A New Four-Scroll Chaotic System with a Self-Excited Attractor and Circuit Implementation

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Abstract

This paper reports the finding a new four-scroll chaotic system with four nonlinearities. The proposed system is a new addition to existing multi-scroll chaotic systems in the literature. Lyapunov exponents of the new chaotic system are studied for verifying chaos properties and phase portraits of the new system via MATLAB are unveiled. As the new four-scroll chaotic system is shown to have three unstable equilibrium points, it has a self-excited chaotic attractor. An electronic circuit simulation of the new four-scroll chaotic system is shown using MultiSIM to check the feasibility of the four-scroll chaotic model.

Keywords: Chaos, chaotic systems, circuit simulation, four-scroll system, Lyapunov exponents

1. Introduction

Chaos theory has applications in several branches of science and engineering. Some of the popular applications can be listed as oscillators ([1]-[6]), population ecology ([9]-[11]), epilepsy [12], neurons ([13]-[16]), random bit generators [17], neural networks ([18]-[20]), text encryption [21], image encryption ([22]-[24]), voice encryption [25], economy ([26]-[27]), weather systems ([28]-[30]), chemical reactions ([31]-[34]), robotics ([35]-[38]), and secure communication systems ([40]-[42]).

The development of multi-scroll chaotic attractors is an active research topic. Some classical examples of two-scroll chaotic attractors can be cited as Lorenz system [43], Li system [44] and Tigan system [45]. Some recent examples of two-scroll chaotic attractors can be mentioned as Lien system [46], Vaidyanathan systems ([47]-[49]), Zheng system [50], etc. Some newly discovered examples of three-scroll chaotic attractors can be described as Dadras system [51], Pan system [52], Vaidyanathan 3-scroll system [53], etc. Many four-scroll attractors have also been reported in the literature such as Wang system [54], Lü 4-scroll system [55], Liu-Chen 4-scroll system [56], Sampath 4-scroll system [57], Akgul system [58], etc. In 2003, Yang and Li showed how to generate n-scroll chaotic system with the help of scaler output feedback control [59].

In this paper, we report the finding of a new four-scroll chaotic system with four nonlinear terms. We show that the system has dissipative and self-excited attractor with three equilibrium points. Furthermore, using MultiSIM, we design the electronic circuit of the new four-scroll chaotic system and verify the feasibility of the new chaotic oscillator.

Circuit design of chaotic systems is important for their practical implementation. Many papers in the chaos literature have studied on the electronic circuit implementation of chaotic systems and the chaotic circuits are useful in many control applications ([60]-[69]).

2. A new four-scroll chaotic system with self-excited attractor

We report a new three-dimensional dynamical system given by

\[
\begin{align*}
\dot{x} &= a(y - x) + byz \\
\dot{y} &= -y - 10y|y| + 4xz \\
\dot{z} &= cz - xy
\end{align*}
\]

(1)

which has a total of four nonlinear terms in the dynamics. We show that the system (1) is chaotic for the parameter values \((a, b, c) = (3, 12, 4)\).

For numerical simulations of phase portraits and for the calculation of Lyapunov chaos exponents, we take the initial values as \(X(0) = (0, 1, 0.1, 0.1)\) and parameter set as \((a, b, c) = (3, 12, 4)\). The Lyapunov chaos exponents are determined as \((L_1, L_2, L_3) = (0.6022, 0, -15.0407)\) using Wolf’s algorithm [70]. Since \(L_1 > 0\), the new system (1) is chaotic. By adding \(L_1, L_2\) and \(L_3\), we get the sum as \(-14.4385\), which is negative. This shows that the new system (1) is dissipative. The Kaplan-Yorke dimension is determined as

\[D_{KY} = 2 + (L_1 + L_2)/|L_3| = 2.0400\]  

(2)
which gives a pointer to the complexity of the four-scroll chaotic system (1).

We note that the four-scroll chaotic system (1) is invariant for all parameter values under the coordinates transformation \((x, y, z) \mapsto (-x, -y, z)\). This pinpoints that the four-scroll chaotic system (1) has rotation symmetry about the z-axis. Hence, every non-trivial system trajectory of the four-scroll model (1) has a twin trajectory of the four-scroll model (1).

The equilibrium points of the new system (1) are tracked by solving the following system:

\[
\begin{align*}
-a(y-x) + byz &= 0 \\
-y - 10y|y| + 4xz &= 0 \\
cz - xy &= 0
\end{align*}
\]  

(3)

For the parameter values \((a, b, c) = (3, 12, 4)\), the system (3) has three equilibrium points given by

\[
Q_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 3.0837 \\ 0.8509 \\ 0.6560 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} -3.0837 \\ -0.8509 \\ 0.6560 \end{bmatrix}
\]  

(4)

A simple calculation using the Jacobian matrix of the system (1) shows that the equilibrium point \(Q_0\) is a saddle point, while the equilibrium points \(Q_1\) and \(Q_2\) are saddle-foci equilibrium points. Thus, all three rest points \(Q_0, Q_1, Q_2\) are unstable. This shows that the new four-scroll system (1) has a self-excited attractor.

The phase portraits of the new chaotic system (1) with four-scroll phase portraits are displayed in Figures 1-4. The Lyapunov chaos exponents of the new chaotic system (1) are displayed in Figure 5.

3. Circuit Simulation

In the process of scaling, we proposed \((X, Y, Z) = (x/5, y, z)\) and then the scaled system is obtained as:

\[
\begin{align*}
\dot{x} &= a(\frac{x}{5} - x) + \frac{1}{5}yz \\
\dot{y} &= -y - 10y|y| + 20xz \\
\dot{z} &= cz - 5xy
\end{align*}
\]  

(5)

Based on Kirchhoff’s law, the corresponding circuit equations can be expressed as

\[
\begin{align*}
\frac{1}{C_1} \dot{y} &= -\frac{1}{C_1} y - \frac{1}{10C_1} xz + \frac{1}{10C_1} yz \\
\frac{1}{C_2} \dot{y} &= -\frac{1}{C_2} y - \frac{1}{10C_2} xy + \frac{1}{10C_2} xz \\
\frac{1}{C_3} \dot{z} &= -\frac{1}{10C_3} xy
\end{align*}
\]  

(6)
where \( x, y, \) and \( z \) are the voltages on capacitors \( C_1, C_2, \) and \( C_3, \) respectively. While the power supply is \( \pm 15 \text{ V}. \) The circuit diagram of new four scroll system (6) raised by the MultiSIM software is presented in Fig. 6. The values of circuit components are \( R_1 = 666.67 \Omega, \) \( R_2 = 133.33 \Omega, \) \( R_3 = 16.67 \Omega, \) \( R_4 = 400 \Omega, \) \( R_5 = 4 \Omega, \) \( R_6 = 2 \Omega, \) \( R_7 = 8 \Omega, \) \( R_8 = R_{10} = R_{11} = R_{12} = R_1 = R_4 = \) \( 16 \Omega, \) \( R_9 = R_{13} = 100 \Omega \) and \( C_1 = C_2 = 3.2 \text{ nF}. \) Obtained MultiSIM results in Fig. 7 indicate that the circuit exhibits chaotic attractors. The oscilloscope results in Fig. 7 show good consistency with the MATLAB simulations in Fig. 1-(3).

4. Conclusion

In this research paper, we found a new four-scroll system with a self-excited attractor. The proposed system is a new addition to existing chaotic systems with multi-scroll attractors in the literature. Lyapunov exponents of the new chaotic system were investigated for verifying chaos properties and phase portraits of the new system via MATLAB are unveiled. An electronic circuit simulation of the new chaotic system with four scroll attractors was carried out using MultiSIM to check the model feasibility. As future research work, application of the new chaotic system for voice or image encryption and secure communication systems can be investigated.

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References


Figure 6: Circuit design of the new chaotic system (1) (a) $x$ signal, (b) $y$ signal, (c) $z$ signal, and (d) $|y|$ signal.

Figure 7: Chaotic attractors of the new chaotic system (1) using Multisim circuit simulation: (a) $x - y$ plane, (b) $y - z$ plane, and (c) $x - z$ plane.