Noisy texture analysis based on higher order spectra

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Abstract

Texture is described in several approaches by 1st and 2nd order statistics which cannot preserve phase information carried by the Fourier spectrum. Besides, these statistics are very sensitive to noise. In this paper, we study features derived from higher order spectra, especially the third order spectrum, namely the bispectrum, known to offer a high noise immunity and to recover Fourier phase information. In this paper, we exploit phase preservation property by using bispectrum phase. We propose wrapped Cauchy distribution to model phase. Wrapped Cauchy parameters are estimated by maximizing the log-likelihood function. Experiments show that the wrapped Cauchy distribution fits our phase information well. Hence, their parameters are used to feed our feature vector in order to classify textures corrupted by Gaussian noise. Classification results using the proposed approach show a good noise immunity compared to a statistical model based on Gabor phase.

Keywords: Texture analysis; Higher order spectra; Phase; Wrapped Cauchy distribution; Texture classification.

1. Introduction

Texture analysis is an important area of study in computer vision that seeks to find an efficient description of textures. Several methods for describing texture have been proposed in literature and they are classified into structural and statistical. Most of these methods do not take into consideration noise, this is the reason why we are interested in this paper by features derived from higher order spectra especially the third order spectrum namely the bispectrum which can be classified as statistical. Algorithms based on bispectrum have proved to be able to deal with non gaussian and noisy images in applications such as: pattern recognition [1], image retrieval [2], texture classification [3] and biomedical [4] [5]. These algorithms exploit higher order spectrum properties. In this work we exploit the insensitivity to gaussian noise and phase restoration properties. Therefore, we modelize the phase matrix recovered from bispectrum using circular parametric model: the wrapped Cauchy model. In order to evaluate our procedure, classification tests using SVM are performed on noisy textures and compared to method described in [6]. The remainder of this paper is organized as follows: The main functions and properties of bispectrum are described in section II. The computation of the phase matrix is detailed in section III. A texture model and feature extraction procedure are presented in section IV. The goodness of fit and classification results are described in section V. Finally, section VI concludes our work.

2. Bispectrum

The bispectrum is the Fourier transform of the triple correlation. With the convolution theorem, it can be defined by:

\[ B(f_1, f_2) = X(f_1)X(f_2)X^*(f_1 + f_2) \]  

where \( X \) denotes the Fourier transform of the signal and \( \ast \) its conjugate.

Unlike the power spectrum, higher order spectra as in the case of bispectrum, satisfy invariance properties [7] and others important properties such as insensitivity to gaussian noise and phase preservation:

- **Noise immunity:**
  The bispectrum of a Gaussian process is null. Furthermore, Gaussian noise is removed in higher order spectra.

- **Phase preservation:**
  The bispectrum is also known to preserve phase information, unlike power spectrum which preserves only amplitude information due to conjugate multiplication:

\[ F(u) = X(u) \ast X(u) \]  

For this reason, we are interested in using bispectrum, especially its phase. The bispectrum could also be expressed by eq (3):

\[ B(f_1, f_2) = |B(f_1, f_2)|e^{i\psi(f_1, f_2)} \]  

where \( \psi \) is the phase and \( |B(f_1, f_2)| \) is the amplitude.

- **Phase restoration:**
  Due to the importance of fourier phase in image processing [8] and the loss of this information through the conjugate multiplication, we use bispectrum phase to recover the phase of the signal lost in the power spectra. The fourier phase recovered from bispectrum was used in different image fields such as: texture classification [3] and speckle imaging [2]. However, this paper is the first to study the
distribution of the phase recovered from the bispectrum as detailed in the next section.

3. Phase matrix computation

Fourier transform is an important tool used in image analysis, it could be represented in terms of magnitude and phase as defined by:

$$F(u) = |F(u)|e^{i\varphi(u)}$$

(4)

where $|F(u)|$ and $\varphi(u)$ are respectively the magnitude and the phase. The importance of phase is shown in the experiment suggested by Openhim and Lim [8]. It consists in swapping the phase and magnitude for two images. Figure 1 shows the result of this experiment:

![Image](image1.png)

Figure 1: The image A is the mixture of the two images with the magnitude of HongKong image and the phase of Barbara image while the image B has the opposite [8].

The revealed image is the one whose fourier phase was used in the reconstruction. This experiment proves that phase contains more important features than magnitude. Unfortunately, the phase information is lost through the conjugate multiplication as defined in the Fourier spectrum equation:

$$F(u) = X(u)^*X(u)$$

(5)

From eq (5), we deduce that power spectrum preserves only information about magnitude and loses phase spectrum information due to the conjugate multiplication. For this reason, we extract fourier phase using bispectrum that is a multiplication of three complex functions corresponding to different frequencies, as shown by eq (6).

$$B(f_1, f_2) = X(f_1)X(f_2)^*X(f_1, f_2)$$

(6)

This triple product retains the fourier phase information that we are interested in. Hence, the first step in our algorithm is calculating the bispectrum of the 2D image that is a 4D function. To reduce the computational complexity, we propose to calculate bispectrum from image lines. Let $l_i$ be 1D $i^{th}$ line and $L_i$ its Fourier transform. The bispectrum of $l_i$ is defined as the triple product of $L_i$ by:

$$B_i(\omega_1, \omega_2) = L_i(\omega_1)L_i(\omega_2)^*L_i(\omega_1 + \omega_2)$$

(7)

where $B_i(\omega_1, \omega_2)$ is the bispectrum of the $i^{th}$ line. Biphase (bispectrum phase) obtained from the bispectrum of each image line could be used to extract the unknown fourier phase. From (3) and (7) biphase could be defined as the sum of three fourier phases $\varphi$ by:

$$\psi(\omega_1, \omega_2) = \varphi(\omega_1) + \varphi(\omega_2) - \varphi(\omega_1 + \omega_2)$$

(8)

Based on Matsuoka algorithm [9], eq(8) can be expressed as the matrix equation:

$$A\varphi = \psi'$$

(9)

where $\psi'$ is the unwrapped bispectrum phase and $A$ is a coefficient matrix. The Fourier phase vector is then computed using the least squares solution:

$$\varphi = (A^TA)^{-1}A'$$

(10)

As described above we determine the fourier phase recovered from bispectrum phase for each horizontal projection of the image. Then, each vector is rearranged to form a global phase matrix $\Phi$ as depicted in the scheme in Figure 3. After that we measure the relative phase of $\Phi$ defined as the phase difference $\Delta\Phi$ of neighboring vectors at each spatial location $(i, j)$. This measure is defined by [10]:

$$\Delta\Phi(i, j) = \Phi(i, j) - \Phi(i, j + 1)$$

(11)

4. Texture model and features extraction

4.1. Texture model

Our aim in this section, is to present an accurate statistical model for the phase matrix computed in section III. Therefore, we study the distribution of $\Delta\Phi$.

![Figure 2: Wrapped Cauchy fitted with $\Delta\Phi$ empirical histogram](image2.png)

As observed in Figure 2, the histogram has a Gaussian shape. Since $\Delta\Phi$ is a circular data, we propose to study the phase information using wrapped cauchy distribution. The wrapped Cauchy distribution is an alternative to the Fisher von Mises distribution obtained by wrapping the Cauchy distribution on the unit circle. The wrapped Cauchy distribution fits better with the peaky empirical histogram as shown in Figure 2. It has density [11]:

$$f(\Delta\Phi; \nu, \rho) = \frac{1}{2\pi} \frac{1}{1 + \rho^2 - 2\rho\cos(\Delta\Phi - \nu)}$$

(12)

where $\nu$ and $\rho$ are respectively the location and concentration parameters.
The density function could also be expressed as [12]:

\[ f(\Delta \Phi; \nu_1, \nu_2) = \frac{1}{2\pi M(1 - \nu_1 \cos \Delta \Phi - \nu_2 \sin \Delta \Phi)} \]

(13)

where \( \nu_1 = 2 \rho \cos(\nu_1 + \rho) \), \( \nu_2 = 2 \rho \sin(\nu_1 + \rho) \) and \( M(\nu_1, \nu_2) = (1 - \nu_1^2 - \nu_2^2)^{-1/2} \).

To obtain the log-likelihood equations, another parametrisation is introduced:

\[ \eta_1 = MV_1, \eta_2 = MV_2 \]

(14)

Differentiating the log-likelihood function with respect to \( \eta_1 \) and \( \eta_2 \) lead to the likelihood equations.

Then, an iterative algorithm is used as detailed in [19] and the final estimates are:

\[ \hat{\nu} = \arctan \frac{\hat{\nu}_1}{\hat{\nu}_2}, \hat{\rho} = \frac{1 - \sqrt{1 - \nu_1^2 - \nu_2^2}}{\sqrt{\nu_1^2 - \nu_2^2}} \]

(15)

4.2. Features extraction Algorithm

As shown in Figure 3, feature extraction procedure can be summarized in the following steps:

1. Decompose texture into separable lines;
2. For each line, compute the bispectrum by eq (7);
3. Extract the Fourier phase vector from biphase using eq (9);
4. Rearrange phase vectors into one matrix \( \Phi \);
5. Compute the phase difference \( \Delta \Phi \) using eq (11);
6. Use the likelihood maximum estimator for identifying the parameters \( \hat{\nu} \) and \( \hat{\rho} \);
7. Modelling \( \Delta \Phi \) by wrapped Cauchy distributions \( WC(\hat{\nu}, \hat{\rho}) \);
8. The features vector contain wrapped cauchy parameters .

5. Classification results

The objective of this section is to evaluate the performance of our procedure under the presence of noise. For this, tests are performed on textures corrupted by gaussian noise, with zero and variance depending on SNR value. Then, the proposed procedure is compared with a statistical model of relative phase in complex wavelet domain when images are decomposed by uniform discrete curvelet transform (UDCT) [13].

The experiment was made using an image database of two classes each with 100 grey textures at the considered values of SNR and classified using SVM [20] with Gaussian kernel.

The correct classification rate for noisy textures of both methods are detailed in table 1.

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>Our method (%)</th>
<th>UDCT method (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td>10</td>
<td>98</td>
<td>94</td>
</tr>
<tr>
<td>0</td>
<td>98</td>
<td>65</td>
</tr>
<tr>
<td>-10</td>
<td>98</td>
<td>63</td>
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<tr>
<td>-20</td>
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<td>62</td>
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<td>-50</td>
<td>93</td>
<td>55</td>
</tr>
<tr>
<td>-60</td>
<td>94</td>
<td>54</td>
</tr>
</tbody>
</table>

5. Degradation of both methods according to the SNR values is illustrated in Figure 4.
From table 1 and Figure 4, we notice that for an SNR=20 dB the two features extraction methods have the same performance as for the noise free images. However, the other method performance decreases as the SNR drops below 10 dB (from 94% to 54%), while our features extraction technique remains insensitive to Gaussian noise until the SNR is below -40 dB with a small difference from 98% to 93%

We conclude that the features vector derived from bispectrum clearly outperforms the other features vector. This is not surprising given that higher order spectra and especially bispectrum suppresses Gaussian noise.

6. Conclusions

In this paper, a texture analysis scheme which exploits the statistical properties of the phase information recovered from bispectrum was presented. The properties of higher order spectra, especially the high immunity to symmetrical noise and the ability to preserve phase information, justify the use of the second order spectra, namely the bispectrum, to extract robust texture features. Therefore, a model for our phase matrix computed from bispectrum was studied. Experiment results show that the wrapped Cauchy distribution fits well with our phase information. Hence, its parameters were used as features to describe and classify textures. The proposed scheme yields good results even for noisy textures.

References


