International Journal of Engineering & Technology, 7 (1) (2018) 135-139

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET
doi: 10.14419/ijet.v7i1.8037
Research paper

Some reliability characteristics of a linear consecutive 2-out-of-4 system connected to 2-out-of-4 supporting device for operation

Ibrahim Yusuf[1*], Bashir Yusuf[2], Mansur Babagana[3], Baffa Sani[3] and Muhammad Auwal Lawan[4]

[1]*Department of Mathematical Sciences, Bayero University, Kano, Nigeria*
[2]*Department of Mathematics, Federal University, Dutse, Nigeria*
[3]*Department of Computer Science, Bayero University, Kano, Nigeria*
[4]*Department of Mathematics, Kano State University of Science and Technology, Wudil, Kano, Nigeria*
[*]*Corresponding author E-mail: iyusuf.mth@buk.edu.ng*

Abstract

This paper presents the Markov model for the reliability analysis of a linear consecutive 2-out-of-4 repairable system operating with the help of a linear consecutive 2-out-of-4 external supporting device. The system is analyzed using first order linear differential equation to develop the explicit expression for steady-state availability, busy period and profit function. Based on assumed numerical values given to system parameters, graphical illustrations are given to highlight important results. In addition, the effect of failure and repair on availability and profit are researched.

Keywords: *availability; external supporting device; Linear consecutive; operation; profit.*

1. Introduction

Systems are usually studied with intention to the evaluation of their reliability measures in terms of busy period of repairman, availability and generated revenue. In real-life situations, we often encounter cases where systems that cannot work without the help of external supporting devices connect to such systems. These external supporting devices are systems themselves that are bound to fail. Such systems are found in power plants, manufacturing systems, and industrial systems. Improving the reliability and availability of such systems with their supporting device is vital in ensuring quality of products. Due to their importance in promoting and sustaining industries and economy, reliability and economic analysis of such systems have become an area of interest. Among the reliability measures of interest are the steady-state availability, busy period, profit function and mean time to system failure (MTSF). Improving the availability of such systems leads to an increase in production and associated profit. Large volumes of literature exist on the issue relating to prediction of reliability characteristics of consecutive k-out-of-n systems. Parashar and Taneja [1] deals with reliability and profit evaluation of hot standby PLC system. Singh and Taneja [2] analyzed the reliability of power generating system with random inspection. Yusuf [3] performed the reliability comparison between redundant systems requiring supporting device for operation. Yusuf and Bala [4] analyzed the reliability characteristics of parallel system with external supporting device for operation. Yusuf and Hussaini [5] studied the reliability characteristics of 2-out-of-3 standby system under perfect repair condition. Aliyu et. al.[6] studied the optimization of availability and profit of series-parallel system with linear consecutive cold standby units. Krishnan and Somasundaram [7] analyzed the reliability and profit of Repairable K-Out-of-N System with Sensor. Bhardwaj and chander [8] analyzed the reliability and profit of 2-out-of-3 redundant system with general repair distribution. Bharwaj and Malik [9] studied cost effectiveness of 2-out-of-3 cold standy system with repair and inspection. Chander and Bhardwaj [10] analyzed the reliability of 2-out-of-3 redundant system with repair priority.

Existing literatures ignores the impact of having more redundant supporting device connected to the system or where each unit is connected to its supporting device to improve the reliability of the system. Such literature works laid emphasis on reliability of the system with one supporting device whose failure retards the operation of the system. More sophisticated models of redundant systems connected to multi redundant external supporting device should be developed to assist in reliability of the system, reducing operating costs and the risk of a catastrophic breakdown.

This paper presents availability and profit analysis of a linear consecutive 2-out-of-4 system that worked with the aid of an external linear consecutive 2-out-of-4 supporting device and derived its corresponding mathematical models. The objectives of our analysis are twofold. First is to capture the effect of failure and repair rates on steady-state availability and profit. Second is to capture the impact of steady-state availability on profit. The organization of the paper is as follows. Section 2 contains the description of the system under study. Section 3 present Material and method while section 4 present formulations of the models. The results of our numerical simulations are presented in section 5. Finally, we make a concluding remark in Section 6.

2. Description and States of the System

In this paper, a linear consecutive 2-out-of-4 redundant system is considered. The system is connected to external 2-out-of-4 supporting devices for its operation. Each unit has its own supporting device independent to the other units. When a unit failed, its supporting will stagnate (an idle supporting device) and vice versa. There two repairmen that attend to the system, one attached to the system and the other to the supporting device. When the first unit failed with rate β_1, immediately is sent for repair with rate α_1 and the next standby unit and its supporting device are switch into operation. Similarly, at the failure of the supporting device, with rate β_2, immediately is sent for repair with rate α_2 and the next supporting device in standby is switch into operation. It is assumed that switching from standby to operation is perfect. System failure occurs when three units/three supporting devices have failed. Priority in repair is considered here.

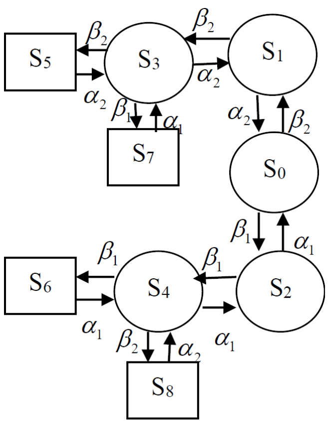

Figure 1: Schematic diagram of the System

State 0: Initial state, units A_1 and A_2, supporting devices P_1 and P_2 are operational; units A_3 and A_4, supporting devices P_3 and P_4 are on standby. The system is working.

State 1: Unit A_1 is idle; supporting device P_1 has failed and is under repair, units A_2 and A_3, supporting devices P_2 and P_3 are operational, unit A_4 and supporting device P_4 are on standby. The system is working.

State 2: Unit A_1 has failed and is under repair, supporting device P_1 is idle, units A_2 and A_3, supporting devices P_2 and P_3 are operational, unit A_4 and supporting device P_4 are on standby. The system is working.

State 3: Units A_1 and A_2 are idle, supporting device P_1 has failed is waiting for repair, supporting device P_2 is under repair, units A_3 and A_4, supporting devices P_3 and P_4 are operational. The system is working.

State 4: Unit A_1 is waiting for repair, unit A_2 is under stage I repair, supporting devices P_1 and P_2 are idle, units A_3 and A_4, supporting devices P_3 and P_4 are operational. The system is working.

State 5: Units A_1, A_2, A_3 and A_4 are idle, supporting devices P_1, P_2 are waiting for repair, supporting device P_3 is under repair, supporting device P_4 is idle. The system is down.

State 6: Units A_1 and A_2 are waiting for repair, unit A_3 is under repair, unit A_4 is idle, supporting devices P_1, P_2, P_3 and P_4 are idle. The system is down.

State 7: Units A_1, A_2 and A_4 are idle, units A_3 is under repair supporting device P_1 is waiting for repair, supporting device P_2 is under repair, supporting devices P_3 and P_4 are idle. The system is down.

State 8: Unit A_1 is waiting for repair, unit A_2 is under stage I repair, supporting devices P_1 and P_2 are idle, units A_3 and A_4, supporting device P_4 are idle, supporting device P_3 is under repair. The system is down.

3. Material and Method

In this study we analyzed the probabilistic analysis of the system by using of the linear first order differential equations and have obtained explicit expressions for the steady state availability, busy period of repairman due to failure of the supporting device, busy period of repairman due to failure of the unit. Profit was calculated as the difference between generated and costs incurred due to repair of both supporting device and unit. Graphical study of the system behaviour has been made using MATLAB computer package.

Notations

S_i:	state of the system
$P_i(t)$:	probability that the system in state S_i at time t
$A_O/A_R/A_W/A_S/A_I/A_F$:	unit in operation/under repair/waiting for repair/in standby/is idle/failed.
$P_O/P_R/P_W/P_S/P_I/P_F$:	supporting device in operation/under repair/waiting for repair/in standby/is idle /failed.
β_1:	failure rate of the unit.
β_2:	failure rate of the supporting device.
α_1:	repair rate of the unit.
α_2:	repair rate of the supporting.
C_0:	revenue generated when the system is in working state and has no income when in failed state.
C_1:	cost of each repair for supporting device.
C_2:	cost of each repair for failed unit.

4. Model formulation

Let $P(t) = [P_0(t), P_1(t), P_2(t), \cdots, P_8(t)]$ be the probability vector for system at time $t \geq 0$. Relating the state of the system at time t and $t + dt$ the steady state differential equations for the system are as follows:

$$
\begin{aligned}
\frac{dP_0(t)}{dt} &= -q_0 P_0(t) + \alpha_2 P_1(t) + \alpha_1 P_2(t) \\
\frac{dP_1(t)}{dt} &= -q_1 P_1(t) + \beta_2 P_0(t) + \alpha_2 P_3(t) \\
\frac{dP_2(t)}{dt} &= -q_2 P_2(t) + \beta_1 P_0(t) + \alpha_1 P_4(t) \\
\frac{dP_3(t)}{dt} &= -q_3 P_3(t) + \beta_2 P_1(t) + \alpha_2 P_5(t) + \alpha_1 P_7(\infty) \\
\frac{dP_4(t)}{dt} &= -q_4 P_4(t) + \beta_1 P_2(t) + \alpha_1 P_6(t) + \alpha_2 P_8(\infty) \\
\frac{dP_5(t)}{dt} &= -\alpha_2 P_5(t) + \beta_2 P_3(t) \\
\frac{dP_6(t)}{dt} &= -\alpha_1 P_6(t) + \beta_1 P_4(t) \\
\frac{dP_7(t)}{dt} &= -\alpha_1 P_7(t) + \beta_1 P_3(t) \\
\frac{dP_8(t)}{dt} &= -\alpha_2 P_8(t) + \beta_2 P_4(t)
\end{aligned}
\tag{1}
$$

This can be written in the matrix form as

$$
P' = MP(t), \quad \text{where}
$$

$$
q_0 = (\beta_1 + \beta_2), q_1 = (\alpha_2 + \beta_2), q_2 = (\alpha_1 + \beta_1),
$$

$$
q_3 = (\alpha_2 + \beta_1 + \beta_2), q_4 = (\alpha_1 + \beta_1 + \beta_2)
$$

$$
M = \begin{pmatrix}
q_0 & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_2 & -q_1 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\
\beta_1 & 0 & -q_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\
0 & \beta_2 & 0 & -q_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 \\
0 & 0 & \beta_1 & 0 & -q_4 & 0 & \alpha_1 & 0 & \alpha_2 \\
0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_1 & 0 & -\alpha_1 & 0 & 0 \\
0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 \\
0 & 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2
\end{pmatrix}
\tag{2}
$$

For the analysis of availability case of system, we use the following procedure to obtain the steady-state availability, busy period and profit function.

$$\begin{pmatrix} P_0' \\ P_1' \\ P_2' \\ P_3' \\ P_4' \\ P_5' \\ P_6' \\ P_7' \\ P'_8 \end{pmatrix} = \begin{pmatrix} -q_0 & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & -q_1 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -q_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & -q_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 \\ 0 & 0 & \beta_1 & 0 & -q_4 & 0 & \alpha_1 & 0 & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_1 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{pmatrix}$$

Let T be the time to failure of the system for system. The explicit expressions for the steady-state availability, state busy period of repairmen due to failure of units and supporting device are as follows:

$$A_T(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) \tag{3}$$

$$B_{T1}(\infty) = P_1(\infty) + P_3(\infty) + P_5(\infty) + P_8(\infty) \tag{4}$$

$$B_{T2}(\infty) = P_2(\infty) + P_4(\infty) + P_6(\infty) + P_7(\infty) \tag{5}$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (2) becomes

$$MP = 0 \tag{6}$$

which is in matrix form

$$\begin{pmatrix} -q_0 & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & -q_1 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -q_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & -q_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 \\ 0 & 0 & \beta_1 & 0 & -q_4 & 0 & \alpha_1 & 0 & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_1 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

Using the following normalizing condition

$$\sum_{k=0}^{8} P_k(\infty) = 1 \tag{8}$$

we substitute (8) in last row in (7) to yield

$$labeleqn9 \begin{pmatrix} -q_0 & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & -q_1 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -q_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & -q_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 \\ 0 & 0 & \beta_1 & 0 & -q_4 & 0 & \alpha_1 & 0 & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_1 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{9}$$

The explicit expressions for (3), (4) and (5) are as follows:

$$A_T(\infty) = \frac{\alpha_1^3\alpha_2^3 + \alpha_1^3\alpha_2^2\beta_2 + \alpha_1^2\alpha_2^3\beta_1 + \alpha_1^3\alpha_2\beta_2^2 + \alpha_1\alpha_2^3\beta_1^2}{\Delta_1 + \Delta_2}$$

$$B_{T1}(\infty) = \frac{\alpha_1^3\alpha_2^2\beta_2 + \alpha_1^3\alpha_2\beta_2^2 + \alpha_1^3\beta_2^3 + \alpha_1\alpha_2^2\beta_1^2\beta_2}{\Delta_1 + \Delta_2}$$

$$B_{T2}(\infty) = \frac{\alpha_1^3\alpha_2^2\beta_1 + \alpha_1\alpha_2^3\beta_1^2 + \alpha_2^3\beta_1^3 + \alpha_1^2\alpha_2\beta_1\beta_2^2}{\Delta_1 + \Delta_2}$$

where

$$\Delta_1 = \alpha_1^3\alpha_2^3 + \alpha_1^3\alpha_2^2\beta_2 + \alpha_1^3\alpha_2\beta_2^2 + \alpha_1^3\beta_2^3 + \alpha_1^2\alpha_2^3\beta_1$$
$$\Delta_2 = \alpha_1^2\alpha_2\beta_1\beta_2^2 + \alpha_1\alpha_2^3\beta_1^2 + \alpha_1\alpha_2^2\beta_1^2\beta_2 + \alpha_2^3\beta_1^3$$

From Figure 1, the units and supporting unit are subjected to corrective maintenance in states 1, 2, 3, 4, 5, 6 and 7 respectively. Let Profit = total revenue generated - cost incurred by the repairman due to when repairing the failed supporting device in the interval $(0,t)$ - cost incurred when repairing the failed units in the interval $(0,t)$. Thus,

$$P_F(\infty) = C_0 A_T(\infty) - C_1 B_{T1}(\infty) - C_2 B_{T2}(\infty) \tag{10}$$

5. Numerical examples

In this section, we numerically obtained the results for system availability and profit function for all the developed models. The following set of parameter values were fixed throughout the simulations for consistency:

$\alpha_1 = 0.6, \alpha_2 = 0.5, \beta_1 = 0.3, \beta_2 = 0.1, C_0 = 10000, C_1 = 1500, C_2 = 2000$

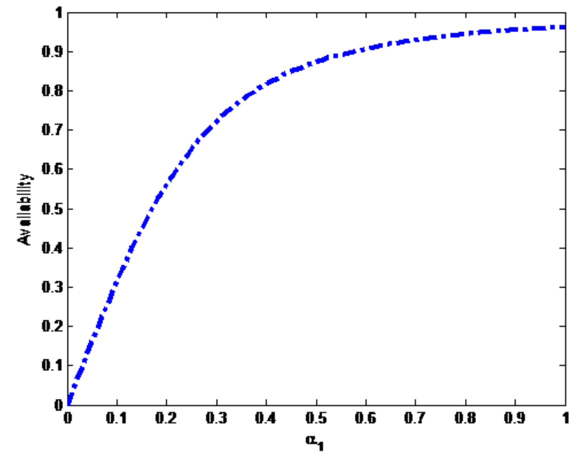

Figure 2: Availability against α_1

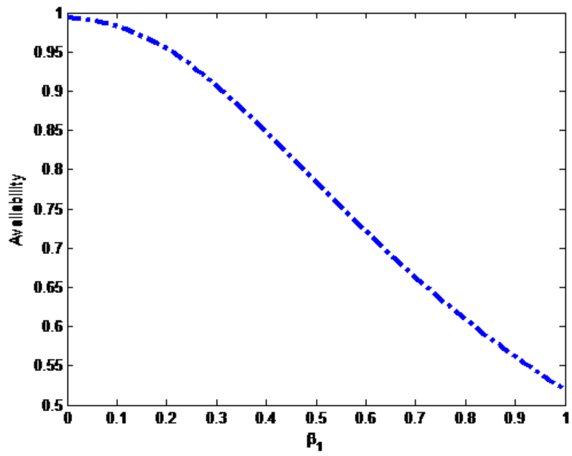

Figure 3: Availability against β_1

Effect of β_1 on steady-state availability and profit can be observed in Figures 3 and 3. From these figures it is evident that the steady-state availability and profit decrease with increase in β_1. Results of steady-state availability and profit with respect to α_1 are given in Figures 2 and 4. It is evident from these Figures that as α_1 increases, the steady-state availability, and profit also increase. The impact of availability on profit, is depicted in Figure 6. In this figure, profit increases with increase in the steady-state availability.

Failure of supporting device has an implication on reliability measures such as system availability and profit. Furthermore, it is a common knowledge that such failure can reduce system performance and production output.

Figures 7 and 9 displays the trends of system availability and profit against the failure rate β_1 for different values of supporting device rate. It is evident from the figures that system availability and profit display decreasing pattern with β_1 for different values of supporting device failure rate. The gaps between the curves in the figures become smaller as β_2 increase. Thus, the system availability and profit are more sensitive to β_2. This sensitivity analysis implies

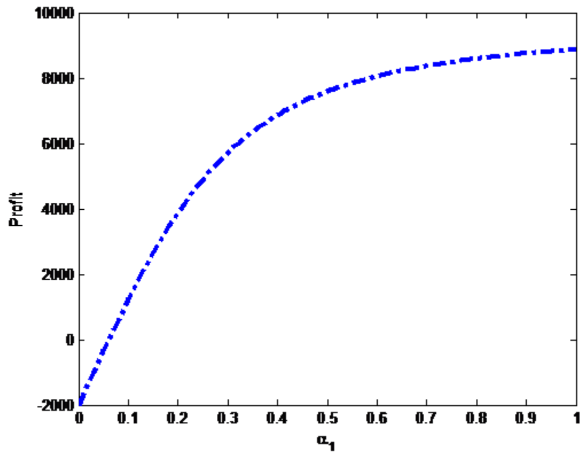

Figure 4: Profit against α_1

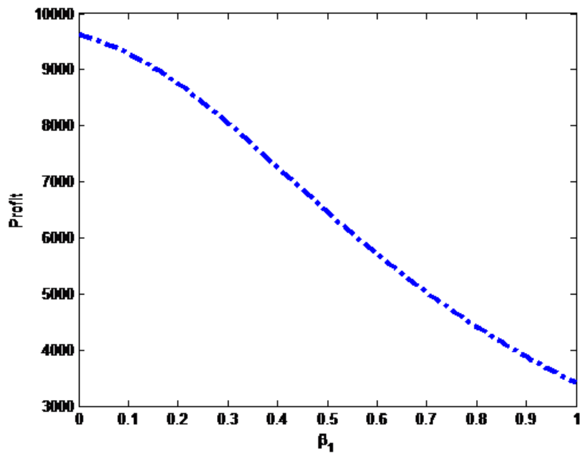

Figure 5: Profit against β_1

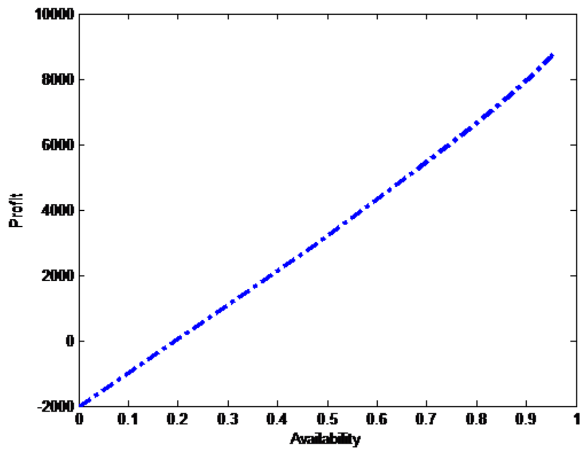

Figure 6: Profit against Availability

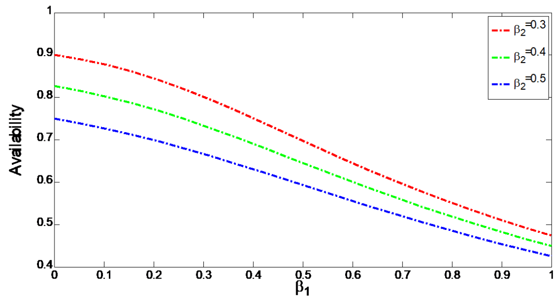

Figure 7: Availability against β_1 different values of $\beta_2(0.3,0.4,0.5)$

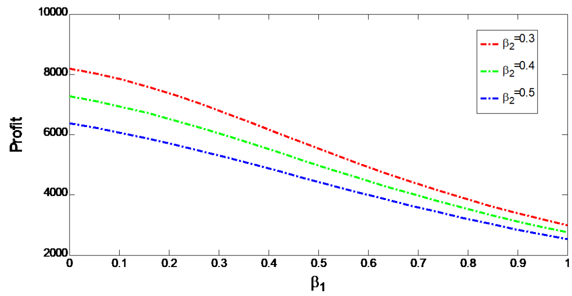

Figure 8: Availability against α_1 different values of $\beta_2(0.3,0.4,0.5)$

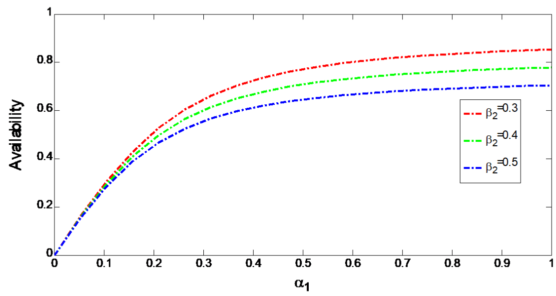

Figure 9: Profit against β_1 different values of $\beta_2(0.3,0.4,0.5)$

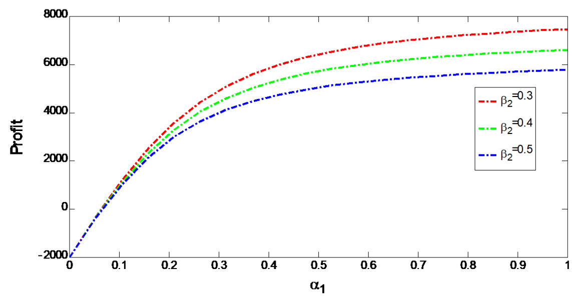

Figure 10: Profit against α_1 different values of $\beta_2(0.3,0.4,0.5)$

that major maintenance should be invoked to minimize the failure of supporting device in order to improve and maximize the system availability and profit.

On the other hand, system availability and profit will also be affected by other parameter such as failure rate. Thus, the higher the unit and supporting device failure rates are, the less the system availability, production output as well the as net profit.

Figures 8 and 10 shows the behavior system availability and profit against the repair rate α_1 for different values of supporting device failure rate. It is clear from these figures that system availability and profit display increasing pattern with α_1. The gaps between the curves in the figures widen as α_1 increases. This sensitivity analysis implies that major maintenance to the units/supporting device/entire system should be invoked to improve and maximize the system availability, production output as well as the profit.

6. Conclusion

In this paper, we analyzed linear consecutive 2-out-of-4 systems with 2-out-of-4 supporting device attached to the systems. Explicit expressions for steady-state availability, busy period and profit function for the system were derived. Graphical analysis was also performed numerically. Maintaining high or required level of availability and profit is often an essential requisite. From the analysis, we conclude that availability and profit of a linear consecutive 2-out-of-4 will be higher if each unit has its own supporting device rather than sharing the supporting device. This will help to minimize system failure and lead to increase in production output as well as the revenue generated. Maintenance managers, reliability engineers and system designers are faced with the challenges of competition and market globalization on maintenance system to improve efficiency and reduce operational costs, this study will serve as a guide in relation to efficiency, reduction of failure and operational costs, increase in production output and revenue mobilized.

References

[1] B. Parashar and G. Taneja. "Reliability and profit evaluation of a PLC hot standby system based on a master slave concept and two types of repair facilities". *IEEE Trans. Reliab*, 56: , (2007), pp.534-539. DOI: 10.1109/TR.2007.903151.

[2] D. Singh and G. Taneja. "Reliability and economic analysis of a power generating system comprising one gas and one steam turbine with random inspection",*J. Math. Stat*, 10 (4), (2014).

[3] I. Yusuf. "Comparison of some reliability characteristics between redundant systems requiring supporting units for their operation". *J Math Comput Sci 3(1)*: (2013), pp.216–232.

[4] 1. Yusuf, and S.I. Bala. "Analysis of reliability characteristics of a parallel system with external supporting devices for operation", *International Journal of Information and Management Sciences*, 24, (2013), pp.297-306.

[5] I. Yusuf and N. Hussaini. (2012). "Evaluation of reliability and availability characteristics of 2-out-of-3 standby system under a perfect repair condition". *American Journal of Mathematics and Statistics*,2(5): pp.114-119

[6] M.S. Aliyu, I. Yusuf and U. A. Ali. "Availability and profit optimization of series-parallel system with linear consecutive cold standby units", *Applied Mathematics*, 6, (2015), pp.332-344.

[7] R. Krishnan and S. Somasundaram. "Reliability and Profit Analysis of Repairable K-Out-of-N System with Sensor". *European Journal of Scientific Research*, 67, (2012), pp.215-222.

[8] R.K., Bhardwaj and S. Chander. "Reliability and cost benefit analysis of 2-out-of-3 redundant system with general distribution of repair and waiting time", *DIAS- Technology review- An Int. J. of business and IT*, (1007) 4(1): pp.28-35.

[9] R.K., Bhardwaj and S.C. Malik. "MTSF and Cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection", *Int. J. of Eng. Sci. and Tech.*, (2010) 2(1): pp.5882-5889

[10] S. Chander and R.K. Bhardwaj. "Reliability and economic analysis of 2-out-of-3 redundant system with priority to repair", *African J. of Maths and comp. sci*, (2009) 2(11): 230-236.