



## Theoretical considerations of machining with grinding wheels

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### Abstract

Grinding is one of the most versatile methods of removing material from machine parts by the cutting action of the countless hard and sharp abrasive particles of a revolving grinding wheel. It works by forcing the abrasive grains into the surface of the workpiece so that each grain cuts away a small bit of material in the form of chips. Abrasive grinding wheel is an expendable wheel that carries an abrasive compound on its periphery. They are made of small, sharp and very hard natural or synthetic abrasive minerals, bonded together in a matrix to form a wheel. The paper presents a review of some of the characteristics as well as theoretical considerations of operations of abrasive grinding wheel. The relationships among the various grinding parameters; the radial force  $f$ , the force on individual grit of grinding wheel  $F$ , velocity of grinding wheel  $V_g$ , velocity of work piece  $V_w$ , the wheel diameter  $D_g$ , and the diameter of the work piece  $D_w$  were established for given grinding operations.

**Keywords:** Abrasive Grains, Expendable, Grinding Parameters, Grinding Wheel, Machining Process,

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## 1 Introduction

Abrasive machining is a grinding process for removing material from a workpiece by using expendable abrasive mineral materials in a wheel, stone, belt, paste, sheet, compound, slurry, or other abrasive products. Abrasives are mineral materials from a selected group of very hard materials used to shape, finish, or polish other materials, while a wheel is a circular device that is capable of rotating on its axis thereby facilitating movement. Thus, a grinding wheel is an expendable wheel which carries abrasive particles on its periphery used for grinding a workpiece. They are composed of thousands of small sharp and very hard natural or synthetic abrasive grains bonded together in a matrix to form a wheel. Each abrasive grain is a cutting edge and as the grain passes over the workpiece, it cuts a small chip, leaving a smooth, geometrical/dimensional accurate surface. As the abrasive grain becomes dull, it breaks away from the bonding material exposing new sharp grains [1]. The grinding of crankshaft and other precision components is only made possible by using the grinding wheels and without grinding wheel the achievement of high precision surface finish to very high tolerance would not have been possible. The shaping and dressing of hard cutting tools such as high speed steel, carbide, diamond cutting and cubic boron nitride (CBN) are only possible with application of grinding using grinding wheels [1].

Abrasive machining as a grinding process predates civilization when sandstone was the only bonded abrasive used for most of the grinding work. The art of grinding dates back many centuries, since man first discovered that he could brighten up and sharpen his tools by rubbing them against certain stones or by plunging them into sand several times. The emery stone appeared when man found that the softer sand stone did not work well on the newly discovered harder materials [2]. In recent years, ceramics, garnets, refractories and composite materials have increased in industrial application and by early nineteenth century, emery (a natural mineral containing iron and corundum) was used to cut and shape metals [3]. Acheson discovered silicon carbide in 1891, while he was attempting to manufacture precious gems in an electric furnace, and a few years later, Jacob developed aluminium oxide from claylike mineral bauxite [2]. Pulson made the first grinding wheel by combining emery with potter's clay and firing it in a kiln. He noted that emery was a natural abrasive of non-uniform texture, so its quality as a grinding wheel varied greatly [2]. However, emery's variable quality and problems with importing it from India prior to its discovery in the United States prompted efforts to

find a more reliable abrasive mineral. By the 1890s, the search had yielded silicon carbide, a synthetic abrasive mineral harder than corundum [4]. There are five characteristics of a grinding wheel and these include: wheel material, grain size, wheel grade, grain spacing, and bond type. These characteristics are usually indicated by codes on the wheel's label.

### 1.1 Principles of grinding

The principles of grinding involve the use of abrasive tool whose cutting elements are grains of abrasive materials. These grains are of high hardness (2200 to 3100 kN/mm<sup>2</sup>) and have a high heat resistance. Grinding is mainly a machining process employed for the attainment of tight tolerance (two dimension tolerance and compatible geometric tolerance), reduced roughness values (Ra from 0.2 to 1.6 μm) and due to the great number of variables involved, grinding is one of the most complex machining processes [5]. Grinding with modern abrasive is a metal cutting process like other conventional metal cutting tools except that the grinding wheel consists of thousands of tiny cutting edges instead of the few large edges possessed by other rotary cutters, such as the milling cutter [6]. The temperature reached by the tip of the abrasive particles when cutting is extremely high and higher than the melting point of steel which is 1,500°C. However, no melting of grains occurs due to brief time of contact, which is often less than 100 x 10<sup>-6</sup>sec.[7]. The different depths of cut on work piece deformation had been discovered to affect the hardness of the abrasive wheel. However, the most generally recognized characteristic wheel hardness is the ability of the wheel to retain dulled abrasive grains. The duller the retained grains, the harder the wheel [8].

## 2 Theoretical considerations

Grinding process is one of the most complex tooling processes, due to the great number of variables involved [1]. In grinding, material is generally removed by shearing and ploughing action in the form of micro sized chips by the abrasive grits of the grinding wheel. Therefore, grinding operation can be regarded as a micro-finishing process and the purpose of theory of grinding is to establish the relationships between the radial force *f*, the force on individual grit of grinding wheel *F*, velocity of grinding wheel *V<sub>g</sub>*, velocity of work piece *V<sub>w</sub>*, the wheel diameter *D<sub>g</sub>*, and the diameter of the work piece *D<sub>w</sub>* [9].

### 2.1 Analysis of number of abrasive particles on a grinding wheel

The orthogonal view of a grinding wheel is presented in Figure 1.

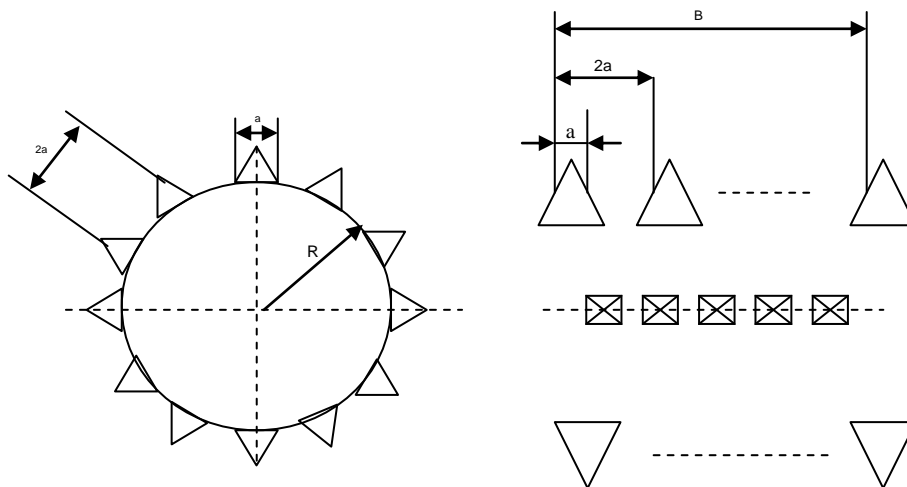


Fig. 1: Orthographic views of a modeled grinding wheel where  
 R = Grinding wheel radius (mm)  
 B = Grinding wheel face width (mm)  
 A<sub>b</sub> = Side of basal square of the pyramidal grain (mm)

The total number of grits on the wheel is given by the product of 'N<sub>w</sub>' and 'N<sub>c</sub>' and the factors 'N<sub>w</sub>' and 'N<sub>c</sub>' are obtained as follows [10].

(1) For grain distribution along the face width of the produced grinding wheel,

$$N_w = \frac{B}{A_b} + 1 \quad (1)$$

(2) For grain distribution along the circumference of the wheel,

$$N_c = \frac{2\pi R}{A_b} \quad (2)$$

Then, the number of grains per unit area of the wheel (the grain density) is given by

$$C_g = \frac{N_w N_c}{2\pi R B} = \frac{B + A_b}{A_b^2 B} \quad (3)$$

Hence

$$A_b = \frac{1 + \sqrt{1 + 4B^2 C_g}}{2BC_g} \quad (4)$$

Where A<sub>b</sub> = side of basal square of the pyramidal grain (mm)

N<sub>w</sub> = number of grains along the face width of the grinding wheel.

N<sub>c</sub> = number of grains along the circumference of the grinding wheel

C<sub>g</sub> = grain density (mm<sup>-2</sup>)

B = grinding wheel face width (mm)

R = grinding wheel radius (mm)

The values of "C<sub>g</sub>" between 10 and 20 are commonly used for fine abrasive wheels, while

values of "C<sub>g</sub>" between 5 and 10 are commonly used for coarse wheels, with lower numbers giving more conservative estimates [11].

## 2.2. Surface grinding operation

The geometry produced in surface grinding is influenced by many variables which include: (i) wheel characteristic, (ii) workpiece characteristic, (iii) machine characteristic and (iv) Operating conditions [12]. Surface grinding on a horizontal spindle machine can be presented as in Figure 2 where the depth of cut has been greatly exaggerated, being only 0.0001 x wheel diameter. The path ABH traced by the tip of a cutting grain is a trochoid generated by a combination of the circular movement of the wheel and the horizontal movement of the workpiece as in Figure 2a. The undeformed chip length L (Figure 2b) and the maximum chip thickness t (Figure 2a), can be obtained as follows;

From Figure 2 (a and b), we have  $L = ABH \approx BH$ .

As  $\theta$  is very small when grinding, curve ABH in Figure 2a  $\approx$  line BH in Figure 2b.

$$\therefore L = BH = \sqrt{(HG^2 + d^2)} \quad (5)$$

But,

$$HG = \sqrt{\left\{ \left( \frac{D}{2} \right)^2 - \left( \frac{D}{2} - d \right)^2 \right\}} \quad (6)$$

$$\therefore L = \sqrt{\left[ \left\{ \left( \frac{D}{2} \right)^2 - \left( \frac{D}{2} - d \right)^2 \right\} + d^2 \right]} \quad (7)$$

$$\therefore L = \sqrt{Dd} \quad (8)$$

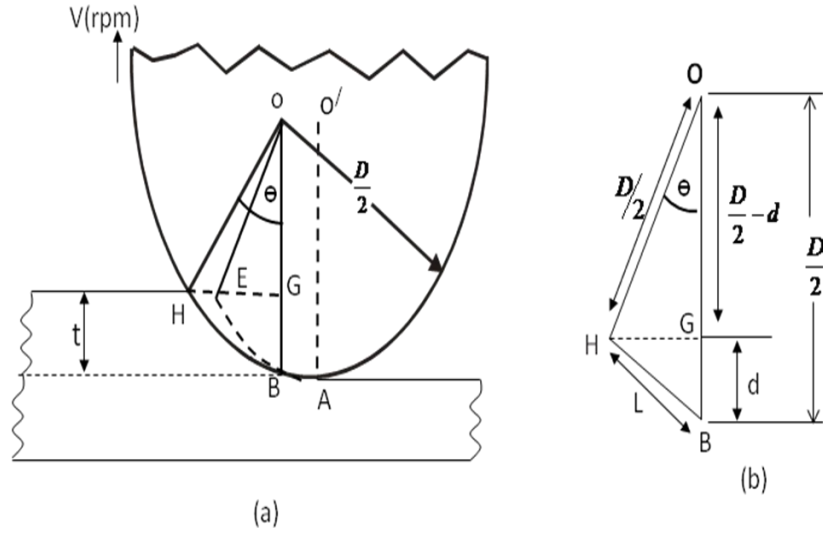


Fig. 2: Surface grinding

### 2.3 Maximum chip thickness in surface grinding (t)

The maximum chip thickness t, can be obtained from the flattened chip in Figure 3,

$$t = EF = HE \sin \theta \tag{9}$$

and

$$\sin \theta = \frac{HG}{D/2} = \frac{2HG}{D}, \text{ from fig 2.(b)} \tag{10}$$

$$\text{but } HG = \sqrt{\left\{ \left( \frac{D}{2} \right)^2 - \left( \frac{D}{2} - d \right)^2 \right\}} \tag{11}$$

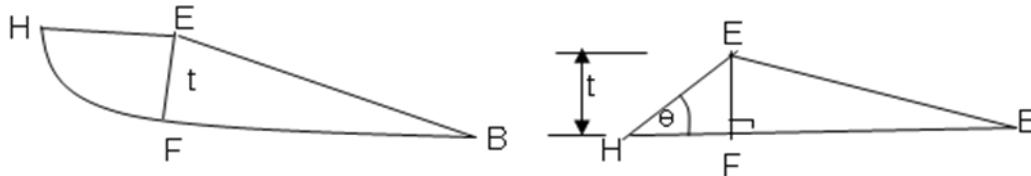


Fig. 3: Chip thickness

$$\text{ie } HG = \sqrt{Dd - d^2} \tag{12}$$

and so

$$\sin \theta = \frac{2(\sqrt{Dd - d^2})}{D}, \tag{13}$$

Hence

$$t = HE \sin \theta = 2HE \frac{\sqrt{Dd - d^2}}{D} = 2HE \sqrt{\left[ \frac{d}{D} - \left( \frac{d}{D} \right)^2 \right]} \tag{14}$$

and since d/D is small, (d/D)<sup>2</sup> can be neglected, ie (d/D)<sup>2</sup> → 0.

$$\therefore t = 2HE \sqrt{(d/D)}. \tag{15}$$

HE is the work feed per grain and if the grains are equally spaced and there are K in line/rev.

$$HE = \frac{f}{KN} \quad (16)$$

where,  $f$  = rate of work feed, mm/min. and  $N$  = rev./min. of wheel.

$$\therefore t = \frac{2f}{KN} \sqrt{d/D} \quad (17)$$

In surface grinding, the chip thickness increases from zero to a maximum  $t$ , unlike the plunge grinding. It is therefore more convenient to consider the ratio of width to depth of cut  $b'/t$  at  $t/2$  instead of at  $t$  and this ratio is represented by  $r$ .

$$\therefore r = \frac{b'}{t/2}, \text{ and so } b' = \frac{tr}{2}. \quad (18)$$

The number of grains per revolution “ $K$ ” is given as;  $K = \pi DCb'$  [7].

but  $b' = \frac{tr}{2}$ , from Equation (18),

$$\therefore K = \pi DC \frac{tr}{2} \quad (19)$$

$$\therefore t = \frac{2f}{\pi DCrN} \sqrt{d/D} \quad (20)$$

$$\text{and } t^2 = \frac{2f}{\pi DCrN} \sqrt{d/D}$$

$$\therefore t = \sqrt{\frac{4f}{\pi DNCr} \left(\frac{d}{D}\right)^{1/2}} \quad (21)$$

## 2.4 External cylindrical grinding

For the theoretical considerations, the operation involving external cylindrical grinding is presented in Figure 4.

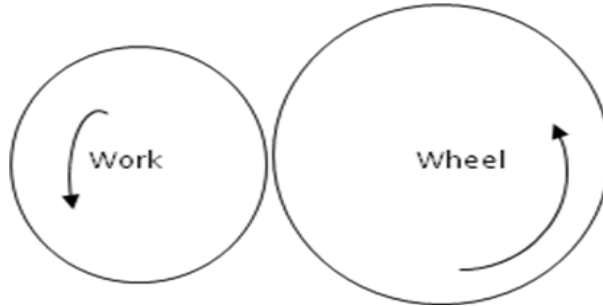


Fig 4: External cylindrical grinding

Figure 5 shows the geometry of chip formation in external cylindrical grinding and it is seen that as an abrasive grain starts to penetrate the material at point A with depth of cut zero and it increases gradually as the wheel and work revolve, and becomes maximum somewhere along the arc of contact of the wheel and work piece. Since the wheel usually rotates much faster than the work, the point of maximum depth of cut is almost at the point where the wheel leaves the work. The maximum depth is known as the grain depth of cut  $t$  (mm).

Let

$D_g$  = diameter of grinding wheel (mm)

$D_w$  = diameter of workpiece (mm)

$V_g$  = surface velocity of grinding wheel. (m/s)

$V_w$  = surface velocity of workpiece. (m/s)

$\phi$  = angle subtended at workpiece centre,

$\alpha$  = angle subtended at wheel centre,

$f_g$  = feed of the workpiece per grit.

$d$  = depth of cut (mm).



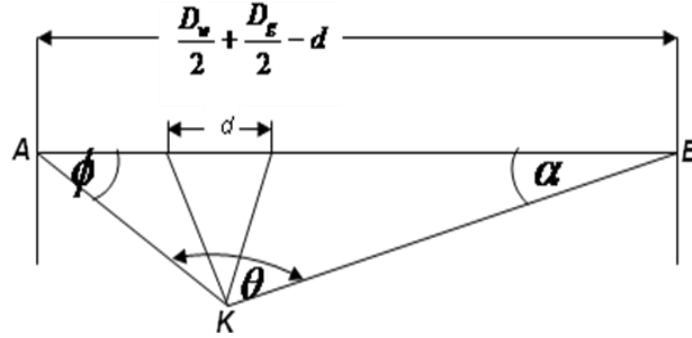


Fig: 6: Section AKB of chip formation geometry

distance moved by the workpiece during the period between the passage of successive grits which is feed per grit and is represented by  $f_g$ . It is seen that the maximum chip thickness  $t_{max}$  is given by,

$$t_{max} = f_g \sin(\alpha + \phi) \quad (27)$$

but the feed per grit

$$f_g = \frac{V_w}{N_g V_g}, \quad (28)$$

$$\therefore t_{max} = \frac{V_w}{N_g V_g} \sin(\alpha + \phi) \quad (29)$$

This implies that the grain depth of cut  $t$ , varies directly as the workpiece speed, inversely as the wheel speed and directly as  $\sin(\alpha + \phi)$ .

$$\text{But } \sin(\alpha + \phi) = 2 \left[ \sqrt{d \left( \frac{D_w + D_g}{D_w D_g} \right)} \right] \quad \text{from Equation 26}$$

$$\therefore t_{max} = \frac{2V_w}{N_g V_g} \left[ \sqrt{\left( \frac{D_w + D_g}{D_w D_g} \right) d} \right]. \quad (30)$$

The force on individual grit is given by,

$$F_g = PA. \quad (31)$$

where  $F_g$  = force on individual grit,

$P$  = pressure on the grit, and

$A$  = area of a grit and  $A = t \times t = t^2$ ,

The pressure  $P$  is constant for the same cutting condition, so the force on individual grit is proportional to the area of the grit.

$$\therefore F_g \propto t^2,$$

But

$$t^2 = \left( \frac{4dV_w^2}{N_g^2 V_g^2} \right) \left( \frac{D_w + D_g}{D_w D_g} \right) = \left( \frac{4dV_w^2}{N_g^2 V_g^2} \right) \left( \frac{1}{D_w} + \frac{1}{D_g} \right) \quad (32)$$

$$\therefore F_g \propto \left( \frac{4dV_w^2}{N_g^2 V_g^2} \right) \left( \frac{1}{D_w} + \frac{1}{D_g} \right) \quad (33)$$

Note that the pitch of the grits ( $N_g$ ) is a constant for the grade of wheel selected,

so  $\frac{4}{N_g^2}$  is a constant.

Thus,

$$F_g \propto \left( \frac{V_w^2}{V_g^2} \right) \left( \frac{1}{D_w} + \frac{1}{D_g} \right) d \quad (34)$$

and

$$F_g = K \left( \frac{V_w^2}{V_g^2} \right) \left( \frac{1}{D_w} + \frac{1}{D_g} \right) d \cdot \quad (35)$$

Thus increasing the workpiece speed ( $V_w$ ) is the most effective way of increasing self – sharpening action if the dull grits are not broken from the bond.

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Important conclusions about the behaviour of a grinding wheel can also be drawn from the above equation as follows:

- (i) For soft wheels, the wheel velocity  $V_g$  should be high.
- (ii) For hard wheels, workpiece speed  $V_w$ , should be high.
- (iii) If  $D$  and  $d$  are nearly equal as in the case of internal grinding, then  $\frac{D_w + D_g}{D_w D_g}$  will be small so that soft wheels are required.
- (iv) In external grinding where  $\frac{D_w + D_g}{D_w D_g}$  is very large, the individual grit force  $F$ , will be more and therefore hard wheels are required to counteract high force per grit.

### 3 Conclusion

Some of the characteristics of a grinding wheel have been discussed. These characteristics include; the wheel material, the grain size, the wheel grade, the grain spacing, and bond type. The most common wheel materials include aluminium oxide, silicon carbide, diamond and cubic boron nitride. The grain size determines the physical size of the abrasive grains in the wheel. A larger grain will cut freely, allowing fast cutting but poor surface finish while very fine grain sizes are for precision finish work. The wheel grade determines how tightly the bond holds the abrasive grains and the grade affects almost all considerations of grinding operation. The wheel bond affects the finish, coolant, and the wheel speed. The wheel grain spacing refers to the density which is the ratio of bond and abrasive to air space. A less-dense wheel will cut freely, and has a large effect on surface finish than a more-dense wheel.

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