Applying Distribution Functions to GWO Algorithm

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Abstract

GWO is an Optimization algorithm. It depends on the different distribution functions. The features of Optimization algorithm are as follows: Convergence, precision, and performance. These characters will generalize this optimization algorithm. In this paper, we explored GWO algorithm for different distributing functions. There are many distribution functions that are kept practical to the GWO algorithm. We evaluated three different distribution functions which are the Gold Stein function, Beale function and the Booth function. To show the effectiveness of the GWO algorithm we have used the above three distribution functions.

Keywords: Grey Wolf Optimization function, Gold Stein function, Beale function, Booth function.

1. Introduction

Software systems have become an important aspect of our lives so there is a need of software systems with high-quality. Some quality factors are needed to be considered for the software. Various models include various factors where all those models incorporate reliability as one of the most important software quality factor. Software reliability is defined [1] as: “the software should correctly give the services as expected by the user in a particular period of time.” A more accurate definition of software reliability is given by [2], [3] as: “the Probability of a failure-free operation over a specified time, in a given environment for the specific purpose”.

In this, we are using various Optimization Test functions like Goldstein test function, Beale’s function and Booth’s function which is based on grey wolf optimization technique. In this paper, we actually compare various Optimization test functions and get the optimized values for those functions, so that we can easily assess the software reliability. The less optimized value gives more reliability.

In this literature, we use some of the test function like Gold stein function, Beale’s function and Booth’s function. These functions are tested actually based on the grey wolf optimizer. Grey wolf optimizer is a meta-heuristics algorithm which was introduced by Mirjalili et al [4] which gives solution to many optimization problems in different fields and successfully could provide highly competitive results. GWO will get the inputs from the algorithm based on the grey wolf hunting process. These Optimization test functions will get the inputs from the positional values of the grey wolf. So, we can get the different optimized values for different Optimization test functions. From this, we can consider the optimization values of these test functions and we can assess software reliability.

2. Literature Review

Grey Wolf Optimization Algorithm is a meta-heuristic algorithm which is classified into three main divisions namely evolutionary, physics based, and Swarm Intelligence algorithms. Evaluation Algorithm’s (EAs) are taken based on the evolution of environment concepts. The well-known algorithm in this division is Genetic Algorithm (GA) which is proposed by Holland. The application is revised by Goldberg. Generally, optimization is done through gradually developed initial random values in EAs. Some of the EAs are DE, EP, and ES, PBIL(Probability-Based Incremental Learning), GP(Genetic Programming), and BBO(Biogeography-Based Optimizer).

The second main category of the Meta heuristic is physics based techniques. Those optimization algorithms will have an imitation of physical rules. Some of them are GLSA, BBBC, GSA, CSS, CFO, ACROA, BH (Black Hole), RO, SWOA, GBSA, and CSO.

The third main category is the Swarm Intelligence (SI) methods. Some of the techniques of Swarm ‘intelligence suggested far are:

- TA
- WSO [5]
- Monkey Search [6]
- BCPA
- Cuckoo Search (CS) [7]
- (DPO)
- (FA) [8]
- BMO
- (KH) 2012 [9]
- FOA

Many of these are based on hunting and search behaviors. As grey wolf's are well known for group haunting, SI technique is the best among all which has known knowledge for hunting of the prey by Grey wolf.. This inspired us to mathematically calculate the grey wolves behavior [10].

The above literature is adopted from the GWO which is an optimizer that was proposed by three different authors that uses different optimization test function and distribution function. These are tested in this and the one which provides a low optimized value can be considered as the best function in terms of the optimization.
in the grey wolf optimization which is a part of the different distributed functions.

3. Grey Wolf Optimization

The Grey Wolf Optimization (GWO) is an algorithm developed by Mirjalili used to get a solution for many optimization problems in various fields that could successfully provide highly optimized results in any environment. The GWO algorithm is very much closely related to the wildlife behavior of the grey wolves during hunting process. Dominance hierarchy states that the GWO splits the total animals' population into four different sections: alpha (α), beta (β), delta (δ), and omega.

Consequently, the optimization process has some order. The alpha (α) wolf will be the leader and the decision maker. The beta (β) and delta (δ) wolves support alpha (α) in decision making. The last category omega wolves will be the followers of the beta (β) and delta (δ) wolves. This is the hierarchy followed by the wolves during the hunting process.

In this GWO, we have a position vector Y represented as:

\[ Y = [y_1, y_2, \ldots, y_n] \] (1)

During the hunting process, the optimized value will be the solution of the problem, which is done by surrounding the prey with wolves. This can be mathematically represented as follows

\[ S_α = |V_1*(Y_α - Y(i))| \] (2)
\[ Y(i+1) = Y(i) + S_α \] (3)

Where Y (ip) is the prey’s position vector, Y (i) is the Grey wolf position vector, i is present iteration, C1 and C2 are coefficient vectors. These coefficient vectors allow the wolf's to change their positions in the environment around the prey. The coefficient vectors C1 and C2 are computed according to

\[ C2 = 2\alpha*r_1 - \alpha \] (4)
\[ C1 = 2*r_2 \] (5)

It is given that the value of element a is linearly decreasing from 2 to 0 over the search process and r1, r2 are random vectors selected in the domain of [0,1]. Then the GWO stores the top three solutions that are alpha, beta and delta wolves which finally allows omega wolf to calculate optimized value based on alpha, beta and delta positions. To compute the difference between the actual position of α, β, and δ the following equations are used respectively.

\[ S_α = |V_1*(Y_α - Y)| \] (6)
\[ S_β = |V_2*(Y_β - Y)| \] (7)
\[ S_δ = |V_3*(Y_δ - Y)| \] (8)

where \( Y_α \), \( Y_β \) and \( Y_δ \) are the positions of the α, β and δ, respectively, Y is current solution position and V1, V2 and V3 are random vectors. Then, the final position of the current solution can be computed mathematically as follows.

\[ Y_1 = Y_α + R_1*S_α \] (9)
\[ Y_2 = Y_β + R_2*S_β \] (10)
\[ Y_3 = Y_δ + R_3*S_δ \] (11)

Thus, Y (i+1) can be computed as follows:

\[ Y(i+1) = Y_1 + Y_2 + Y_3 \] (12)

where i is the number of iterations. R1, R2 and R3 are random vectors. By using this random vectors we can move wolves towards the prey. Finally, we get the solution when the prey is completely surrounded by the wolves and it cannot move, then the hunting process ends.

4. Optimization Test Functions:

Optimization test function is test function that is applied to mathematical test function which is used to evaluate characteristics of the optimization algorithm. In this, we use these optimization test functions for the characterization of grey wolf optimization algorithm. The characteristics of the optimization algorithm as follows:

1. Convergence algorithm
2. Precision
3. Robustness
4. General performance

These test functions are used to find the minimum time by which the prey can be caught by the wolves using this algorithm.

Some of the Optimization test functions we consider are:

- Gold-Stein price Function
- Beale’s Function
- Booth’s Function

5. Gold Stein Function

Gold Stein function is benchmark function which are having some features like continues, not convex, defined on two-dimensional space, multimodal, it is differentiable and non-scalable. Gold Stein Function is a global optimization test function which is having limits \( a \in [-2, 2] \) and \( i \) values will be various from 1 to 2. Gold Stein function is defined as follows:

\[ F(a) = \frac{1}{2} + a_1^2 + \left[a_1^2 + 2a_2^2\right] + \left[2\left(a_1^2 + a_2^2\right) - 1\right]^2 \] (13)

The gold stein function will have the several local minima. The global minimum for the gold stein function is \( F(a^*) = 0 \), at \( a^* = (0, 0) \)

6. Beale’s Function

Beale’s Function consists sharp peaks along the corners of the input domain. Beale’s function is also a test function for the optimization. It has limits \( -4.5 < a[i] < 4.5 \), for all \( i = 1, 2 \).

The Beale’s function is multimodal, not convex and continuous. The be ale’s function has the global minimum of \( F(a^*) = 0 \), at \( a^* = (3, 0.5) \).

The be ale’s function is as follows

\[ F(a) = (1.5 - a_1 + a_2)^2 + (2.25 - a_1 + a_2^2)^2 + (2.625 - a_1 + a_2^3)^2 \] (14)

7. Booth’s Function

Booth’s function is also an optimization test function. This function is used as a test function that evaluates the performance of optimization algorithm.

The Booth’s function is convex, continuous, and differentiable. The function \( f(a) \) is given as:

\[ f(a) = (a_1 + 2a_2 - 2)^2 + (2a_1 + a_2 - 5)^2 \] (15)

The function has limits \( -10 < a[i] < 10 \) for all \( i = 1,2 \).

The global minimum for this function is \( F(a^*) = 0 \) at \( a(1, 3) \).
8. Test/Debug

<table>
<thead>
<tr>
<th>Opt function</th>
<th>Optimization value</th>
<th>Xα</th>
<th>Xβ</th>
<th>Total time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold Stein</td>
<td>3.00000694</td>
<td>-4.49027490</td>
<td>-0.99995</td>
<td>0.469</td>
</tr>
<tr>
<td>Beale</td>
<td>0.65625000</td>
<td>2.12499290</td>
<td>-1.6254520</td>
<td>0.549</td>
</tr>
<tr>
<td>Booth</td>
<td>4.37559826</td>
<td>0.999847</td>
<td>3.000142</td>
<td>0.422</td>
</tr>
</tbody>
</table>

9. Conclusion

In this paper, we implemented a GWO-Based methodology to estimate the different optimization test functions like Gold Stein function, Beale function, and Booth function. Out of all these functions, we observed that Beale function is giving the optimized value and this is obtained by using this configuration Intel® Core™ i5-4210u CPU @1.70GHz 2.40. The time span for execution of Beale function is 0.549. The time span for execution is different for different configuration systems. By this, we can conclude that Beale function gives more optimized value for hunting the prey using Grey Wolf Optimization(GWO).

References