The Combining Technique of Calculating the Sections of Reinforced Concrete Bending Elements Normal to its Longitudinal Axis, Based on the Deformation Model

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Abstract

Methodical foundations for calculating the strength of normal sections of various types of steel-concrete bending elements are proposed, which allow calculating in dependence on the stress-strain state (SSS) at the moment of destruction of its components (concrete and structural reduced steel profile). The basis of the calculation allows solving two problems: the problem of determining the optimal section of reduced structural steel profile (RSSP), which reinforced the section of steel-concrete bending elements; the task of verifying the load-bearing capacity of a normal reduced section of various types of steel-concrete bending elements.

Keywords: deformation model; load-bearing capacity; reinforced concrete beam; stress-strain state.

1. Introduction

For this days, current calculation of steel-reinforced concrete (SRC) bending structures (elements) [1, 2, 3] are based on a new concept of calculation, introducing into practice a method of limiting deformations that will make it possible to approach the real stress-strain state of SRC structures (elements). At the same time, it was not possible to completely abandon the calculation method for limiting stresses, so in the simplified analytical calculations, the calculated models of the limiting states of the elements sections with rectangular stress diagrams for both materials (concrete and steel) are used. The calculated positions proposed in the norms [1, 2, 3] do not fully identify the dependence of the load-bearing capacity of the bending SRC element with its SSS at the moment of destruction, which leads to re-reinforcement of its individual sections, that is, to the use of strength properties of its structural metal component not in full volume. Therefore, it is necessary to improve the calculations of the current norms [1, 2, 3], which allowed creating a common methodology for calculating and designing the bending SRC elements depending on the SSS of its sections at the time of destruction.

2. Methodology for Calculating the Bearing Capacity of a Normal Section of Steel-Reinforced Concrete Beams

The purpose of the study is to develop a general methodology for calculating the load-bearing capacity of the normal section of steel-reinforced concrete (SRC) beams depending on the stress-strain state (SSS) at the time of the destruction of its composite materials (concrete, structural steel I-beam profile).

The bearing capacity of the SRC beams is directly related to the bonding conditions between its constituent materials: concrete and structural steel profile. Johnson R.P. in the article [4], notes three cases of stress-strain state of the SRC beams: non-linear-composite, when there is no cohesion between the concrete and the steel profile; partially-composite, when the connection between the concrete and the profile is partial; fully-composite, when there is complete cohesion between the concrete and the steel profile. Therefore, in order to carry out further scientific research on the improvement of the calculations of the current norms [1, 2, 3], a classification of SRC beams was proposed by the type of its general reduced section and cases of the stress-strain state that are inherent in its specific composite properties (Table 1). In this article, the authors developed a methodical approach to the calculation of the sections of bending SRC elements (beams) normal to the longitudinal axis, which have a complete cohesion between concrete and the structural steel profile (cases I-B, II-B, III-B in Table 1).

The general methodology for calculating the load-bearing capacity of the normal section of the bending SRC elements depending on the SSS at the time of destruction of its composite materials involves solving two problems: the selection of the section of the structural reduced steel I-beam profile (SRSIP), which reinforces the normal reduced section of the bending SRC element, that is can be a problem of optimization design; check the strength of the normal reduced section of the bending SRC element.

As a result of the typological analysis of various options for reinforcing the sections of the bending SRC elements, variants of its general reduced calculating sections were adopted: for the section of the SRC element with the concrete upper shelf (case I) in Fig.1; for the rectangular section of the SRC element (case III) in Fig. 2.
Table 1: Classification of the deformation cases of SRC beams depending on the type of its reduced section and the conditions of cohesion between its components

<table>
<thead>
<tr>
<th>Type of total reduced section of the SRC beam</th>
<th>The deformed state of the SRC beams during the destruction stage depending on the type of cohesion between its components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>absence of cohesion</td>
<td>partial cohesion</td>
</tr>
<tr>
<td>I</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
</tr>
<tr>
<td>Option does not exist</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1: Total reduced section of the SRC element with a concrete top shelf

Fig. 2: Total reduced section of the rectangular SRC element, the reduced section of the SRC element

The tasks of checking the strength and selection of the section of the SRSIP (rigid reinforcement) of the normal section of the bending SRC element are based on the following criteria:

- the task of selecting the optimal section \( A_0 \) SRSIP, which reinforced the normal section of the bending SRC element, is solved on the basis of the criterion:

\[
A(\varepsilon_{cu}; \varepsilon_{au}) = A_0 = \min, \tag{1}
\]

where \( A_0 = 2 \times A_t + A_u \) - the sectional area of the SRSIP, which accordingly consists of the sum of the areas of its shelves and the edge; \( \varepsilon_{cu} \) - limiting relative deformations of compression in the uppermost fiber of the compressed zone of concrete of the normal section of the SRC element, taken equal to \( \varepsilon_{cu} = 0.0035 \) (within \( f_{c} = 58 \ldots 60 \) MPa) or in accordance with the norms [1, 2, 3]; \( \varepsilon_{au} \) - the limiting relative deformation strains in the extreme lower fiber of the extended zone of the SRSIP, which reinforced the normal section of the SRC element, the values of which are taken in accordance with the norms of [1, 2, 3].

- the task of checking the strength of the normal section of the SRC element is based on the criteria:

\[
M(e_{cu}; \varepsilon_{cu} < \varepsilon_{au}) = \max; M(e_{au}, \varepsilon_{au}) = \max; M(e_{cu}; \varepsilon_{cu} < \varepsilon_{au}) = \max, \tag{2}
\]

where \( M \) - the maximum value of the bending moment, which can perceive the normal reduced section of the SRC element; \( \varepsilon_{cu} \) - relative deformation in the lowermost fiber stretched zone SRSIP.

To solve the above problems, the prerequisites for calculation have been adopted, which are given in [5, 6, 7]. The prerequisites are common for all calculations of the bending SRC elements (beams) that have a complete cohesion between concrete and the structural steel profile (cases I-B, III-B in Table 1). Some prerequisites for calculation were used on the basis of the assumptions of the calculation, which are set out in the provisions of the current norms [1, 2, 3], other ones were proposed by the authors for the first time.

Below are the calculated analytical dependencies for solving the problems of checking the strength and selection of the section of the SRSIP (rigid reinforcement) of the normal section of the bending SRC element, which was obtained as a result of numerical solutions [7, 8, 9].

2.1. Selection of the Required Section, Which Reinforces the Normal Section of the Bending Element

The objective of the problem is to determine the optimal section of the SRSIP of the SRC element, in which the deformations in the upper (concrete) and lower (steel) fibers of its normal section simultaneously reach the limiting values \( \varepsilon_{cu} \) and \( \varepsilon_{au} \) respectively.

The optimal sectional area of the SRSIP \((A_a)\) for the rectangular SRC element (case III-B) is proposed to be determined by the following dependencies:

\[
A_a = A_0 \times \mu_a; \quad \mu_a = (1 + \Delta_L)/(1 + \Delta_L), \quad \mu_a = (0.85 \times \beta \times \Delta_L)/(1 + \Delta_L), \tag{3}
\]

where \( \alpha = E_c/E_s \) - coefficient of relation of the modulus of elasticity of structural steel and concrete; \( \Delta_L = \varepsilon_{cu}/\varepsilon_{su} \) - coefficient of relation of the boundary relative deformations of concrete \((\varepsilon_{su})\) and SRSIP \((\varepsilon_{su})\); \( \Delta_L = h \times \beta/2 \) - coefficient of relation of height SRSIP \((h)\); \( \Delta_L = C/2 \times h \) - coefficient of relation of the height of the protective layer of concrete \((C)\) to the total height of the SRC element \((h)\); \( \mu_a = A_0/ A_a \) - the optimal coefficient reinforcing structural reduced steel 1-beam profile by normal section SRC element; \( A_0 = h \times b \) - the area of the normal rectangular section of the SRC element.

As a result of the transformations, dependence (3) makes it possible to obtain dependencies with relation to the quantities \( \Delta_L \) and \( \Delta_L \):

\[
\Delta_L = [1 + \Delta_L] \times (2 - \Delta_L)/[(1 + \Delta_L) \times (1 - \Delta_L)], \tag{5}
\]

\[
\Delta_L = [1 + \Delta_L] \times (2 - \Delta_L)/[(1 + \Delta_L) \times (1 - \Delta_L)], \tag{6}
\]

Coordinates of the neutral line in height \((h)\) of the rectangular section of the SRC element can be determined by the dependencies for \( h = Y_b + Y_t \):

\[
Y_b = h \times [(1 + \alpha \mu)/(2 - \Delta_L)] \times [(2 + (1 + \alpha \mu))], \tag{7}
\]

\[
Y_t = h \times [(1 + \alpha \mu)/(2 - \Delta_L)] \times [(2 + (1 + \alpha \mu))], \tag{8}
\]

As a result, numerical dependencies between the dimensionless coefficients of the relations \( \Delta_L \) and \( \Delta_L \) and calculating the coefficients \( \alpha \mu \) were obtained. So the value of calculation \( \alpha \mu \) depending on the values of the coefficients of the relations \( \Delta_L \) and \( \Delta_L \) for the normal reduced rectangular section of the SRC...
element in the absence of a protective layer of concrete in its lower stretched zone, that is when \( C_r = 0 \) and coefficient \( \Delta_c = 0 \) are shown in Table 2. Correcting value of calculating the coefficients \( a_{\text{opt}} \) with \( C_r > 0 \), \( \Delta_c > 0 \) depending on the height of the protective layer of concrete \( C_r \) by using the coefficient \( k_c \), the value of which is given in Table 3:

\[
a_{\text{opt}}(\text{within } \Delta_c > 0) = k_c \times a_{\text{opt}}(\text{within } \Delta_c = 0).
\]

(9)

**Table 2**: Calculation value \( a_{\text{opt}} \) within a coefficient of relation \( \Delta_c = 0 \) depending on the value of the coefficients of relations \( \Delta_c \) for SRC beams of rectangular section

<table>
<thead>
<tr>
<th>( \Delta h )</th>
<th>( 0.1 )</th>
<th>( 0.2 )</th>
<th>( 0.3 )</th>
<th>( 0.4 )</th>
<th>( 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.501</td>
<td>0.531</td>
<td>0.564</td>
<td>0.601</td>
<td>0.644</td>
</tr>
<tr>
<td>0.10</td>
<td>0.476</td>
<td>0.506</td>
<td>0.539</td>
<td>0.577</td>
<td>0.621</td>
</tr>
<tr>
<td>0.15</td>
<td>0.451</td>
<td>0.480</td>
<td>0.514</td>
<td>0.552</td>
<td>0.596</td>
</tr>
<tr>
<td>0.20</td>
<td>0.426</td>
<td>0.455</td>
<td>0.488</td>
<td>0.526</td>
<td>0.571</td>
</tr>
<tr>
<td>0.25</td>
<td>0.400</td>
<td>0.429</td>
<td>0.462</td>
<td>0.500</td>
<td>0.545</td>
</tr>
<tr>
<td>0.30</td>
<td>0.374</td>
<td>0.402</td>
<td>0.435</td>
<td>0.473</td>
<td>0.519</td>
</tr>
</tbody>
</table>

Optimum sectional area \( S_{\text{RSIP}} \) (\( A_\text{opt} \)) of the SRC element of rectangular section is determined by dependencies (3), (4), (9), preset by the initial values: beam section dimensions \( h \), \( b \); \( C_r \); strength characteristics of concrete and steel: \( E_c \), \( E_p \), \( f_{cu} \) and \( f_{eu} \); relation of both heights section \( S_{\text{RSIP}} \) and element: \( \Delta_\text{h} = \text{h}/h \).

Optimum sectional area \( S_{\text{RSIP}} \) (\( A_\text{opt} \)) of the SRC element with a concrete top shelf (Case I-B) is proposed to be determined by the following dependencies:

\[
A_{\text{opt}} = \frac{A_\text{opt}}{A_\text{c}} = \frac{A_\text{opt}}{A_{\text{c}}} = \frac{h}{h} \times \frac{(2A_\text{opt} + 1)}{(1 + h^2/\Delta_c^2)} \times \frac{(2 - \Delta_c^2)(1 - \Delta_c)}{(1 - \Delta_c)^2} \quad (10)
\]

where the coefficient values \( \mu_c \), \( \Delta_c \) see the explanations to dependencies (3) and (4); \( \Delta_\text{h} = \text{h}/T_\text{f} \) – coefficient of relation of height \( S_{\text{RSIP}} \) (\( h \)) to the height of the upper concrete belt (\( T_\text{f} \)) of the SRC element; \( \mu_{\text{opt},c} = A_\text{opt}/A_c \) – the optimal coefficient reinforcing structural reduced steel 1-beam profile by normal section SRC element; \( A_{\text{c}} = A_\text{c}/A_{\text{c}} \) sectional area of the upper concrete shelf of the SRC element.

As a result of the transformations, dependence (10) makes it possible to obtain dependencies with respect to the quantities \( \Delta_\text{c} \) and \( \Delta_c \):

\[
\Delta_\text{c} = \frac{(1 + \mu_c \times (2 - \Delta_c^2)/(1 + \Delta_c^2))}{(1 + \Delta_c^2)} \times (2 - \mu_{\text{opt},c}) - 2\times (1 - \Delta_c^2) \quad (11)
\]

With the coordinates on the neutral line in the height of the section (h) of the SRC element with a concrete upper shelf can be determined by the dependencies within \( h = Y_\text{c} + Y_\text{f} = T_\text{f} + h_i \):

\[
Y_\text{c} = (T_\text{f} + \mu_{\text{opt},c} \times (1 + \Delta_c^2))/\Delta_c^2 \times (1 - \mu_{\text{opt},c}) \quad (12)
\]

As a result, numerical dependencies between the dimensionless coefficients of the relations \( \Delta_\text{h} \), \( \Delta_\text{c} \) by calculation of coefficients \( a_{\text{opt},c} \), were obtained. So calculation value \( a_{\text{opt},c} \) depends on the values of the coefficients of the relations \( \Delta_\text{c} \) and \( \Delta_\text{h} \) for a normal reduced section of the SRC element with a concrete top shelf are shown in Table 4.

**Table 4**: Calculation value \( a_{\text{opt},c} \) depending on the coefficients of the relations \( \Delta_\text{c} \) and \( \Delta_\text{h} \) for the section of the SRC I-beam with a concrete top belt

<table>
<thead>
<tr>
<th>( \Delta_\text{h} )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.097</td>
<td>0.098</td>
<td>0.101</td>
<td>0.103</td>
<td>0.105</td>
</tr>
<tr>
<td>0.10</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.20</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.25</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.30</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Optimum sectional area \( S_{\text{RSIP}} \) (\( A_\text{opt} \)) for SRC I-beam elements with a concrete upper belt, it is determined the dependence (10), first setting the initial values: the size of the section of its concrete upper belt \( B \), \( T_\text{f} \); strength characteristics of concrete and steel: \( E_c \), \( E_p \), \( f_{cu} \) and \( f_{eu} \); relation of the heights of the sections of the \( S_{\text{RSIP}} \) and element: \( \Delta_\text{h} = \text{h}/T_\text{f} \).

2.2. Checking the Strength of the Normal Section of a Bending Element

The aim of the problem is to determine the limit value of the bending moment parameter \( M_\text{pl} \) of the given normal section of the bending SRC element and its comparison with the moment \( (M) \) acting on it from external loads:

\[
M_\text{pl} \geq M \quad (15)
\]
As a result of the generalization, three separate cases were identified for the SSS of bending SRC elements at the fracture stage or in its extreme state, depending on the position of the neutral axis: for SRC I-beams with a concrete upper belt (Figure 3); for SRC beams of rectangular section (Figure 4):

- case "a": when in the uppermost fiber of the compressed concrete section of the section the relative deformations of the concrete reach the value of the limiting compression deformations \( \varepsilon_{\text{cu}} \), and in the boundary lower stretched fiber, the relative deformations of the SRSIP vary within \( \varepsilon_{\text{s}} = \varepsilon_{\text{pl}} \), that is, there is a zone of plastic deformation;
- case "b": when in the boundary fibers of the section the relative deformations of the concrete reach a value \( \varepsilon_{\text{pl}} \), and the relative deformations of the SRSIP - value \( \varepsilon_{\text{s}} = \varepsilon_{\text{pl}} \);
- case "c": when in the uppermost fiber of the compressed section of the SRC element (case III)

Calculation of the load-carrying capacity of a normal reduced rectangular section of the SRC element (case III-B) for specified parameters \((x_{\text{opt}}, \varepsilon_{\text{opt}}, C_{\text{f}}, E_{\text{f}}, f_{\text{pl}}; \alpha_{\text{f}} = b \times t_{\text{f}}; A_{\text{f}} = 2 \times h_{\text{f}} \times b_{\text{f}} + h_{\text{f}} \times t_{\text{f}})\) is conducted in the following sequence:

- the first stage of a condition check:
  \[ \alpha_{\text{f}} \geq k_{C} \times \alpha_{\text{f, opt}}, \]  
  if the condition is satisfied, then the SSS of the normal rectangular section of the SRC element corresponds to the SSS for the case "a", and if not - then SSS for the case "b", and under condition \( \alpha_{\text{f}} = k_{C} \times \alpha_{\text{f, opt}} \) – the SSS of the rectangular section of the SRC element corresponds to the SSS for the case "c" (Fig. 4).

- a second calculation step determines the position of the neutral axis with respect to the condition for SRSIP:
  \[ h_{\text{y}} \leq h_{\text{y}} + C_{\text{f}}, \]  
  where the value of the distance \( Y_{\text{B}} \) is determined by the dependence (7), and the value \( h_{\text{y}} = 2 \times h_{\text{f}} + h_{\text{c}} \), if the condition (21) is satisfied, then the neutral axis crosses the section of the SRSIP (case 3), if not, then the neutral axis passes above the section of the SRSIP (case 1), and under condition \( h_{\text{y}} = h_{\text{y}} + C_{\text{f}} \) - the neutral axis in the normal rectangular section of the SRC element passes along the upper boundary of the section of the SRSIP, that is, case 2;

- in the third stage of the calculation, it is composed the equations of equilibrium of the bending moments of the previously determined case SSS of the normal rectangular section of the SRC element, and then checked the dependence (15) for compliance with the condition of its strength.
h - Y = h - the neutral axis passes along the upper boundary of the section of the SRSIP, (case 2);

• in the third stage of the calculation, it is composed the equations of equilibrium of the bending moments of the previously determined case SSS of the normal section of the SRC element, and then checked the dependence (15) for compliance with the condition of its strength.

3. Conclusion

The main steps of the calculation technique of the load-carrying capacity of the normal reduced section of solid rectangular SRC elements and SRC I-beam elements with a concrete top shelf depending on the SSS of the concrete and SRSIP are described. The proposed dependencies will make it possible to distinguish between the cases of calculation of the load-carrying capacity of the bending SRC elements that have a complete cohesion between the concrete and the structural steel profile, which in turn will make it possible to simplify the process of its calculating by the deformation model.

References


