A study on the range & deflection distance observation based on latitude position of artillery gun system

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Abstract

Background/Objectives: In this paper, instead of using the six degree of freedom trajectory for a projectile, we consider its four degree of freedom trajectory which is the NATO standard trajectory model. We consider the range and deflection distance observation based on latitude position of Artillery Gun System due to Coriolis acceleration.

Methods/Statistical analysis: We obtain the trajectory of a projectile with 155 mm diameter using the Runge-Kutta method. We obtain the range and deflection distances associated with fire elevation angle when the launch position is located at different position of the artillery gun system. Its position is a function of the latitude.

Findings: By changing the latitude of gun position, variation of range and the deflection distances are measured due to Coriolis force. Under the condition that the fire elevation is fixed, the range and deflection distances are calculated by numerical analysis. The maximum of the range is achieved at the equator. The minimum of deflection distance is observed near the Antarctic while the maximum value of the deflection is observed at the North Pole.

Improvements/Applications: This approach will be a good resource for analysis of delivery accuracy of the artillery gun system based on its position which is a function of latitude.

Keywords: Range; Deflection Distance; Coriolis Acceleration; Four Degree of Freedom; Trajectory; Projectile; Latitude Position.

1. Introduction

For the anti-aircraft artillery system, the point mass trajectory is used for fire power performance analysis [1], [2]. For the indirect artillery system, the modified point mass trajectory model with four degree of freedom or five degree of freedom 3 may be used. There are two methods of the modified point mass trajectory model with four degree of freedom [4], [5]. The first model is the standard NATO model [4]. The second model is Bradley model [5]. Drag force, Magnus force, lift force, gravitational force and Coriolis forces are exerted during the travel of projectiles. The three translational motion equations and one rotational force play a major role for the trajectory equation with four degree of freedom. In this paper, we use the standard NATO trajectory model. By numerical analysis for the standard NATO trajectory model, we calculate the range and deflection distances when the latitude of the launch position of the artillery gun system varies from the South Pole to the North Pole. In Section 2, the standard NATO trajectory model is described. In Section 3, Coriolis acceleration is presented which results in the variation of the range and deflection distances for the artillery gun system. In Section 4, simulation is given and finally conclusions are followed.

2. The nato standard trajectory model

The vector of the yaw of repose (\(\alpha_e\)) as shown in figure 1, consists of velocity and the acceleration components for the NATO standard form [6-8].

Fig. 1: Yaw of Repose (\(\alpha_e\)).
The equation of the x-axis is represented as
\[
\frac{dx}{dt} = -\frac{\pi\rho d^2}{8m} \left[ C_D u + C_D a_x (Q a_x)^2 \right] v v_x + \frac{\pi\rho d^2 f_k}{8m} \left[ C_L u + C_L a_x^2 \right] v^2 a_x + g_x \frac{z t}{R} - \frac{\pi\rho d^2 Q a_x^2 a_x}{8m} (\alpha x a_x^2 - \alpha y a_y^2) - 2\Omega (-u_x \cos(lat) \sin(az) - u_z \sin(lat))
\]
(7)

The equation for the movement of the y-axis is expressible as
\[
\frac{dy}{dt} = -\frac{\pi\rho d^2}{8m} \left[ C_D u + C_D a_x (Q a_x)^2 \right] v v_y + \frac{\pi\rho d^2 f_k}{8m} \left[ C_L u + C_L a_x^2 \right] v^2 a_x + g_y \frac{x t}{R} - \frac{\pi\rho d^2 Q a_x^2 a_x}{8m} (\alpha x a_x^2 - \alpha y a_y^2) + 2\Omega (u_y \sin(lat) + u_z \cos(lat) \cos(az))
\]
(8)

The equation for the movement of the z-axis is expressible as
\[
\frac{dz}{dt} = \frac{dQ}{dt} \sin(lat) - u_2 \cos(lat) \cos(az)
\]
(9)

The equation for spin velocity is as follows
\[
\frac{dp}{dt} = -\frac{\pi\rho d^2 v x}{4u_x}
\]
(10)

Here, \(I_x\) represents the rotational mass of the bullet and \(C_{spin}\) stands for the spin count. An equation regarding the orientation of the yaw angle x, y, and z axis is
\[
\alpha x_1 = -\frac{b_x p (v_x u_x - v_x u_x)}{\pi d^2 C_{mag-f}}
\]
(11)
\[
\alpha x_2 = -\frac{b_x p (v_x u_x - v_x u_x)}{\pi d^2 C_{mag-f}}
\]
(12)
\[
\alpha x_3 = -\frac{b_x p (v_x u_x - v_x u_x)}{\pi d^2 C_{mag-f}}
\]
(13)

The speed at which the calibration point is plotted appears in the following equation (15):
\[
v = \sqrt{v_x^2 + v_y^2 + v_z^2}
\]

Here, \(u_i\) and \(w_i\) represent the speed of the wind and the velocity of wind.

### 3. Coriolis acceleration

Trajectory equations are based on the equation
\[
m \ddot{u} = DF + LF + MF + mg + mA
\]
(15)

Where \(DF, LF, MF, mg\) and \(mA\) mean the drag force, lift force, Magnus force, gravity force and the Coriolis force. In Figure 1, the vector of yaw of repose is given by
\[
\alpha_x = -\frac{\rho d^2 p (v_x u_x)}{\pi d^2 (C_{mag-f} + C_{mag-f} a_x^2)^3}
\]
(16)

And the Coriolis acceleration in equation (15) is given by
\[ \Lambda_x = 2\Omega\left(-u_y \cos(L) \sin(AZ) - u_z \sin(L)\right) \]
\[ \Lambda_y = 2\Omega(u_x \cos(L) \sin(AZ) + u_z \cos(L) \cos(AZ)) \]
\[ \Lambda_z = 2\Omega(u_x \sin(L) - u_z \cos(L) \cos(AZ)) \]

The rotational motion of the projectile is given by
\[ \frac{dp}{dt} = \frac{\pi d^4 \rho C_{\text{spin}} \Omega}{I_{x}} \]

where \(I_x\) is the projectile’s axis moment of inertia, \(d\) is the projectile’s diameter, \(\Lambda_x, \Lambda_y, \Lambda_z\) are Coriolis’s acceleration vector component \([6], [7]\), \(\Omega\) is the angular velocity of the Earth around its polar axis and \(C_{\text{spin}}\) is the spin damping moment coefficient.

### 4. Simulation results

The characteristic of the projectile and its initial conditions\(^8\) tabulate in table 1.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Projectile mass (Kg)</th>
<th>Projectile diameter(m)</th>
<th>Spin angular velocity (rad/s)</th>
<th>Elevation angle(mil)</th>
<th>Muzzle velocity(m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0</td>
<td>47.5</td>
<td>0.155</td>
<td>1654.3</td>
<td>1000(56.25deg)</td>
<td>347m/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lift factor</th>
<th>QM (Magnus force factor)</th>
<th>I(form factor)</th>
<th>Gravity(m/s(^2))</th>
<th>Qd(Yaw drag factor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>9.8005</td>
<td>1.2</td>
</tr>
</tbody>
</table>

### Table 2: Various Aerodynamic Coefficients \([8]\)

<table>
<thead>
<tr>
<th>(M)</th>
<th>(C_{d_{0\Lambda}})</th>
<th>(C_{d_{\Lambda}})</th>
<th>(C_{m})</th>
<th>(C_{s_{\Lambda}})</th>
<th>(C_{\text{mag-f}})</th>
<th>(C_{\text{M}})</th>
<th>(C_{\text{spin}})</th>
</tr>
</thead>
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<tr>
<td>0.40</td>
<td>0.138</td>
<td>4.1710</td>
<td>1.302</td>
<td>20</td>
<td>-0.510</td>
<td>3.725</td>
<td>-0.01320</td>
</tr>
<tr>
<td>0.60</td>
<td>0.138</td>
<td>4.1710</td>
<td>1.302</td>
<td>20</td>
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<td>3.733</td>
<td>-0.01320</td>
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<tr>
<td>0.70</td>
<td>0.139</td>
<td>4.4305</td>
<td>1.311</td>
<td>20</td>
<td>-0.510</td>
<td>3.849</td>
<td>-0.01280</td>
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<tr>
<td>0.75</td>
<td>0.140</td>
<td>4.5500</td>
<td>1.430</td>
<td>20</td>
<td>-0.510</td>
<td>4.220</td>
<td>-0.01260</td>
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<tr>
<td>0.80</td>
<td>0.141</td>
<td>4.6895</td>
<td>1.439</td>
<td>20</td>
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<td>4.276</td>
<td>-0.01240</td>
</tr>
<tr>
<td>0.85</td>
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<td>4.9560</td>
<td>1.452</td>
<td>20</td>
<td>-0.545</td>
<td>4.553</td>
<td>-0.01210</td>
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<tr>
<td>0.875</td>
<td>0.152</td>
<td>5.0840</td>
<td>1.458</td>
<td>20</td>
<td>-0.560</td>
<td>4.692</td>
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<tr>
<td>0.90</td>
<td>0.156</td>
<td>5.2320</td>
<td>1.474</td>
<td>20</td>
<td>-0.575</td>
<td>4.830</td>
<td>-0.01180</td>
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<tr>
<td>0.925</td>
<td>0.177</td>
<td>5.4915</td>
<td>1.423</td>
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<td>-0.650</td>
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<tr>
<td>0.950</td>
<td>0.199</td>
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<td>1.371</td>
<td>20</td>
<td>-0.725</td>
<td>4.542</td>
<td>-0.01155</td>
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<tr>
<td>0.975</td>
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</tr>
<tr>
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<td>6.2950</td>
<td>1.490</td>
<td>20</td>
<td>-0.665</td>
<td>4.587</td>
<td>-0.01150</td>
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<tr>
<td>1.025</td>
<td>0.309</td>
<td>6.5755</td>
<td>1.551</td>
<td>20</td>
<td>-0.635</td>
<td>4.522</td>
<td>-0.01160</td>
</tr>
<tr>
<td>1.050</td>
<td>0.329</td>
<td>6.8555</td>
<td>1.621</td>
<td>20</td>
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<td>4.457</td>
<td>-0.01175</td>
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<tr>
<td>1.10</td>
<td>0.326</td>
<td>7.4470</td>
<td>1.694</td>
<td>20</td>
<td>-0.575</td>
<td>4.516</td>
<td>-0.01150</td>
</tr>
<tr>
<td>1.20</td>
<td>0.318</td>
<td>8.0510</td>
<td>1.802</td>
<td>20</td>
<td>-0.510</td>
<td>4.572</td>
<td>-0.01150</td>
</tr>
<tr>
<td>1.35</td>
<td>0.305</td>
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<td>1.945</td>
<td>20</td>
<td>-0.510</td>
<td>4.599</td>
<td>-0.01130</td>
</tr>
<tr>
<td>1.50</td>
<td>0.291</td>
<td>7.1545</td>
<td>2.089</td>
<td>20</td>
<td>-0.510</td>
<td>4.708</td>
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<td>1.75</td>
<td>0.269</td>
<td>6.7155</td>
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<td>-0.510</td>
<td>4.723</td>
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<tr>
<td>2.00</td>
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<td>2.411</td>
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<td>2.25</td>
<td>0.233</td>
<td>6.0135</td>
<td>2.517</td>
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<td>-0.510</td>
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<tr>
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<td>0.216</td>
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<td>2.614</td>
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<td>-0.510</td>
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</tr>
<tr>
<td>3.0</td>
<td>0.194</td>
<td>5.2330</td>
<td>2.576</td>
<td>20</td>
<td>-0.510</td>
<td>4.289</td>
<td>-0.01070</td>
</tr>
</tbody>
</table>

Table 2 tabulates the aerodynamic coefficients \([8]\) used for ballistic equations.

Assume that the fire elevation angle is 1000mil. We obtain the trajectory of a projectile with 155 mm diameter using the Runge-Kutta method. Under this condition, we obtain the range and deflection distance for every latitude from the South Pole to the North Pole. As shown in figure 4, the maximum value of the range is given at the equator. In figure 5, the minimum value of deflection distance is given near the latitudes of 70 degree in the southern hemisphere and the maximum value of the deflection distance is achieved at the North Pole.

Fig. 4: Range Versus Latitude Angle.

Fig. 5: Deflection Distance versus Latitude Angle.

### 5. Conclusions

For the modified point mass trajectory model, the Coriolis acceleration vector results in variation of range and deflection distance. We observe the relationship between the latitude and distances such as range and deflection distance. When the latitude varies, the variations of range and deflection distance are measured. This result will be useful resources for analysis of fire power performance of the indirect artillery system [9], [10].
Acknowledgment

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References


