

Linear programming technique based optimal relay coordination in a radial distribution system

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Abstract

Now-a-days to ensure power continuity and system's reliability the protection system is to be designed properly for distribution systems to handle the faults to avoiding the damage to the equipments and to the service engineers. Different types of relays with different working principles are used to detect different types of faults in the system. In order to avoid mal operation of relays, proper coordination is to be carried out. The objective of this paper is to maintain the relay coordination as well as to decrease the working time of relays by optimizing the values of time dial setting (TDS) using linear programming problem technique (LPP). The inequality constraints guarantee the coordination margin for each primary or backup relay pairs having a fault very close to the primary relay. Simulation is carried out on a IEEE 15 bus balanced radial distribution system with 3 different types of relays namely standard inverse, extremely inverse and very inverse relay and the results are presented and analyzed.

Keywords: Inverse Relay; Linear Programming Problem; Optimal Relay Co-Ordination; Time Dial Setting.

1. Introduction

Protective system plays a very vital role in a power system, used to detect the abnormalities in the operation and to execute appropriate steps to isolate the faulty part from the healthy region to maintain continuity. A protective system can be considered as a reliable system if only it provides a guarantee of system stabilization as well as continuity of customer's supply. Protection system mainly consists of various types of relays with different working principle to detect different types of faults in the system. In a distribution system, primary relays along with backup relays are used to improve the system protection. Whenever primary relay fails to operate the backup relay comes into operation to maintain reliability so that no instrument is going to be damaged and also system engineers are safe. To achieve this objective, relays have to operate properly and have to coordinate with each other perfectly otherwise protection miscoordination will occur. Broadly relays are classified as static relay, electromagnetic relay and micro-processor based relays. Usage of micro-processor based relays is more frequent due to their fast operations and the design flexibilities. Normally relays having inverse characteristics are used, because these are simple in operation and also reliable. In this paper three different types of inverse relays are considered namely standard inverse relay, extremely inverse relay and very inverse relay. Standard inverse relays operate when the current exceeds its pick up value and the working time depends on magnitude of working current inversely. Extremely inverse relay gives the steepest time current characteristic. These relays are used for protection of machines against overheating. Very inverse relay gives better selectivity as compared to standard inverse relays [1].

Practically it is almost impossible to obtain such a protection strategy given above. On the other hand, the reliability of these networks can be greatly increased if the exact location of fault is determined using modified protection systems or any advance techniques. Several Authors [2-10] applied techniques only to minimize working time while the pickup currents are selected based on experience. To make the coordination process faster, Knable [11] innovated the break points concept, which represent the relays from which the coordination process will done. Several authors [13-14] implement nature inspired algorithms called as genetic algorithm and firefly algorithms to maintain relay coordination.

In this paper, distribution load flow is carried out for IEEE 15 Bus radial system consisting of three different types of relays namely standard inverse relay, extremely inverse relay and very inverse relays. Pre fault voltages at all the buses are determined using load flow analysis. Fault is created at different locations in the system and faults currents are determined. A linear programming problem is formulated with the objective to minimize working time and with primary / backup coordination time interval, TDS and minimum working time of each relay as constraints. Depending on the relay characteristics the coefficients of the objective function is determined. Simulation is carried out and the optimal values of TDS are obtained for different types of relays and the results are presented and described.

2. Problem formulation

The coordination of over-current relays is considered as an optimization problem where the sum of working times of all the relays is to be minimized and is given in equation (1).

Minimize

$$OF = \sum_{i=1}^m t_{pri_near}^i + t_{pri_far}^i \quad (1)$$

Where m denotes the total number of relays $t_{pri_near}^i$ is the working time of the primary relay at I for near end fault at i, $t_{pri_far}^i$ is the working time of primary relay at I for far end faults at i.

2.1. Coordination constraint

The primary and backup relay pairs in the system have to be coordinated by the criterion given in equation (2)

$$t_{j,k} - t_{i,k} \geq CTI \quad (2)$$

Where $t_{i,k}$ is the working time of the primary relay at k and $t_{j,k}$ is the working time of the back-up relay for the same fault at k and CTI is the Coordination time interval.

2.2. Bounds on TDS and working time

The minimum and maximum limits for TDS value for all relays is given in equation (3). This constraint is given in equation (4)

$$TDS_{min}^i \leq TDS^i \leq TDS_{max}^i \quad (3)$$

$$t_{min} \geq 0.2 \quad (4)$$

Where TDS_{min}^i is the minimum TDS and TDS_{max}^i is the maximum TDS value of relay 'i', t_{min} is the minimum working time of relays.

2.3. Relay characteristics

The working time of the relay depends on the characteristics of the relay. The general expression for working time is given in equation 5. The coefficient for three relays considered is given in equations 6 to 8.

$$t_{op} = a_p (TMS) \quad (5)$$

For Standard inverse relay (SIR)

$$a_p = \frac{0.14}{psm^{0.02} - 1} \quad (6)$$

For Extremely inverse relay (EIR)

$$a_p = \frac{80}{psm^2 - 1} \quad (7)$$

For Very Inverse Relay (VIR)

$$a_p = \frac{13.5}{psm - 1} \quad (8)$$

3. Linear programming problem

Linear Programming is applied to the coordination problem with 26 relays for calculation of total working time for 3 different types of relays and given in the equations 9- 13. 39 constraints are there as relay pairs are taken as one. The Linear programming problems for three types of relays are

i) Standard inverse relay (SIR)
Objective Function

$$Z = 2.3140 X_1 + 2.9787 X_2 + 2.9787 X_3 + 3.4280 X_4 + 3.423 X_5 + 4.1056 X_6 + 2.314 X_7 + 3.5088 X_8 + 3.5088 X_9 + 4 X_{10} + 3.5088 X_{11} + 4.0698 X_{12} + 2.3140 X_{13} + 3.2440 X_{14} + 3.2440 X_{15} + 3.9899 X_{16} + 2.9787 X_{17} + 3.7758 X_{18} + 3.7758 X_{19} + 4.9491 X_{20} + 4.9491 X_{21} + 6.0454 X_{22} + 3.4230 X_{23} + 4.4191 X_{24} + 3.4230 X_{25} + 3.9454 X_{26}; \quad (9)$$

Subjected to

$$G(1) = 2.9787 X_4 - 2.9787 X_2 \geq 0.5$$

$$G(2) = 2.3140 X_1 \geq 0.2;$$

$$G(3) = 2.9787 X_2 \geq 0.2; \quad (10)$$

ii) Extremely inverse relay (EIR)
Objective Function

$$Z = 0.2266 X_1 + 0.8183 X_2 + 0.8183 X_3 + 1.4845 X_4 + 1.4845 X_5 + 2.9134 X_6 + 0.2266 X_7 + 1.6253 X_8 + 1.6253 X_9 + 2.64225 X_{10} + 1.6253 X_{11} + 2.811 X_{12} + 0.2266 X_{13} + 1.1873 X_{12} + 1.1873 X_{15} + 2.6263 X_{16} + 0.8183 X_{17} + 2.1552 X_{18} + 2.1552 X_{19} + 5.2379 X_{20} + 5.2379 X_{21} + 9.0199 X_{22} + 1.4845 X_{23} + 3.6998 X_{24} + 1.4845 X_{25} + 2.5249 X_{26} \quad (11)$$

Subjected to

$$G(1) = 0.8183 X_4 - 0.8183 X_2 \geq 0.5$$

$$G(2) = 0.2266 X_1 \geq 0.2;$$

$$G(3) = 0.8183 X_2 \geq 0.2; \quad (12)$$

iii) Very inverse relay (VIR)
Objective Function

$$Z = 0.7577 X_1 + 1.5104 X_2 + 1.5104 X_3 + 2.1065 X_4 + 2.1065 X_5 + 3.1144 X_6 + 0.7577 X_7 + 2.2179 X_8 + 2.2179 X_9 + 2.9396 X_{10} + 2.2179 X_{11} + 3.049 X_{12} + 0.7577 X_{13} + 1.8571 X_{14} + 1.8571 X_{15} + 2.9291 X_{16} + 1.5104 X_{17} + 2.6091 X_{18} + 2.6091 X_{19} + 4.4495 X_{20} + 4.4495 X_{21} + 6.3039 X_{22} + 2.1065 X_{23} + 3.5939 X_{24} + 2.1065 X_{25} + 2.8619 X_{26} \quad (13)$$

Subjected to

$$G(1) = 1.5104 X_4 - 1.5104 X_2 \geq 0.5$$

$$G(2) = 0.7577 X_1 \geq 0.2;$$

$$G(3) = 1.5104 X_2 \geq 0.2; \quad (14)$$

4. Case study

A IEEE 15 Bus radial distribution system is considered with 14 Lines fed from the substation having voltage level of 11 kV. There are 26 over current microprocessor based relays that are installed at both ends of each feeder as primary and backup relay pairs. The plug settings and the CT ratios of all the relays are taken as 1 and 300:1 respectively. Minimum working time for each relay is considered as 0.2s and CTI (coordination time interval) is taken as 0.5. The typical layout of the test system is given in Figure 1.

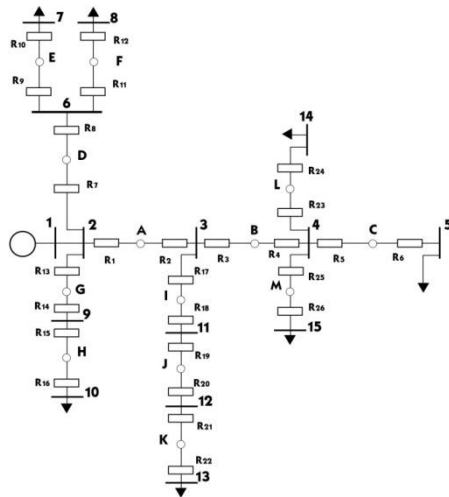


Fig. 1: 15 Bus Radial Distribution System.

Table I gives the detail view about fault current values at each location along with idea about primary relay and its corresponding backup relay. From the table it can be clearly said that fault is severe at the location A, D and G as the fault current is equal to 5644.8A, so attention is to be made towards these three locations along with other locations to maintain relay coordination as any mis-coordination leads to severe damage to the system. Even it is observed that fault current value is more nearer to the source node. Table II gives coefficient values of TDS for different types of relays used in the objective function to minimize working current using linear programming technique. Figure 2 show coefficient value of relays in graphical format from that it can be easily recognize that the coefficient value is more for standard inverse relay as compared to other 2 relays when connected nearer to substation and its value decreases while moving far from the station but at that point coefficient value is more for very inverse relays.

Table 1: Fault Currents at Different Locations for Primary and Backup Relays

Fault location	Primary relay	Fault current (amp)	Backup relay	Fault current (amp)
A	2	5644.8	4	2222.6
B	3	2981.4	1	5644.8
C	5	2222.6	3	2981.4
D	7	5644.8	6	1600.4
E	9	1600.4	7	5644.8
F	11	1600.4	7	5644.8
G	13	5644.8	2	2981.4
H	15	2480.8	13	5644.8
I	17	2981.4	1	5644.8
J	19	1852.2	17	2981.4
K	21	1210.2	19	1852.2
L	23	2222.6	3	2981.4
M	25	2222.6	3	2981.4

In Table III TDS values that obtained from the MATLAB programming is given for different relays. To minimize the working time of the relays irrespective of its value whether it is increasing or decreasing, TDS values are adjusted. From figure 3 it can be concluded that the TDS value is more for relay 4, 1 and 2, when standard inverse type relay is taken. Again when extremely inverse relay are considered the TDS value is increased for relay 4, 2 and 1. Similarly TDS value is more for relay 2 and relay 4 while very inverse relay is taken into consideration. So it can be easily said that the time dial setting value is more for relay 4 and 2 for all 3 types of relays. TDS values are less for relay 5, 10, and 25 etc while taking standard inverse, very inverse or extremely inverse relay to get a minimum working time.

Table 2: Coefficient for Standard Inverse, Extremely Inverse and Very Inverse Relay

Relay no.	Coefficients for relays		
	SIR	VIR	EIR
1	2.314	0.7577	0.2266
2	2.9787	1.5104	0.8183
3	2.9787	1.5104	0.8183
4	3.423	2.1065	1.4845
5	3.423	2.1065	1.4845
6	4.1056	3.1144	2.9134
7	2.314	0.7577	0.2266
8	3.5088	2.2179	1.6253
9	3.5088	2.2179	1.6253
10	4	2.9396	2.6425
11	3.5088	2.2179	2.6453
12	4.0698	3.049	2.811
13	2.314	0.7577	0.2266
14	3.244	1.8571	1.1873
15	3.244	1.8571	1.1873
16	3.9899	2.9291	2.6263
17	2.9787	1.5104	0.8183
18	3.7758	2.6091	2.1552
19	3.7758	2.6091	2.1552
20	4.4191	4.4495	5.2379
21	4.4191	4.4495	5.2379
22	6.0454	6.3039	9.0199
23	3.432	2.1065	1.4845
24	4.4191	3.5939	3.6998
25	3.423	2.1065	1.4845
26	3.9454	2.8619	2.5249

Table 3: Time Dial Setting Values for Standard Inverse, Extremely Inverse and Very Inverse Relay

Relay No.	SIR	EIR	VIR
1	0.4417	1.0826	0.6801
2	0.4319	3.0891	1.0368
3	0.2045	0.4715	0.3323
4	0.5997	3.7002	1.3679
5	0.0584	0.1347	0.0949
6	0.0487	0.0686	0.0642
7	0.1995	0.8826	0.3156
8	0.057	0.1231	0.0902
9	0.057	0.1231	0.0902
10	0.05	0.0757	0.068
11	0.057	0.1231	0.0902
12	0.0491	0.0711	0.0656
13	0.2158	0.8826	0.3769
14	0.0617	0.1684	0.1077
15	0.0617	0.1684	0.1077
16	0.0501	0.0762	0.0683
17	0.2739	0.3656	0.349
18	0.053	0.0928	0.0767
19	0.1414	0.1336	0.1574
20	0.0404	0.0382	0.0449
21	0.0404	0.0382	0.0449
22	0.0331	0.0222	0.0317
23	0.0584	0.1347	0.0949
24	0.0453	0.0541	0.0556
25	0.0584	0.1347	0.0949
26	0.0507	0.0792	0.0699

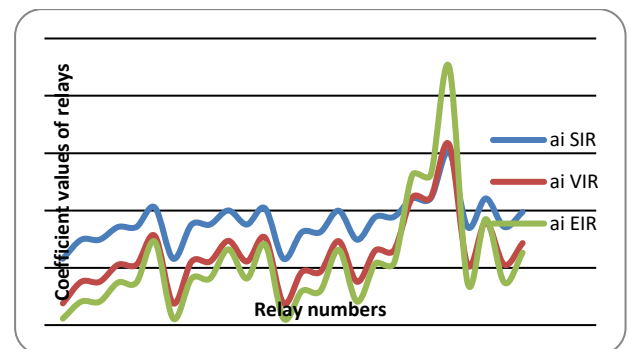


Fig. 2: Coefficients Value of Different IDMT Relays.

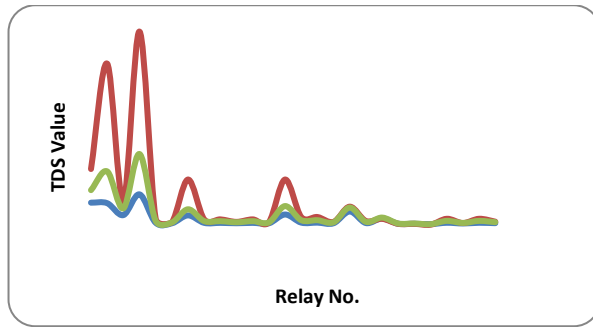


Fig. 3: Time Dial Setting Values of Different Relays.

4.1. Standard inverse relay-(SIR)

These are the relays, whose working times are approximately inversely proportional to fault currents near pickup value and become substantially constant slightly above the pickup value of the relay. The working time depends on the magnitude of working current and it decreases as the current increases. The working time of standard inverse relay is given in Table IV for different fault locations. From this table it can be concluded that while standard inverse relays are taken into consideration the working time of relays are decreasing for location E, F, H, K, L, and M to 0.2s, which is taken as a optimal working time for relays so these are the best location for sensitive devices to place.

Table 4: Working Time of Standard Inverse Relay

Location of Fault	Working Time (Sec)		Location of Fault	Working Time (Sec)	
	Primary Relay	Backup Relay		Primary Relay	Backup Relay
A	R2	R4	H	R15	R13
	1.29	2.05		0.20	0.81
B	R3	R1	I	R17	R13
	0.61	1.02		0.82	1.34
C	R5	R3	J	R19	R17
	0.20	0.72		0.53	1.42
D	R7	R2	K	R21	R19
	0.46	1.29		0.20	1.13
E	R9	R7	L	R23	R3
	0.20	0.76		0.20	0.91
F	R11	R7	M	R25	R3
	0.20	0.76		0.20	0.91
G	R13	R2			
	0.50	1.29			

4.2. Very inverse relay-(VIR)

For Relay with this characteristic, saturation of core occurs at a later stage as compared to standard inverse relay. The time current characteristic is inverted over a greater range and after that saturation tends to definite time. These types of relays are employed in feeders and long sub transmission lines due to its characteristics. The working time for very inverse relay is given in Table V, for different fault locations. From the table it is clearly told that among 13 different locations, working time is decreasing at 8 locations and the value is now 0.2s by application of LPP. Now maximum relays are working optimally so this type of relays can protect the feeders and transmission lines properly by the help of LPP.

4.3. Extremely inverse relay- (EIR)

This is the steepest time characteristic and gives a time-current characteristic that is more inverse than the very inverse relay. Table VI shows the working time of extremely inverse relay. It is clearly visible that as compared to standard inverse and very inverse relay, this relay is the best one as the working time is minimum at maximum locations and the value is equal to 0.2. So this relay is most applicable for costly instruments like transformers and also used to protect machines from overheating.

Table 5: Working Time of Very Inverse Relay

Location of Fault	Working Time (Sec)		Location of Fault	Working time(sec)	
	Primary Relay	Backup Relay		Primary Relay	Backup Relay
A	R2	R4	H	R15	R13
	1.57	2.88		0.2	0.89
B	R3	R1	I	R17	R13
	0.5	1.52		0.53	1.17
C	R5	R3	J	R19	R17
	0.2	0.72		0.41	0.98
D	R7	R2	K	R21	R19
	0.24	1.57		0.2	0.81
E	R9	R7	L	R23	R3
	0.2	0.74		0.2	0.75
F	R11	R7	M	R25	R3
	0.2	0.74		0.2	0.75
G	R13	R2			
	0.29	1.57			

Table 6: Working Time of Extremely Inverse Relay

Location of Fault	Working Time (Sec)		Location of Fault	Working Time (Sec)	
	Primary Relay	Backup Relay		Primary Relay	Backup Relay
A	R2	R4	H	R15	R13
	2.53	5.49		0.20	0.77
B	R3	R1	I	R17	R13
	0.39	0.25		0.30	0.82
C	R5	R3	J	R19	R17
	0.20	0.79		0.29	0.82
D	R7	R2	K	R21	R19
	0.20	2.53		0.20	0.72
E	R9	R7	L	R23	R3
	0.20	0.67		0.20	0.71
F	R11	R7	M	R25	R3
	0.20	0.69		0.20	0.71
G	R13	R2			
	0.20	2.53			

5. Conclusion

The problem of optimal coordination to solve the problems for 3 different types of relays placed in different locations of an IEEE 15 bus radial distribution system as primary and backup relay pairs to maintain relay coordination to protect a distribution system using the linear programming technique. It is observed that the relay working time is decreasing with the optimal setting of TDS value irrespective of the values of TDS, whether it is increasing or decreasing. It is concluded that extremely inverse relay is the best one when compared to the standard inverse relay and very inverse relays as working time are least for these relays at maximum locations of the distribution system. Even extremely inverse relay is the only over current relay which is applicable for fuse graded systems. The results obtained here show that the linear programming technique is an easiest and an effective method for finding an optimal solution.

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