u) \cdot (2N)^n + \varepsilon_y \cdot (2N)^c \]  

(4)

where \( \sigma_u \) takes positive sign for tensile stress and negative sign for compression stress. Smith-Watson-Topper (SWT) proposed another relationship based on strain life test data at fracture obtained with various mean stress as described in Equation (5):

\[ \sigma_{max} \cdot \varepsilon_s \cdot E = (\sigma_y)^2 (2N)^{2n} + \sigma_y \cdot \varepsilon_y \cdot (2N)^{2c} \]  

(5)

where \( \sigma_{max} = \sigma_u + \sigma_y \) and \( \varepsilon_s \) is the alternating strain.

The Palmgren-Miner linear cumulative damage rule developed independently by A. Palmgren in 1924 and M.A. Miner in 1945, as reported in [10], is extensively used to calculate the cumulative fatigue damage of a variable amplitude loading (VAL) block. According to Palmgren-Miner linear damage rule, the cumulative fatigue damage can be computed from the summation of damage from each damaging cycle identified. The rule is defined as:

\[ D = \sum \left( \frac{n_i}{N_i} \right) \]  

(6)

When a component experiences 100% of fatigue damage, Equation (6) can be rewritten as:

\[ \sum \frac{n_i}{N_i} = 1 \]  

(7)

2.2. Signal Processing Approaches

2.2.1. Frequency Domain

Fourier transform \( \hat{f}(\omega) \) can be obtained by the inner product of the signal \( f(t) \) with a sinusoidal wave \( e^{j\omega t} \) [6]:

\[ \hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \]  

(8)

In addition to Fourier transform, the analysis in frequency domain can also be performed using Power Spectral Density (PSD). PSD is a statistical method that takes into account of the “power” (mean square value) within the unit frequency band and can be expressed as follows [11]:

\[ S_f(f)df = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{T} x^2(t)dt \]  

(9)

Where \( x(t) \) is the random signal in time, \( x(f) \) is the amplitude of the frequency spectrum and \( S_f(f) \) is its power spectral density.

2.2.2. Time-Frequency Domain

Short time Fourier transform (STFT) is a common approach used in time-frequency analysis of a signal. The basic of STFT is to introduce a sliding window function \( g(t) \) to the Fourier transform and obtain the localized time-frequency atom \( \phi \) [6]:

\[ \phi_{a,t}(t) = e^{j\omega t}g(t-u) \]  

(10)

Thus, STFT of a signal \( f(t) \) can be expressed as:

\[ (STFT)\{f(t)\} = \left\{ \int_{-\infty}^{\infty} f(t)g(t-u)e^{-j\omega t}dt \right\} \]  

(11)
STFT is presented in a spectrogram which is the magnitude squared of STFT, \([STFT f(t)]^2\) [12]. A deficiency of STFT is its poor frequency resolution at high time resolution.

Continuous wavelet transform (CWT) emerged after STFT to solve the resolution problem of STFT. Similar to STFT, CWT also uses a window function but with variable size in each window. The window size is larger at lower frequency and vice versa. Fourier transform breaks up the signal into sinusoidal waves of various frequencies and phases. In CWT, the signal is broken into shifted and scaled version of the mother wavelet \(\psi_s(t)\) as described in following:

\[
\psi_s(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right)
\]

Where \(s\) is the scaling parameter and \(u\) the position parameter. The mother wavelet must fulfill the following conditions:

\[
\int_{-\infty}^{\infty} \psi(t)dt = 0, \quad \int_{-\infty}^{\infty} \psi^2(t)dt = 1
\]

The CWT of a signal \(f(t)\) is defined as [6]:

\[
W_{\psi} f(s,u) = \{f(t), \psi_{s,u}(t)\} = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-u}{s}\right)dt
\]

CWT and complex-value wavelets have advantages in the detection of transients and singularities [7]. Thus, the load histories of car suspension which consists of large amount of transients are best studied using CWT. Morlet is one of the mother wavelet involved in the CWT and can be described as follows [10]:

\[
\psi(t) = \exp(-\frac{\beta^2 t^2}{2})\cos(\pi t)
\]

where \(\beta\) is the shape parameter.

3. Methodology

Figure 1 shows the process flow of methodology in this study. The random fatigue loading history (also known as strain data) of a coil spring under driving conditions had been collected in this study. A strain gauge was attached to the surface of the coil spring and connected to data logging instruments for data record. The sampling frequency rate must be greater than 400 Hz [10] for automotive applications and 500 Hz was selected in this study. A strain data with a length of 100 seconds had been collected.

The strain signal was first analysed statistically to determine its statistical parameters, i.e. mean, standard deviation, root mean square (r.m.s), skewness and kurtosis. The strain data collected was observed in time domain to determine the changes of strain amplitude in time. Moreover, fatigue damage was computed from the time domain signal using commercial software. Coffin-Manson, Morrow and SWT relationships as expressed in Equation (3), (4) and (5) respectively, were employed in the fatigue damage assessment. Next, the strain signal was transformed into frequency domain using the FFT algorithm. Frequency components that existed in the strain signal were identified. With the aid of commercial software, the PSD of the signal was also plotted to find the energy content in the signal. The energy content was determined from the area under the PSD graph obtained via integration of the graph. Furthermore, the strain data was analysed in time-frequency domain through STFT and wavelet transform. In STFT, a Gaussian window function was used as \(g(t)\) in Equation (10) since this function is optimally concentrated in both time and frequency. The window length in STFT has significant effect on the time and frequency resolution. Shorter window length will yield higher time resolution but lower frequency resolution. Hence, window lengths of 64, 256 and 1024 were used in the analysis to obtain the optimal time-frequency resolution. For wavelet transform, Morlet wavelet function as described in Equation (15) was selected in this study. This is because the advantage of Morlet wavelet in detecting singularities and discontinuities in a signal which is suit to the case in this study.

4. Results and Discussion

4.1. Time Domain

Fatigue damage and life of the strain data was computed using the Coffin-Manson, Morrow and SWT relationships as expressed in Equations (3), (4) and (5) respectively and presented in Table 1. The three relationships yielded similar fatigue damage and life from the signal but the Coffin-Manson relationship recorded the highest damage (1.58 x 10^4) and the shortest predict life (6.31 x 10^4 cycles). This may because Coffin-Manson relationship did not consider the mean stress effect in the strain history. The strain signal in time domain and its running damage in Coffin-Manson relationship are plotted and shown in Figure 2. From the figure, eight shocks or transients can be observed at the time positions of 4s, 16s, 31s, 42s, 51s, 60s, 71s and 89s. The shocks can be attributed to the uneven road surface profile such as small bumps. These shocks contributed to high fatigue damage as they matched the time positions in the running damage where high damages were indicated. These transient segments are defined as high damaging segments and named with S1-S8 as shown in Figure 2. The highest damage (2.36 x 10^4) was recorded in S4 at 42s which was exactly when largest strain amplitude change took place (from 335.4 to -375.9 in microstrain). Figure 3 shows the damage histogram of the strain signal. The histogram indicates that the fatigue damage increases with the strain range. Highest strain range of

![Fig. 1: Process flow of methodology](image-url)
4.722 x 10^6 occurs at a strain range of 7100 µε. This indicates that larger changes in the strain amplitude can contribute to higher damage because the signal contains higher energy level.

Statistical analysis was also performed on the strain signal and the statistical parameters are presented in Table 2. The strain signal is a non-zero mean signal, the positive value indicates that the signal is a tensile loading. The high standard deviation of 81.38 µε shows that the data is distributed far away from the mean value, this is due to the large changes in strain amplitude that were identified as the shocks in the signal. The signal also contains high vibrational energy as the r.m.s value is 81.4 µε. Higher r.m.s indicates high energy content which also indicates higher fatigue damage [13]. The skewness measures the asymmetry of the data. A normal distribution should yield zero in skewness. The positive skewness value indicates that the distribution is skewed to the left of the mean and vice versa. The skewness of 0.1595 obtained from the strain data indicates that the data is a mildly non-normal distribution and the distribution is more concentrated to the compressive range of strain. Kurtosis measures the peakedness and the Gaussianity of a distribution. The strain signal should be categorised as non-Gaussian or non-stationary data since its kurtosis is more than 3 [13]. It is common to experience non-stationary loading in automotive applications. Palmieri et al. [3] remarked that non-stationary loading may cause higher fatigue damage compared to stationary loading.

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Life (Cycles to failure)</th>
<th>Fatigue damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coffin-Manson</td>
<td>6.31 x 10^3</td>
<td>1.58 x 10^-7</td>
</tr>
<tr>
<td>Morrow</td>
<td>7.48 x 10^7</td>
<td>1.34 x 10^-1</td>
</tr>
<tr>
<td>SWT</td>
<td>8.26 x 10^7</td>
<td>1.21 x 10^-7</td>
</tr>
</tbody>
</table>

Table 1: Fatigue Damage and Life of Strain Signal

![Fig. 2](image)

**Fig. 2:** Strain data in time domain (upper) and its Coffin-Manson running damage (lower)

<table>
<thead>
<tr>
<th>Mean (µε)</th>
<th>r.m.s (µε)</th>
<th>Standard Deviation (µε)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9676</td>
<td>81.40</td>
<td>81.38</td>
<td>0.1595</td>
<td>6.304</td>
</tr>
</tbody>
</table>

Table 2: Statistical Parameters of the Strain Signal

4.2. Frequency Domain

Signal in time domain can identify the time position of damaging segments (or transients) and fatigue damage. In fatigue assessment, the information about the frequencies of the signal is also important especially the frequencies of damaging segments. Through Fourier transform, the time domain signal can be analysed in frequency domain. Figure 4 depicts the frequency spectrum of the strain signal obtained through FFT algorithm. The frequency spectrum shows that the frequency components that existed in the signal are mostly ranged from 0-5 Hz. A closer view into the frequency spectrum (Figure 4b) found that the frequency components in the signal can be grouped into two frequency ranges: 0-0.3Hz and 1-2 Hz. However, the frequencies of the damaging segments remain unclear since the frequency spectrum did not provide the local time property [6]. Hence, FFT is a fast and convenient approach to analyse the frequency components in a signal but it is not suitable in some cases that are interested in time localization. The signal can also be analysed in frequency domain using PSD. Figure 5 shows that PSD of the strain signal in frequency domain. PSD identifies the frequencies in a signal that contain high power. It can be seen that the PSD matched the frequency spectrum obtained through FFT (Figure 4a). High power was observed in low frequency range of 0-0.5 Hz. The total area under the graph of PSD represents the energy content in the signal that contributes to fatigue damage. In this study, the strain signal contains a total energy of 4.0872x10^7 µε². This approach can quantify the energy content in a signal and determine if fatigue damage is likely to happen. The higher the energy content of the signal brings higher fatigue damage and will result in shorter fatigue life.

4.3. Time-Frequency Domain

A major limitation of time and frequency domain approaches is that the local properties of time and frequency cannot be obtained at the same time. Thus, it is difficult to identify the frequencies of the damaging segments in time domain. For example, in the case of this study, the frequency properties of the eight damaging segments in the time domain were desired but neither the time domain signal nor the frequency spectrum could provide the information. Thus, a time-frequency analysis is needed under such circumstance.

![Fig.3](image)

**Fig.3:** Damage histogram of strain signal computed with Coffin-Manson relationship
In this study, STFT had been performed with four window lengths to obtain a suitable time-frequency resolution. Figure 6 depicts the STFT analysis of the strain signal which is presented in spectrograms. The spectrograms consist of two axes which represent the time and frequency at x and y axis, respectively. When STFT was analysed with a short window length, for example window length of 64 (Figure 6a), a good time resolution was obtained. It can be seen that the position of bright stripes that represented high amplitude events became clearer in time domain. The time positions of the damaging segments can be identified in the spectrogram. Bright colour stripes can be seen at time positions of 4s, 16s, 31s, 42s, 51s, 60s, 71s and 89s which indicates the damaging segments in the signal. However, owing to the low frequency resolution associated with a short window length, the bright stripes were found to have a large frequency range of 0-20 Hz. As reveal by the FFT, the frequencies of the strain signal should fall between 0 to 3 Hz. Hence, the window length of 64 is unacceptable since it cannot give a precise frequency range. It can be seen that the frequen
cy resolution gradually increased with larger window length (Figure 6b, c and d) as the frequency range detected at the shocks became smaller. With window length of 512, a frequency range of 0-2 Hz had been identified among the damaging segments of the signal. Nonetheless, a long window length gave a poor time resolution that the time position of the damaging segments became unclear. In this study, window length of 256 is a suitable window length for STFT because it gave precise time positions (±0.5 s) with an acceptable frequency range (0-3 Hz) for the damaging segments.

The frequency range of the transients in time domain had been identified through FFT. However, another problem still remains. As revealed by FFT, the frequencies in the signal can be divided into two groups with ranges of 0-0.3 Hz and 1-2 Hz. STFT was unable to accurately identify which frequency range the damaging segments belong to, due to its limitation in time-frequency resolution. Hence, Morlet wavelet transform was performed to solve this problem because wavelet transform has an adaptive time-frequency resolution [7]. Figure 7 shows the time-frequency representation of the strain signal through Morlet wavelet transform. The white dashed line with a shape of cone in the scalogram is known as the cone of influence (COI). COI is the region in wavelet spectrum in which the edge effects are significant. From Figure 7, eight segments with high magnitude were found at the time positions corresponding to each damaging segment identified in time domain. Furthermore, all the damaging segments were in the frequency range of 1-2 Hz. The frequency range of 0-0.3 Hz in FFT is some high amplitude structural responses. Thus, the results show that the wavelet transform has a more accurate time-frequency resolution compared to STFT.

![Magnitude Scalogram](image)

**Fig. 7:** Scalogram representation of the strain signal obtained by Morlet wavelet transform

5. Conclusion

This paper demonstrated the characterization of a strain signal with several approaches in signal processing. Signal processing approaches that were discussed in this study including time domain, frequency domain and time-frequency approaches. Fatigue damage can be computed from time domain signal from the Rainflow cycle counting algorithm and strain-life relationships. Through time domain signal, high damaging segments can be easily identified. For frequency domain analysis, FFT is a very convenient and fast way to identify the frequency components in a signal. The PSD is also a suitable frequency domain approach when the energy content of the signal is interested. However, both time and frequency domain cannot provide local properties of time and frequency at the same time. For applications which are interested in time-frequency properties, STFT can present the signal in time-frequency domain. However, STFT has a limitation of its constant time-frequency resolution. Wavelet transform has the same function as STFT but it can provide a better time-frequency resolution.

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References


