Comparison of beamforming algorithms based on localization of calibrating sound sources and air jet noise

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Abstract

The paper considers study of beamforming algorithms for localization of noise sources. The mathematical formulations are briefly described for the following algorithms: Delay-and-Sum Beamforming, Cross-spectral Beamforming, Deconvolution Approach for the Mapping of Acoustic Sources (DAMAS). Based on the mentioned algorithms the program codes were developed. Operability of the program codes was tested on virtual localization of the point sources. All algorithms demonstrated good ability to distinguish these sources at different frequencies at their close position relative to each other. Initially, experiments were based on localization of calibrating static sound sources (beepers) using Bruel & Kjaer 54-microphone array. The measured data were processed both in the Bruel & Kjaer software and in the developed software. For static point sources, all algorithms have shown good work quality. The experiments were also carried out for the localization of noise sources in a turbulent air jet. In this case, the best results were demonstrated by Cross-Spectral Beamforming algorithm.

Keywords: Anechoic chamber; beamforming; microphone array; noise pollution; sound sources.

1. Introduction

With the advance of scientific and technological progress, the noise pollution of the environment has increased significantly. This primarily affects human health, as well as the animate nature. To develop the effective means for suppressing the noise generated by various power plants, industrial and transportation facilities, it is necessary to have information about the spatial distribution of the acoustic sources. For these purposes, noise measurements are carried out by phased microphone arrays and various beamforming algorithms process experimental data. This gives the knowledge on the position and sound pressure level (SPL) of the noise sources at frequencies of interest. The fundamentals of the beamforming method are detailed in the works [1-2].

In the study, the possibilities of Delay and Sum Beamforming, Cross-Spectral Beamforming and Deconvolution Approach for the Mapping of Acoustic Sources (DAMAS) algorithms for localization of stationary point sources and jet noise sources were considered. Here we briefly touch only the mathematical formulations of these algorithms. The Delay-and-Sum method is a classical beamforming algorithm for the far-field case, which is described by the response of a microphone array in the frequency domain [1-3]:

\[ B(\omega, \vec{r}) = \frac{1}{M} \sum_{m=1}^{M} P_m e^{i \Delta_n} \]

where \( P_m \) is the sound pressure in Fourier space, \( \Delta_n \) is the time delay, \( \vec{r} \) is the vector that determines the distance between the position of the microphone and the point of a discrete grid in space, \( M \) is the microphone number in the array.

The squared response of a microphone array can be expressed as:

\[ |B|^2(\omega, \vec{r}) = \frac{1}{M} \left| \frac{v^T C v}{w^T w} \right|^2 \]

Here \( C \) is the matrix of cross-spectrum elements \( C_{nn} = P_n P_m^* \), \( v \) is the steering vector with elements \( v_m = e^{-i k \vec{r}} \), \( w \) is the steering vector containing the squared absolute values \( v_m(\vec{r})^2 \). \( 1 \) is the all-ones matrix.

The Cross-Spectral Beamforming algorithm [3] is characterized by the exclusion of autospectral elements from the spectral matrix containing the self-noise of individual microphones:

\[ |B|^2(\omega, \vec{r}) = \frac{1}{M(M-1)} \left| \frac{v^T C v}{w^T w} \right|^2 \]

Here \( C \) and \( 1 \) are the modified cross-spectral matrix and the all-ones matrix, respectively, where all the diagonal elements are replaced by zeros. The superscripts \( T \) and * are the operation of transposition and complex conjugation respectively.

The DAMAS algorithm [4] allows increasing the spatial resolution limit of the Delay-and-Sum method. The problem reduces to a system of linear equations, where it is assumed that incoherent monopoles with different complex amplitudes form the source region:

\[ \begin{bmatrix} |B_1|^2 \ \ldots \ |B_L|^2 \end{bmatrix} = \frac{1}{M^2} \begin{bmatrix} \left| v_1^* v_1 \right|^2 \ldots \left| v_1^* v_L \right|^2 \left| v_1 \right|^2 \ldots \left| v_L \right|^2 \end{bmatrix} \begin{bmatrix} |H_1|^2 \ \ldots \ |H_L|^2 \end{bmatrix} \]
Here $|q_j|^2$ and $|B_l|^2$ are the unknown signal power and the standard Delay-and-Sum power estimate at $l$-th scanning point respectively, $v_j$ is the element of steering vector at $l$-th scanning point. The resulting system of linear equations is solved iteratively using a Gauss-Seidel-type relaxation. All negative source power approximations are equated to zero. For further studies based on experimental data, these algorithms have been programmed. The next sections of the paper present the results of testing these algorithms.

2. Tests of beamforming algorithm operation based on virtual point sound sources

Test of program codes was based on localization of virtual point sound sources. The wave front of sound pressure with a given amplitude at a particular radiation frequency was given in the form of monopole source:

$$p_{in}(r, t) = A \cdot e^{j(\omega t - kr)}$$

Here $A$ is the source amplitude, $\omega$ is the angular frequency, $t$ is the time, $k$ is the wave number, $r$ is the vector defining the distance from source to the microphone in the array. A different number of sources were set at different distances from each other. The ability of algorithms to localize these sources for different frequencies of sound radiation was tested. Signals from the sources were received at points corresponding to the actual locations of the microphones in the 54-channel microphone array Bruel & Kjaer [5, 6], which is used in natural experiments on the localization of sound sources (Figure 1).

![Fig. 1: Microphones position in 54-channel microphone array B&K](image)

In order to find the values of the dynamic range of the microphone array Bruel & Kjaer depending on the frequency of the source, an array pattern was constructed using the formula:

$$W(K) = \sum_{m=1}^{M} e^{jK r_m}$$

Here $K = \hat{k} - \hat{k}_0$ is the difference of the projections of the plane wave with wave number vector $k_0$, incident from a direction different from the focus direction $k$. Based on the array pattern, you can build the maximum sidelobe level (MSL) function:

$$\text{MSL}(K) = 10 \cdot \log_{10} \left( \max_{K_{min} \leq |K| \leq K} |W(K)|^2 / M^2 \right)$$

Here $K_{min} = 1.22(2\pi / D)$ is the first zero in the array pattern corresponding to the circular aperture with diameter $D$. In order to pass from the absolute value of the difference between the projections of the wave vectors to the frequencies, one can use the expression $|K| \leq 2k = 2\omega / c = 4\pi f / c$ and construct the MSL function, depending on the source radiation frequency, as shown in Figure 2. It has been established that the location of microphones in the array is optimized for measurements with a dynamic range of 8.5 dB for frequencies up to 4300 Hz.

![Fig. 2: Array pattern and MSL function for the array depicted in Figure 1](image)

The results of the virtual experiments are shown in Figure 3. It can be seen that all algorithms reliably localize point sources of sound in given positions. The error in determining SPL for localization of several sources is less than 1%. Thus, the next step in verifying the operation of the program code was performing natural tests in anechoic chamber on various types of sound sources.

![Fig. 3: Localization of point sources at frequency of 2000 Hz and SPL of 100 dB](image)
3. Comparison of beamforming algorithms based on natural experiments data

The experiments on localization of two calibrating sound sources (beepers) were performed by 54-channel microphone array B&K in the acoustic anechoic chamber at PNRPU [7, 8]. The tone frequencies of calibrating sound sources were accurately known and equal to 2400 and 2700 Hz respectively. The microphone array was directed at the sound sources at the distance of 4.2 m. In measurements, the position of the beepers was static. The results of the experiments were processed in the B&K software PULSE Array Acoustics Post-processing in the frequency range up to 6400 Hz. The map of the distribution of calibrating noise sources at frequencies 2408 and 2668 Hz is presented in Figure 4. The dynamic range is 5 dB.

The measurement data being processed in the B&K software give the SPL of the beepers 49 and 51.96 dB for the frequencies 2408 and 2668 Hz respectively (Figure 4a). The processing these data with the Delay-and-Sum Beamforming algorithm determines that the SPL is equal to 49.33 and 52.21 dB for the same frequencies (Figure 3b). The Cross-Spectral Beamforming method was more accurate, showing SPL of 49.27 and 52.21 dB, respectively (Figure 3c). When using the DAMAS algorithm, the SPL of the sources is equal to 47.28 and 49.42 dB, but the localization map of sound sources does not contain a wide range of signal distribution relative to the peak value of its amplitude (Figure 3d), which is the main feature of this method.

Thus, the method Cross-spectral Beamforming demonstrates the best agreement with the localization results obtained by the B&K software. As early studies have found, in the localization map of the classical Delay-and-Sum algorithm, with increasing dynamic range, the side lobes are more pronounced due to the influence of self-noise generated by individual microphones. The localization of statistically independent sound sources by the DAMAS algorithm is effective, but there is a difference in the measured SPL. The solution to this problem is probably the use of the DAMAS-C algorithm (with the increase in the spatial resolution limit of the Cross-spectral Beamforming method) and a more careful selection of the number of discrete grid elements that has a significant influence on the efficiency of this method.

The noise of air turbulent jet was also measured by 54-channel microphone array B&K. The measurements were carried out for a conical nozzle with a diameter of 40 mm. The jet velocity in this experiment was 0.3 M. The microphone array was located parallel to the axis of the jet and was distanced from it by 3.83 m. The results of the experiments were processed in the B&K software in the frequency range up to 25600 Hz. The noise sources map for the frequency of 3520 Hz is presented in Figure 5a. The dynamic range is 5 dB.

The maximum SPL obtained by processing the noise of the jet with the B&K software is 27.61 dB (Figure 5a). In the Delay-and-Sum Beamforming algorithm, with SPL of 29.52 dB, the presence of side lobes is clearly seen (Figure 5b). Cross-Spectral Beamforming shows the greatest matching with results of B&K software both in distribution of sound sources and SPL of 27.7 dB (Figure 5c). The operation of the DAMAS algorithm in this study is satisfactory and associated, as mentioned above, with the relatively low density of the discrete grid, and with the wide area of distribution of the acoustic sources in the turbulent air jet at the frequency considered. As in previous experiments, the method shows SPL of 25.63 dB with a significant understatement relative to results of B&K software (Figure 5d).

Fig. 5: Results of experimental data processing for jet noise sources at frequency 3520 Hz. (a) B&K software, (b) Delay-and-Sum, (c) Cross-Spectral Beamforming, (d) DAMAS.
A good initial qualitative localization of the jet noise sources, it is necessary taking into account the effect of the shear layer on the noise propagation (for both intensity and time delay) from sources located outside the jet to microphones located outside the jet [9]. Also, aeroacoustic tests are often conducted in open jet wind tunnels. Typically, the model is placed inside the potential core of the jet, and the microphones locations are outside the flow. Hence, the sound is propagated by the aeroacoustic sources of the model located through the turbulent shear layer of the wind tunnel to the microphones at a distance from the jet. This sound propagation through the turbulent shear layer is accompanied by several effects, which have a negative influence on the beamforming accuracy [10].

4. Conclusion

In the study, the following results have been found. All considered beamforming algorithms demonstrate good operability and reliable localization of sound sources with a relatively uniform radiation region. The Cross-spectral Beamforming method demonstrates the best agreement with the localization results obtained by the B&K software. The presence of side lobes due to the influence of self-noise created by individual microphones is presented in the localization map of the classical Delay-and-Sum algorithm. The operation of the DAMAS algorithm is satisfactory when a uniform sound sources, which is directly related to the size of the discrete grid, which has a significant effect on the resolution and the signal-to-noise ratio. For better localization of sound sources in the turbulent air jet, where the region of distribution of noise sources in space is wide, an increase in the density of the discrete grid is required, which in turn leads to increase in the processing time of the experimental data. In localization of the jet noise sources, the Cross-spectral Beamforming demonstrates the best agreement with B&K software.

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