Design of Second Order Sliding Mode for Glucose Regulation Systems with Disturbance

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Abstract

In this work, Design of second order sliding mode control has been developed to control the diabetic glucose concentration level under disturbing meal has been controlled using three sliding mode controllers. A comparative study of three sliding mode controllers is made in terms of robustness characteristics due to meal feeding. The first is the classical sliding mode controller, the second is integral sliding mode controller and the third is the second order sliding mode controller. Due to their characteristic features of disturbance rejection, all the three sliding mode controllers are presented here for comparison. The Bergman minimal mathematical model is used to describe the dynamic behavior of blood glucose concentration due to insulin regulator injection. Simulations, based on MATLAB/Simulink, were performed to verify the performance of each controller. It has been shown that integral and second order sliding mode controllers are the best of all in terms of disturbance rejection capability.

Keywords: Sliding mode control, Integral sliding mode control, Second order sliding mode control.

1. Introduction

Diabetes mellitus is the human disease which results from the presence of the high level of blood sugar for prolonged period due to inadequate generation of insulin in blood [1].

In the human body, the beta cells in the pancreas are responsible for reproducing the insulin, which regulates glucose consumption. In diabetes, beta cells fail to produce enough insulin concentration in blood of the human body, which becomes incapable of controlling the glucose level.

With type 1 diabetes, the patient body cannot produce sufficient insulin and doses of insulin have to be injected in the human body to regulate blood glucose level. The patient serves to complete the glucose level control system. Unless the patient with diabetes mellitus gave sufficient insulin, serious problems may arise such as damage to nerve or brain, amputation and probably causes death [2].

The concentration of glucose level in the blood of normal human body is in the range (70-110) mg/dL. Diabetes can be recognized in a human being if its body is unable to control the normal interaction between glucose and insulin. As such, it is necessary to regulate the level of blood glucose by injecting the insulin [3], [4].

In general, the closed loop glucose regulation system consists of three main elements; glucose sensor, insulin pump and control techniques for generating the necessary insulin dosage based on the glucose measurements [5]. Figure (1) shows the block diagram of closed loop system for glucose level control.

Several approaches have been earlier considered the design feedback controllers for insulin-glucose control. Recent research in the orbit of insulin regulation system contained highly advanced control theory, that orbit sample represented by Reinforcement Learning Algorithm as in [7], new module for the multivariable adaptive like in [8], Model-based falsification has dealt with the problem in [9], Robust glucose control via μ-synthesis has presented in [10], Hybrid Newton Observer has been assessed in Analysis of Glucose Regulation System in [11], Terminal Synergetic Control has been employed in the field as in [12], Backstepping sliding mode Gaussian insulin injection control also presented in [13] and lastly Super twisting control algorithm has presented in [14].

![Figure 1. Block diagram of closed-loop insulin regulation system [6].](image)

Biomedical applications managed by advanced control theory such the insulin regulation systems are still struggling for new aspects for improving and enhancing the performance and robustness and that was the motivation of the present work [15],[16]. This work contribution is to introduce a design and comparison study among robust Insulin regulation system based on three sliding mode controllers under meal disturbance. The compared controllers are the classical sliding mode, integral sliding mode and the recent second order sliding mode.
2. Mathematical Model

Bergman minimal mathematical model is the most common referenced model in the literature. It approximates the dynamic behavior of a diabetic patient’s blood glucose concentration to the insulin injection. The main advantage of using Bergman minimal model is that the number of parameters is minimum and it describes the relation between main two factors, insulin and glucose concentrations, without getting into biological complicated details.

In the present work, the nonlinear three-state minimal model of Bergman is considered [1, 17, 18]:

\[
\begin{align*}
    \dot{G}(t) &= -p_1 G(t) - X(t) \left( G(t) + G_b \right) + h(t) \\
    \dot{X}(t) &= -p_2 X(t) + p_3 Y(t) \\
    \dot{Y}(t) &= -p_4 \left( Y(t) + Y_b \right) + i(t)/V_L
\end{align*}
\]

(1)

where \( G(t) \) is plasma glucose deviation, \([\text{mg/dL}]\), \( X(t) \) is remote compartment insulin utilization \((1/\text{min})\) and \( Y(t) \) is plasma insulin deviation \((\text{mU/dL})\). The control variable \( i(t) \) is the exogenous insulin infusion rate \((\text{mU/min})\), while the disturbance \( h(t) \) represents the exogenous glucose infusion rate \((\text{mg/dL min})\).

The physical parameters \( G_b \) and \( Y_b \) are the basal glucose level \((\text{mg/dL})\), and basal insulin level \((\text{mU/dL})\), respectively, and \( V_L \) is the insulin distribution volume \((\text{dL})\). The control parameters are \( p_1 \) \((1/\text{min})\), \( p_2 \) \((1/\text{min})\), \( p_3 \) \((\text{dL/ (mU min)})\) and \( p_4 \) \((1/\text{min})\).

If the unmeasurable variable \( Y(t) \) is assumed a slow variable, then \( \dot{X}(t) = 0 \). From Eq. (1), the expression \( \dot{X}(t) = (p_3/p_2) Y(t) \) can be found. Substitution this expression into the first equation, the model of Eq. (1) is reduced to the following [17]:

\[
\begin{align*}
    \dot{G}(t) &= -p_1 G(t) - \frac{p_3}{p_2} Y(t) \left( G(t) + G_b \right) + h(t) \\
    \dot{Y}(t) &= -p_4 \left( Y(t) + Y_b \right) + i(t)/V_L
\end{align*}
\]

(2)

The linearization of Eq.(2) is performed by taking the variation of \( G(t) = G_0 + \delta G(t) \) and \( Y(t) = Y_0 + \delta Y(t) \) around equilibrium points \( (G_0, Y_0, h_0, i_0) \). The perturbed version of Eq. (2) is given by:

\[
\begin{align*}
    \delta \dot{G} &= \left( -p_1 - \frac{p_3}{p_2} Y_0 \right) \delta G(t) - \frac{(G_0 + G_b) p_3}{p_2} \delta Y(t) \\
    \delta \dot{Y} &= -p_4 \delta Y(t) + \delta i(t)/V_L
\end{align*}
\]

(3)

If the variation function is defined as the first state variable \( x_1(t) \), \( \delta Y(t) \) is set as second state variable \( x_2(t) \) and \( u(t) \) is assigned to control input function \( \delta i(t) \). Then the previous equation can be written in the following state space form, the model of Eq. (3):

\[
\begin{align*}
    x(t) &= \begin{bmatrix} -p_1 - \frac{p_3 Y_0}{p_2} - \frac{(G_0 + G_b) p_3}{p_2} & 0 \\ 0 & -p_4 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1/V_L \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} h(t) \\
    y(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t)
\end{align*}
\]

(4)

For control objectives, the linearization in state space of the above model is taken at the equilibrium points with the specified values: \( h_0 = 0 \), \( i_0 = p_4 Y_0 V_L \), \( G_0 = 0 \). Therefore, the obtained linearized model can be written as [1]:

\[
\begin{align*}
    \dot{x}(t) &= \begin{bmatrix} -p_1 G_b p_3/p_2 \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\
    y(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t)
\end{align*}
\]

(5)

where \( u(t) = [h(t) \; u(t)]^T \). Equation (5) can be written in compact form as:

\[
\dot{x}(t) = A x(t) + B u(t)
\]

(6)

\[
y(t) = C x(t)
\]

It is easy to show that the linearized model of Eq.(5) is completely controllable.

3. Sliding Mode Control Design

Three structures of sliding controllers will be presented and designed for controlling the glucose level in human blood under meal disturbance. These suggested controllers are classic, integral and second sliding mode controllers. Later, the performance of such controllers will be verified and compared to each other using Matlab/Simulink.

3.1. Classical Sliding Mode Controller

Sliding mode control is a discontinuous feedback control strategy which forces the system states to reach and remain on a specific surface within the state space (called sliding surface). The first stage of design is the selection of the discontinuity surface such that sliding motion would exhibit desired properties [19]. Let us define a surface \( s \) in the state space as follows:

\[
s = x_2 + c x_1
\]

(7)

A controller design is required to enforce the system trajectories follow the surface \( s = 0 \), then Eq.(7) becomes

\[
x_2 + c x_1 = 0
\]

(8)

From Eq. (5), one can find that

\[
\dot{x}_1 = -p_3 x_1 - (G_b p_3/p_2) x_2
\]

(9)

Rearranging the above equation results in

\[
x_2 = - p_2 (p_1 x_1 + \dot{x}_1)/(G_b p_3)
\]

(10)

Substituting \( x_2 \) from the above equation into Eq.(8) results in

\[
- p_2 (p_1 x_1 + \dot{x}_1)/(G_b p_3) + c x_1 = 0
\]

(11)

or,

\[
x_1 + (p_1 - c G_b p_3/p_2) x_1 = 0
\]

(12)

The time solution of the above solution is given by

\[
x_1(t = \infty) = 0 \rightarrow G(t) = 0
\]

(13)

such that \( (p_1 - c G_b p_3/p_2) > 0 \).

Equation (13) tells that if state trajectories of the system are enforced to stay moving on the surface \( s = 0 \), then \( x_1 \) will tend to zero exponentially after a finite time interval forced to move, then \( x_1 \) will tend exponentially to zero after a finite time interval; i.e.,

\[
x_1(t = \infty) = 0 \rightarrow G(t) = 0
\]

(14)

To guarantee the approach of state trajectories to the surface \( s = 0 \), the following reaching condition has to be achieved

\[
s s < 0
\]

(15)

Since \( s = x_2 + c x_1 \), then

\[
s s = s [\dot{x}_2 + c \dot{x}_1] = s [-p_3 u_2 + u(t)/V_L - c x_1 - p_b x_1 - c \dot{h}(t)]
\]

(16)

If one assumes that the control defined as a discontinuous function for the surface \( s \) as below
Substituting for the control \( u(t) \) in Eq.(16), we have
\[
\dot{s} = -p_4 s x_2 - k_s \text{sign}(s) V_L - c p_1 s x_1 - c G_B (p_3/p_2) s x_1 + s c h(t) \quad (18)
\]
Using the fact that \( s \times \text{sign}(s) = |s| \) and from linear algebra, the inequality \( ab \leq |a||b| \) holds and results in the following;
\[
\dot{s} \leq |s| \left[ p_4 |x_2| - k_s V_L + c |p_1| |x_1| + c G_B |p_3/p_2| x_1 + c |h(t)| \right] < 0 \quad (19)
\]
Solving for \( k \), we have
\[
k > V_L \left[ |p_4| |x_2| + c |p_1| |x_1| + c G_B |p_3/p_2| x_1 + c |h(t)|_{\text{max}} \right] \quad (20)
\]

### 3.2. Integral Sliding Mode Control

In what follows, integral sliding mode controller is designed for control and disturbance rejection of glucose systems. Starting with rewriting the control law as follows [19]
\[
u = u_0 + u_1 \quad (21)
\]
where the design of controller component \( u_0 \) is devoted to make the system trajectory track a specified trajectory \( x_0 \) in the state space which is based on optimal LQR controller, while the design of \( u_1 \) is dedicated to cancelling the disturbance \( h(t) \).

Rewriting Eq.(4) by separating the control input \( i(t) \) from disturbance \( h(t) \) and then substituting the control signal from Eq.(21) to have;
\[
\dot{x} = A x(t) + B u_0(t) + B u_1(t) + L h(t) \quad (22)
\]
where \( L = [1 \text{ 0}]^T \) and \( B_u = [0 \text{ 1}/V_L]^T \).
The first stage of design is to select the discontinuity surface \( s \) such that sliding motion would exhibit desired properties. The such surface is defined in the state space as follows
\[
s = s_0 + z \quad (23)
\]
where the variable \( z \) stands for the integral part which attempts to cancel the disturbance and \( s_0 \) represents the desired path in the state space given by
\[
s_0 = C x \quad (24)
\]
In order to hold the system trajectories moving on a specified trajectory \( s_0 \), the time derivative of the surface \( s \) has to be equal zero. This means that the system trajectory will stay on surface \( s_0 \) and never leave it.
\[
\dot{s} = s_0 + \dot{z} = C \dot{x} + \dot{z} = 0 \quad (25)
\]
or,
\[
\dot{s} = C [A x(t) + B_u u_0(t) + B_u u_1(t) + L h(t)] + \dot{z} = 0 \quad (26)
\]
To ensure that \( s(t) = s_0(t) \) for all \( t > 0 \), the following condition should be satisfied
\[
B_u u_1(t) = -L h(t) \quad (27)
\]
Substituting for \( z \) we get
\[
\dot{z} = -C [A x(t) + B_u u_0(t)] \quad (28)
or,
\[
z = -C \int_{t_0}^{t} [A x(t) + B_u u_0(t)] \text{ dt} \quad (29)
\]
Substituting for \( z \) into surface equation \( s \) will guarantee all system trajectories to remain on the surface \( s \) even with existence of external disturbances. If \( u_1 \) is considered as a nonlinear function such as given by,
\[
u_1 = -k_1 \text{sign}(s) \quad (30)
\]
where \( k_1 \) is the discontinuous controller gain, then the control law can be rewritten as follows;
\[
u = u_0 + u_1 \quad (31)
\]
To ensure \( \dot{s} = 0 \) for \( \forall t \geq 0 \) in Eq. (26), the following condition has to be satisfied
\[
C B_u u_1(t) = -C L h(t) \quad (32)
\]
Multiplying out the matrices in the above equation results in,
\[
-k_1 \text{sign}(s)/V_L = h(t) \quad (33)
\]
Taking the worst case disturbance \( h(t)_{\text{max}} \), the integral sliding mode gain \( k_1 \) can be evaluated as follows
\[
k_1 = 245 \quad (34)
\]

### 3.3. Second Order Sliding Mode Control

The derivation of the second order sliding mode controller (2-SMC) is based on the direct Lyapunov method, which is also used to prove the asymptotic stability. The advantages of the 2-SMC over the first order one are that it enhance the overall performance and reference tracking even when external disturbances are present. It has faster reaching phase, which means less effect of parameters uncertainties and disturbances, which leads to stability improvement [19]. The 2-SMC has a smoother control action when compared to first-order sliding mode control, 2-SMC has small chattering and faster convergence while maintains robustness [20], [21].

The problem in 2-SMC is not to keep the sliding function equal to zero, but also its second derivative as well; this means that the control action is still acting in the second derivative of the sliding surface [20].

Direct Lyapunov approach is used to prove the stability of the closed loop system. According to Lyapunov, one can ensure the global asymptotic stability if the derivative of the Lyapunov functions is negative. In sliding mode theory, this condition is also known as reaching condition [19]:
\[
\dot{V} < 0, \quad s(t) \neq 0, \quad \dot{s}(t) \neq 0 \quad (35)
\]
Put the system in the following form
\[
\dot{x}_1 = -p_2 x_1 - (G_B p_3/p_2) x_2 + h(t) \\
\dot{x}_2 = -p_4 x_2 + u/V_L \quad (36)
\]
The proposed surface function is
\[
s = x_1 \quad (37)
\]
Differentiate \( s \) twice
\[
\dot{s} = x_1 = -p_4 x_2 - (G_B p_3/p_2) x_2 + h(t) \quad (38)
\]
\[
\ddot{s} = -p_4 \dot{x}_1 - (G_B p_3/p_2) \dot{x}_2 + h(t) = -p_4 [ -p_1 x_1 - (G_B p_3/p_2) x_2 + h(t) - \left( G_B p_3/p_2 \right) x_2 - p_1 x_1 + h(t) \left( G_B p_3/p_2 \right) u ] \quad (39)
\]
Eq.(38) can be written as;

\[ \ddot{s} = \psi(t) - gu \]  

(39)

where,

\[
\psi(t) = p_2 x_2 + (G_b p_3 (p_1 + p_4)/p_2) x_2 - p_1 h(t) + \dot{h}(t)
\]

and \( g = (G_b p_3 / p_2) \dot{h}(t) \). The proposed control action is

\[
u = \lambda_1 s + k_2 \text{sign}(\dot{s})
\]

(40)

Substituting Eq. (40) into Eq. (39) gives

\[ \ddot{s} = \psi(t) - \lambda_1 g s - k_2 g \text{sign}(s) \]

(41)

Let \( \lambda_4 = \lambda_1 g, \dot{k}_2 = k_2 g \) then Eq. (41) becomes

\[ \ddot{s} = \psi(t) - \lambda_4 s - \dot{k}_2 \text{sign}(\dot{s}) \]

(42)

Let the candidate Lyapunov function be

\[ V = \frac{1}{2} \lambda_4 s^2 + \frac{1}{2} \dot{k}_2 \dot{s}^2 \]

(43)

To have an asymptotic stable system, \( V \) should be less than or equal to zero; i.e.,

\[
V = \lambda_4 s \dot{s} + \dot{k}_2 \dot{s} \left[ \psi(t) - \dot{\lambda}_4 s - \dot{\dot{k}}_2 \text{sign}(\dot{s}) \right] \\
= \lambda_4 s \dot{s} + \dot{k}_2 \dot{s} \left[ \psi(t) - \dot{\lambda}_4 s - \dot{\dot{k}}_2 \text{sign}(\dot{s}) \right] \\
\leq \lambda_4 |s||\dot{s}| + \dot{k}_2 |\psi(t)||\dot{s}| - \dot{\lambda}_4 |s||\dot{s}| - \dot{\dot{k}}_2 |\dot{s}| \\
\leq -|s| \left| \left( \lambda_4 \dot{k}_2 \dot{s} - \dot{\lambda}_4 \right) + \dot{k}_2 |\psi(t)| \right| 
\]

(44)

The above equation is satisfied if the following conditions holds

\[ \lambda_4 \dot{k}_2 - \dot{\lambda}_4 \geq 0 \Rightarrow \lambda_4 \geq 1 \]

(45)

\[ \dot{k}_2 - |\psi(t)| \geq 0 \Rightarrow \dot{k}_2 \geq |\psi(t)| \]

(46)

By the above conditions, the negative semi-definite of Lyapunov function derivative is assured and the global asymptotic stability is guaranteed. In the control action law a hyperbolic tangent function has been used instead of sign function to avoid chattering [19]-[21].

### 4. Sliding Mode Control Design

The effectiveness of suggested controllers against meal disturbance is verified via simulations based on MATLAB-Simulink. Figure (2) shows meal disturbance function behavior, which represents the exogenous glucose infusion. Figure (3) referred in Appendix shows the Simulink modeling of insulin regulator system based on sliding mode controllers for glucose level control against disturbing meal. The behavior of meal disturbance is saved inside a look-up table (see Appendix A). The model parameters are listed below [1]:

**Figure 2.** Disturbance meal function (exogenous glucose infusion).

\[ p_1 = 0.028, \quad p_2 = 0.025, \quad p_3 = 0.00013, \quad p_4 = 0.093, \quad G_b = 110, \quad Y_b = 1.5, \quad i_0 = 16.67. \]

Based on the above values, the design parameter \( K \) is calculated using Eq.(20). Then, the value of \( K \) is equal to 2.307.

The design parameters for the second order sliding mode controller are

\[ \lambda_4 = 1.8, \quad K_2 = 8 \]

Figure (4) shows the glucose level under meal disturbance of Figure (2) with the three controllers. Classical sliding mode controller (SMC) gives performance of maximum glucose level does not exceed 123 (mg/dL); i.e., the controller permits 11.18% change over the basal glucose level. The glucose level of integral sliding mode controller is 119.2 (mg/dL) which means a percentage change of 8.36% above the basal level. It is evident from the figure that second order sliding mode controller shows the best robustness characteristics than all the above controllers with glucose level of 119 (mg/dL) which indicate a percentage change of 8.18% above the basal level, this percentage is the minimum one as compared to others.

Figure (5) shows the insulin rate resulting from each controller. It has been shown that the more robust controller, the higher level of insulin rate which is taken by human body. This physically indicates that more robust controller of requires more insulin rate to be injected; this is the price of robustness. It can be noticed from Figure (5) that the 2-SMC has less insulin rate than ISMC even though the basal level of 2-SMC is less than that of ISM.

This shows 2-SMC is able to achieve better performance while maintaining control action at a reasonable level.

**Figure 4.** Plasma glucose level

**Figure 5.** Insulin rate resulting from controllers

### 5. Conclusion

In this paper, three different types of controllers were presented for the problem of blood glucose concentration level control. The simulated results have shown that the designed controllers could successfully control the glucose level successfully and retain the glucose level back to its basal level. However, the robustness of the controllers against meal variation differs from a controller to another. Both the integral and second order sliding mode controllers seem to have approximately the same performance characteristics in terms of disturbance rejection capability. These controllers are both preventing the glucose level to exceed 120 mg/dL. However,
the second order sliding mode controller could keep the glucose level to lower value than the others under the same meal characteristics. On the other hand, the results have been demonstrated that the classical sliding mode controller delivered the worst performance at the glucose level exceeds the value of 122 mg/dl. Even so, it has been shown that if the second order sliding mode controller has been well designed it would have the lowest insulin infusion compared to other controllers. Thus the second order controller outperforms others in terms of robustness and insulin saving.

6. References


Appendix (A)

Figure (3) shows the Simulink modeling of sliding mode controllers for glucose level control system against disturbing meal. Table below lists the data of disturbing meal behavior.