Surface deformation on thermocapillary convection in a binary fluid with internal heat generation and temperature dependent viscosity

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Abstract

The effects of temperature dependent viscosity and internal heat generation on the onset of steady Bénard-Marangoni convection in a horizontal binary fluid layer heated from below is investigated theoretically. The upper free surface is assumed to be deformable and the lower boundary is considered to be rigid and perfectly insulated to temperature perturbations. The asymptotic solution of the long wavelength is obtained using regular perturbation method with wave number as a perturbation parameter. It is found that the surface deformation of a binary fluid layer enhances the onset of thermocapillary convection while increasing the value of internal heat generation and temperature dependent viscosity will destabilize the binary fluid layer system.

Keywords: Binary Fluid; Convection; Internal Heat Generation; Surface Deformation; Temperature Dependent Viscosity.

1. Introduction

Bénard-Marangoni convection also known as buoyancy-thermocapillary convection refers to the tendency of a hotter material to rise and colder material to sink due to thermal buoyancy and surface tension. The problem of Bénard-Marangoni instability was first studied by [1] where the study shows that buoyancy and the variation with temperature of a surface tension will cause instability when it reinforces with each other. Later, [2] studied the onset of Bénard-Marangoni convection with a deformable free surface in an ordinary fluid layer. The pioneer study to analyzed the stability of both stationary and oscillatory mode for a thermosolutal convective binary fluid layer induced by thermal and solutal gradients is presented in [3]. [4] analysed the effect of evaporation on Bénard-Marangoni convection in a binary fluid layer, where the liquid-gas interface was assumed undeformable. Few researchers also study the binary fluid layer instability by applying other physical influences such as the Soret number [5], [6], nonlinear Soret effect [7] and with throughflow and Soret effect [8]. It is crucial to control the Bénard-Marangoni convection in a binary fluid since thermosolutal problem (behaviour of convection with both temperature and salinity (salt) gradients) is very important in oceanography. At present, numerous models have been developed to investigate the effect of temperature dependent viscosity to a fluid such as by [9]. They found that the effect of temperature dependent viscosity towards the Rayleigh number depends on the type of the fluid, a contrast to the prediction in [10] where it state that the critical Rayleigh number will decrease when temperature dependent viscosity increased. [11] analysed the non-linear stability of convection for a pure fluid with exponentially temperature-dependent-viscosity. Other researchers have studied the temperature dependent viscosity in various models such as [12] investigated the onset of Bénard-Marangoni convection in a pure fluid overlying a solid plate and [13] studied the effect in a ferrofluid heated from below. Some research has been reported to discuss the effects in Rayleigh-Bénard instabilities [14], [9] and in Marangoni instabilities [15], [16]. Internal heat generation also plays an important role in controlling the onset of convection. [17] have studied the onset of Bénard-Marangoni convection with internal heat generation in a pure fluid. They considered the upper to be deformable and they used Runge-Kutta-method with the shooting technique. [18] studied the effect of internal heating on the onset of Bénard-Marangoni ferroconvection meanwhile [19] studied the same effect in a binary fluid.

We are interested to investigate the influence of surface deformation on the onset of Benard-Marangoni convection in a binary fluid layer with the presence of internal heat generation and temperature dependent viscosity. We set the lower rigid and upper deformable free boundaries to be perfectly insulated to temperature perturbations. The resulting eigenvalue problem is solved analytically by using regular perturbation method with wave number as a perturbation parameter.
2. Mathematical formulation

We consider an infinite x- and y- directions of an incompressible binary fluid layer of thickness, d, heated from below with a presence of uniformly internal heat generation and temperature dependent viscosity. The layer of thickness, d is assumed not too thin, so that the effect of buoyancy can be included. The temperature at the lower-boundary rigid (z = 0) is kept at T_r and upper free deformable boundary (z = 1) is kept at T_a (T_r < T_a). The surface tension at the upper-free deformable surface is assumed to vary linearly with temperature and solute concentration gradient in the form

\[ \sigma = \sigma_0 - \tau_s (T - T_0) + \tau_c (C - C_0) \]  

where \( \sigma_0 \) is the unperturbed value, \( T_0 \) is the reference value of temperature, \( C_0 \) is the reference value of concentration, \( \tau_s \) is the rate of change of surface tension with temperature and \( \tau_c \) is the rate of change of surface tension with concentration. The fluid density, \( \rho \) take the form

\[ \rho = \rho_0 [1 - \alpha(T - T_0) + \alpha_c (C - C_0)] \]  

where \( \rho_0 \) is the reference value of density at \( T = T_0 \), \( \alpha \) and \( \alpha_c \) are the coefficients of thermal and solute expansion respectively.

We assumed that the Soret and Dufour effects on heat and mass diffusion to be negligible. The viscosity, \( \eta \) of the binary fluid vary exponentially with temperature in the form

\[ \eta = \eta_0 \exp[\zeta (T - T_0)] \]  

where \( \eta_0 \) is the reference value at the reference temperature and \( T_0 \) and \( \zeta \) are both positive constant. Following [17], the non-dimensionalize equations that based on the Boussinesq approximation are

\[ \overline{T} \left( D^2 - a^2 \right) W - a^2 R a \theta + \frac{R_s}{L_e} a^2 \Phi + 2 D \left( D^2 - a^2 \right) D W \\
+ D^2 \overline{T} \left( D^2 - a^2 \right) W = 0, \]  

\[ \left( D^2 - a^2 \right) \theta - \left[ Q(1 - 2z) - 1 \right] W = 0, \]  

\[ W + \frac{1}{L_e} \left( D^2 - a^2 \right) \Phi = 0, \]  

where \( R_a = \frac{\alpha g d^3 V T}{\nu k} \) is the Rayleigh number, \( R_s = \frac{\alpha_c g d^3 V C}{\nu k} \) is the solutal Rayleigh number, \( Q \) is the internal heat generation, \( L_e = \frac{\kappa}{\nu_c} \) is the Lewis number, \( \nu \) is the kinematic viscosity, \( \frac{d}{dz} \) is the differential operator, \( a = \sqrt{l^2 + m^2} \) is the total horizontal wavenumber, \( W \) is the amplitude of vertical velocity, \( \Theta \) is the amplitude of vertical temperature, \( \Phi \) is the amplitude of vertical concentration of the solute. According to [15], \( \overline{T}(z) \) is given by

\[ \overline{T}(z) = \exp \left[ B \left( z - \frac{1}{2} \right) + \frac{\left( T_r - T_a \right)}{\Delta d} \right] \]  

We assumed that the lower rigid and upper deformable free boundaries are set to be perfectly insulated to temperature perturbations. The boundary conditions are given by

\[ W = DW = D \theta = D \Phi = 0 \] at \( z = 0 \) and

\[ W = D \theta = D \Phi = 0 \] at \( z = 1 \)

where \( C_r = \frac{\kappa V}{\sigma d} \) is the crispation number and \( B = \frac{\Delta \rho d^2}{\sigma} \) is the bond number. Here, the overall Marangoni number is expressed as

\[ M_a = M_{a_1} + M_{a_2} = \frac{\partial \sigma}{\partial T} \frac{\Delta T_d}{\mu k} + \frac{\partial \sigma}{\partial C} \frac{\Delta C_d}{\mu k} \] (see Refs. 21) where \( M_a \) is a measure of the relative strengths of surface tension and viscous forces. The surface deflection given by equation (11) is

\[ E = \frac{C_r \left( D^2 - a^2 \right) D W}{a^2 \left( a^2 + Bo \right)} \]  

and equation (12) will be substituted in equation (10), in order to get rid of E. Now, equations (4)-(6) together with the boundary conditions (8)-(10) are solved analytically by using regular perturbation method. The variables \( W, \theta, \Phi \) and \( \Phi \) are expanded in the power of \( a^2 \) as

\[ (W, \theta, \Phi) = (W_0, \theta_0, \Phi_0) + a^2(W_1, \theta_1, \Phi_1) + ... \]  

Substituting equation (13) into equations (4) – (6) and equations (8) – (10), the zeroth and first order are solved using MAPLE.

3. Results and discussion

The linear stability analysis is carried out to investigate the effect of surface deformation on binary fluid in the presence of internal heat generation and temperature dependent viscosity on the onset of Bénard-Marangoni convection. The lower rigid and the upper deformable free boundaries are assumed to be perfectly insulated to temperature perturbations. The critical Marangoni number, \( M_{a_k} \) is computed analytically by using regular perturbation method for different values of \( R_s, Q, B, C_r, Bo \) and \( Le \). We obtained the graph of various physical parameters to investigate the behaviour of Bénard-Marangoni convection in a binary fluid. Figure 1 represents the variation of critical Marangoni number, \( M_{a_k} \) as a function of internal heat parameter, \( Q \) for different values of crispation number, \( C_r \). From the figure, it can be inferred that an increase in the values of \( C_r \) will decrease the values of critical Marangoni number, \( M_{a_k} \) and hasten the onset of convection in the binary system. The reason being that an increase in \( C_r \) is to elevate the deflection of the upper free surface which in turns promotes insta-
bility in the binary fluid layer. The crucial influence of internal heat parameter, $Q$ on the onset of Marangoni convection in a binary fluid is to decrease the critical Marangoni number as seen clearly in the figure. The deficit of the Marangoni number indicates that the system is unstable because of the positive thermal efficiency that comes from the internal heat source that exists in the system. This finding agrees well with the results of [17], [13] and [19].

The effect of the gravitational forces to surface tension forces or known as Bond number, $Bo$ is illustrated in Figure 2. The graph reveals that an increase of $Bo$ elevates the critical Marangoni number, and makes the system more stable. The reason is due to an increase of $Bo$ leads to an increase in the gravity effect which keeps the free surface flat against the effect of surface tension. The results are in a good agreement with [17]. As for the influence of temperature dependent viscosity, we note that the viscosity parameter, $B$ act as a destabilizer in the system. It is observed that $Ma_c$ decreases rapidly when $B$ increases.

A plot of critical Marangoni number, $Ma_c$ as a function of Bond number, $Bo$ is shown in Figure 4 for different values of Lewis number, $Le$. Lewis number is a ratio of thermal diffusivity to mass diffusivity and it has a significant effect on the fluid flows with simultaneous heat and mass transfer. From the graph, it reveals that the effect of increasing the $Le$ promotes instability in the binary fluid layer.

Figure 5 shows the simultaneous effect of $Bo$ and $Q$ on the onset of Marangoni convection in a binary fluid. From the figure it is obvious that increasing of $Bo$ can increase the stability of the binary system. The highest value of $Bo$ indicates that the system is relatively unaffected by surface tension forces. However, an increase in the internal heat source accelerates the onset of convection in the system.

The combined effects of solutal Rayleigh number, $Rs$ and internal heat source, $Q$ is plotted in Figure 6. Although the effect of $Q$ still remains the same which is destabilize the system, however enlarging the solute concentration in the binary fluid layer assist to hold up the stability in the system.
4. Conclusion

The investigation on the effects of surface deformation, internal heat generation and temperature dependent viscosity on the onset of Bénard-Marangoni convection in a binary fluid layer has been carried out theoretically. From the investigation, the following conclusions can be drawn:

(1) The effect of surface deformation on thermocapillary convection in a binary fluid layer enhances the deformation of the upper free surface and thus making the system more unstable.

(2) Elevating the values of Bond number, Bo can delay the onset of convection in a binary fluid system.

(3) The effects of increasing the viscosity parameter, B and internal heat generation Q are to hasten the onset of Bénard-Marangoni convection in a horizontal binary fluid layer.

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References