



# A New Type of <sup>a</sup>Contra Continuous Functions

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## Abstract

In this article, the author present the another class of functions which is called a contra generalized  $ab$  - continuous function in topological spaces. A few portrayals and properties identified with contra  $gab$  - continuous functions are to be obtained.

**Keywords:**  $gab$  - SetO,  $gab$  -continuity, contra  $gab$  -continuity.

## 1. Introduction

Dontchev presented the thoughts of <sup>a</sup>contra continuous function on 1970. Another class of capacity called contra b-continuous presented by Nasef. The researchers M. S. M. Noorani and A. A. Omari have examined advanced properties of b-<sup>a</sup>function on 2009. This article, the presentation of the new idea of contra  $gab$  - continuous and concentrate a portion of the utilizations of this function. We additionally present and concentrate two new spaces called  $gab$  -<sup>H</sup>Hausdorff spaces,  $gab$  - normal spaces and get some new outcomes. Through this article  $(X, \tau)$  and  $(Y, \sigma)$  stands for the non-empty topological spaces. Let  $X \supseteq A$  will be indicated by  $\text{int}(A)$  and  $\text{closer}(A)$  separately, the set  $X$   $gab$  - is union of all Set<sup>O</sup> in set<sup>C</sup> with  $A$  is called  $gab$  - interior of  $A$  then which is signified  $gab$  -  $\text{int}(A)$ , the point of intersection all  $gab$  - sets of  $X$  is in  $A$  called  $gab$  - closure of  $A$  and it is meant by  $gab$  -  $\text{closure}(A)$ .

Assumption: <sup>a</sup>function – Contra Continuous Function, Set<sup>O</sup>–Open Set, spaces –<sup>H</sup>Hausdorff spaces, closed set-set<sup>C</sup>

## 2. Preliminary

**Definition 2.1:** Assume that  $A$  be the subset  $A$  for the space  $(X, \tau)$ , which is known as

- (i) Semi-Set<sup>O</sup> when  $\text{clo}(\text{in}(A)) \supseteq A$
- (ii)  $\alpha$  -Set<sup>O</sup> when  $\text{in}(\text{clo}(\text{in}(A))) \supseteq A$
- (iii)  $b$  -Set<sup>O</sup> when  $\text{clo}(\text{in}(A)) \cup \text{in}(\text{clo}(A)) \supseteq A$
- (iv) Pre-Set<sup>O</sup> when  $\text{in}(\text{cl}(A)) \supseteq A$
- (v)  $\text{clo}(A) \subseteq U$ , whatever  $A \subseteq U$ ,
- (vi)  $b$  -set<sup>C</sup>, when  $\text{bclo}(A) \subseteq U$  if  $A \subseteq U$ ,  $U$  is open in  $X$ .

- (vii) Pre-set<sup>C</sup> generalized semi- set, when  $\text{spr}(A) \subseteq U$ ,  $A \subseteq U$  with  $U$  is open in  $X$ .
- (viii) Generalized b-semi set<sup>C</sup> set when  $\text{bclo}(A) \subseteq U$ . if  $A \subseteq U$  and  $U$  is semi open in  $X$ .
- (ix) Pre-regular generalized set<sup>C</sup> set when  $\text{pclo}(A) \subseteq U$ ,
- (x) Set<sup>C</sup>  $\alpha$  b-generalized when  $\text{bclo}(A) \subseteq U$ ,
- (xii) Semi generalized set<sup>C</sup> set with the condition  $\text{scl}(A) \subseteq U$ .

**Definition 2.2:** Assume that  $(Y, \sigma) \rightarrow f : (X, \tau)$  is known as a

- (1) Contra pre, when  $f^{-1}(V)$  is pre-set<sup>C</sup> in  $(X, \tau)$  any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .
- (2) b-<sup>a</sup>function when  $f^{-1}(V)$  is b-set<sup>C</sup> in  $(X, \tau)$  any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .
- (3) gpr-<sup>a</sup>function when  $f^{-1}(V)$  is to be gpr-set<sup>C</sup> in  $(X, \tau)$  meant any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .
- (4) Contra gb-continuous when  $f^{-1}(V)$  is to be gb-set<sup>C</sup> in  $(X, \tau)$  any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .
- (5) <sup>a</sup>function when  $f^{-1}(V)$  is to be set<sup>C</sup> in  $(X, \tau)$  any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .
- (6) Contra  $g\alpha$  -continuous with the condition  $f^{-1}(V)$  is  $g\alpha$  -set<sup>C</sup> in  $(X, \tau)$  any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .
- (7) Contra gsp-continuous when  $f^{-1}(V)$  is to be gsp-set<sup>C</sup> in  $(X, \tau)$  any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .
- (8) Semi <sup>a</sup>function when  $f^{-1}(V)$  is semi-set<sup>C</sup> in  $(X, \tau)$  any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .

### 3. Contra generalized $gab$ - continuous functions

In this area, The presentation of a contra generalized  $gab$  - continuous and explore a portion of their properties.

**Definition 3.1:** Let a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is known as a generalized contra  $gab$ -continuous function and with the condition  $f^{-1}(V)$  is  $gab$ -set<sup>C</sup> in  $(X, \tau)$  any Set<sup>O</sup>  $V$  of  $(Y, \sigma)$ .

**Example 3.2:** Assume that the function  $X = Y = \{a, b, c\}$  by way of the function  $\{X, \varphi, \{a\}, \{b\}, \{a, b\}\} = \tau$  and  $\sigma = \{Y, \varphi, \{a, b\}\}$ . Let us consider another function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = b, f(b) = c, f(c) = a$ . Obviously,  $gab$  - continuous.

**Definition 3.3:** Assume that the subset  $A$  of the space  $(X, \tau)$  then we define the set

(i) Set  $\cap \{F \subset X : A \subset F\}$ ,  $F$  is  $gab$ -set<sup>C</sup> then the  $gab$ -closure of  $A$ , which is written by  $gab-cl(A)$ .

(ii) Set  $\cup \{G \subset X : G \subset A\}$ ,  $G$  is Set<sup>O</sup> and it is known as interior -  $gab$  set  $A$ , which is written by  $gab-int(A)$ .

**Lemma 3.4:** For  $x \in X, x \in gab$  iff  $U \cap A = \varphi$  for any  $gab$  - Set<sup>O</sup>  $U$  is in  $x$ .

*Vital Part:* Assume that  $\forall$  a  $gab$ -Set<sup>O</sup>  $U$  is in  $x$  with the end goal that  $U \cap A = \varphi$ . Hence  $A \subset X - U, gab-cl(A) \subset X - U$ .

This implies that  $x \notin gab-cl(A)$ . This is an inconsistency.

*Adequacy part:* Let assume that  $x \notin gab-cl(A)$ . Now  $\forall$  a  $gab$ -set<sup>C</sup> subset  $F$  in  $A$  such that  $x \notin F$ . Then  $x \in X - F$  is  $gab$ -open,  $[(X - F) \cap A] = \varphi$ . This is an inconsistency.

**Theorem 3.5:** Assume that function  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the map. The subsequent conditions are also the same:

- (i) The function  $f$  were contra in  $gab$  - continuous,
- (ii) The quash image of every set<sup>C</sup> set in  $(Y, \sigma)$  are also  $gab$  - Set<sup>O</sup> in  $(X, \tau)$ .
- (iii) Any set  $x \in X$  and for every  $F \in C[Y, f(x)]$  then  $\forall U \in [gab-op(X)]$ , with the end of  $f(U) \subset F$ .

**Proof:** In case (i)  $\Leftrightarrow$  (ii) and case (ii)  $\Rightarrow$  (iii) are self-evident.

(iii)  $\Rightarrow$  (ii): Assume that the function  $F$  be any set<sup>C</sup> set of  $Y$ , then  $x \in f^{-1}(F)$ . If  $f(x) \in F$  then  $\forall U_x \in gab-op(X, x)$  at extent that  $f(U_x) \subset F$ . So, to obtain  $f^{-1}(F) = \cup \{U_x / x \in f^{-1}(F)\} \in gab-op(X, x)$ . Hence the each inverse of each set<sup>C</sup> set in  $(Y, \sigma)$  is  $gab$ -open in  $(X, \tau)$ .

(ii)  $\Rightarrow$  (iv): Suppose that  $A$  be a subset of  $X$ . Assume that the set  $y \notin Ker[f(A)]$ . By result, the function  $F \in C(Y, y)$  if  $\varphi = f(A) \cap F$ . We obtain the set  $\varphi = A \cap f^{-1}(F)$  and the set  $\varphi = gab-cl(A) \cap f^{-1}(F)$ . Hence we get

$\varphi = f[gab-cl(A)] \cap F$  and  $y \notin f[gab-cl(A)]$ . We get the result  $f[gab-cl(X)] \subset Ker[f(A)]$ .

**Definition 3.6:** Let assume that the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is known as  $gab$  - continuous if the preimage of each open arrangement of  $Y$  is -  $gab$  open in  $X$ .

**Example 3.7:** Suppose that set  $X = Y = \{a, b, c\}$  through that  $\tau = \{X, \varphi, \{a\}, \{a, b\}, \{a, c\}\}$  with  $\sigma = \{Y, \varphi, \{a, b\}\}$ . Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = f(b) = f(c) = a = b = c$  respectively. Without a doubt  $f$  is contra  $gab$ -continuous, also  $f$  is not  $gab$ -continuous. Since  $f^{-1}[\{a, b\}] = \{b, c\}$  is unable to belongs to  $gab$ -open in  $(X, \tau)$ , whenever  $\{a, b\}$  is open in  $(Y, \sigma)$ .

**Example 3.8:** Consider the set  $X = Y = \{a, b, c\}$  and  $\sigma = \{X, \varphi, \{ac\}, \{bc\}, \{b\}, \{c\}\}$  with  $\sigma = \{Y, \varphi, \{b\}, \{b, c\}\}$ . Then the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by uniqueness function. Obviously  $f$  is a  $gab$ -continuous function, also it is not a contra  $gab$ -continuous. Since the reason that  $\{b, c\} = f^{-1}[\{b, c\}]$  is not a contra  $gab$ -set<sup>C</sup> set in  $(X, \tau)$ . Somewhere  $\{a, b\}$  is open in  $(Y, \sigma)$ .

**Theorem 3.9:** Assume that  $f : (X, \tau) \rightarrow (Y, \sigma)$  is contra  $gab$  - continuous and  $(Y, \sigma)$  is regular then  $f$  is  $gab$ -continuous.

**Proof:** Let  $(X, \tau)$  and  $V$  be an Set<sup>O</sup> of  $(Y, \sigma)$  containing  $f(x)$ . Hence  $(Y, \sigma)$  were regular, then a Set<sup>O</sup>  $W$  of  $(Y, \sigma)$  with  $f(x)$  way that  $clo(W) \subset V$ . Since  $f$  is contra  $gab$  - continuous, by result, There is a  $U \in gab-op(X, x)$  that  $f(U)$  is subset of  $clo(W)$ . Next  $f(U) \subset clo(W) \subset V$ . Since  $f$  is  $gab$  - continuous.

**Theorem 3.10:** Each  $^a$ function is a contra  $gab$  continuous function.

**Proof:** Let  $V$  be an Set<sup>O</sup> in  $(Y, \sigma)$ . Hence  $f$  is  $^a$ function,  $f^{-1}(V)$  be set<sup>C</sup> when  $(X, \tau)$ . By result, Any set<sup>C</sup> is to be  $gab$  - set<sup>C</sup> set. Therefore  $f^{-1}(V)$  were  $gab$ -set<sup>C</sup> when in  $(X, \tau)$ . Hence  $f$  is contra  $gab$ -continuous.

**Example 3.11:** Assume that  $\{a, b, c\} = A = B$  among  $\tau = \{\{a, c\}, \{a\}, \{c\}, X, \varphi\}$  and  $\sigma = \{\{a, b\}, \varphi, Y\}$ . To Describe the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  through  $a = f(c)$ ,  $b = f(a)$ , and  $c = f(b)$ . Undoubtedly  $f$  is contra  $gab$ -continuous function and  $f$  is also not  $^a$ function. Since  $\{a, c\} = f^{-1}[\{a, b\}]$  were not set<sup>C</sup> in  $(X, \tau)$  wherever  $\{a, b\}$  is open in  $(Y, \sigma)$ .

**Theorem 3.12:**

- (i) Any sg-  $^a$ function is also a  $gab$  -  $^a$ function.
- (ii) Any  $gab$  -  $^a$ function is also a gs- $^a$ function.

**Example 3.13:**

(1) Assume that the set  $\{a, b, c\} = X = Y$ , along  $\{\{a, b\}, \{a\}, \{b\}, \{X, \varphi\}\} = \tau$  and  $\{\{a\}, \{Y\}, \{\varphi\}\} = \sigma$ . To describe the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  through  $a = f(c)$ ,  $b = f(a)$ , and  $c = f(b)$ . undoubtedly  $f$  is contra  $gab$  - continuous function and  $f$  is also a not  $^a$ function. Since  $\{b\} = f^{-1}[\{a\}]$  were not set<sup>c</sup> in  $(X, \tau)$  wherever  $\{a, b\}$  is open in  $(Y, \sigma)$ .

(2) Assume that the set  $\{a, b, c\} = X = Y$ , along  $\{\{a, b\}, \{b, c\}, \{b\}, c\{X, \varphi\}\} = \tau$  and  $\{\{a\}, \{a, c\}, \{Y\}, \{\varphi\}\} = \sigma$ . To describe the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  through  $b = f(c)$ ,  $c = f(a)$ , and  $a = f(b)$ . undoubtedly  $f$  is contra  $gab$  - continuous function and  $f$  is also a not  $^a$ function. Since  $\{b\} = f^{-1}[\{a\}]$  were not set<sup>c</sup> in  $(X, \tau)$  wherever  $\{a, c\}$  is a Set<sup>o</sup> in  $(Y, \sigma)$ .

**Example. 3.14:** Suppose that  $\{a, b, c\} = X = Y$ , along  $\{\{a, b\}, \{b\}, \{a\}, \{X, \varphi\}\} = \tau$  and  $\{\{b\}, Y, \varphi\} = \sigma$ . Then to describe the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  through  $c = f(a)$ ,  $b = f(b)$ , and  $a = f(c)$ . Undoubtedly  $f$  is contra  $sg$  - continuous function and  $f$  is also a not  $gab$  -  $^a$ function. Since  $\{b\} = f^{-1}[\{a\}]$  were not set<sup>c</sup> in  $(X, \tau)$  wherever  $\{b\}$  is a Set<sup>o</sup> in  $(Y, \sigma)$ .

**Example. 3.15:** Assume  $\{a, b, c\} = X = Y$ , along  $\{\{a, c\}, \{c\}, \{a\}, \{X, \varphi\}\} = \tau$  and  $\{\{a, c\}, \{c\}, Y, \varphi\} = \sigma$  To describe the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  through  $a = f(c)$ ,  $b = f(b)$  and  $a = f(c)$ . Undoubtedly  $f$  is contra  $gab$  continuous function and  $f$  is also a not  $sg$  -  $^a$ function. Since  $\{a, c\} = f^{-1}[\{a, c\}]$  were not set<sup>c</sup> in  $(X, \tau)$  wherever  $\{a, c\}$  is a Set<sup>o</sup> in  $(Y, \sigma)$ .

## 4. Applications

**Definition:** Assume that  $(X, \tau)$  be the topological space, then which is called a  $gab$  - Hausdorff space. When for every different points  $(x, y)$  in  $X$ , then  $U \in Op(X, x)$  and  $V \in gab - Op(X, y)$  with  $U \cap V = \varnothing$ .

**Egs:** Suppose that set  $\{a, b, c\} = X$  along  $\{\{a, c\}, \{b, c\}, \{a, b\}, \{a\}, \{b\}, \{c\}, X, \varphi\} = \tau$  and assume that  $x$  and  $y$  are the two different points of  $X$ , then  $gab$  - neighborhood of  $x$  and  $y$  correspondingly, then  $\varphi = \{x\} \cap \{y\}$ . Therefore  $(X, \tau)$  is a  $gab$  -<sup>H</sup>space.

**Theorem:** Consider the topological space  $X$ . whos pairs are distinct point  $x_1$  point  $x_2$  then  $f \rightarrow X$  into with the  $f(x_2) \neq f(x_1)$  then  $f$  is  $gab$   $^a$ function at  $x_2$  and  $x_1$ . Also  $X$  is -  $gab$  <sup>H</sup>space.

**Proof:** Suppose that  $X$  having the two distinct points  $x_2$  and  $x_1$ . With the assumption that,  $Y$  be the Uryshon space and there is a function  $f : X \rightarrow Y$  with the condition  $f(x_2) \neq f(x_1)$  and  $f$  is

$gab$  -  $^a$ function at the points  $x_2$  and  $x_1$ . Assume that  $f(x_i) \neq y_i$  with  $i = 1, 2, 3, \dots$  and  $y_2 \neq y_1$ . Hence  $Y$  is Uryshon space and there is an Set<sup>o</sup>s  $U_{y_2}, U_{y_1}$  containing with  $y_2$  and  $y_1$  in  $Y$  which is in  $\varphi = Clo(U_{y_2}) \cap Clo(U_{y_1})$ . Hence  $f$  is  $gab$   $^a$ function at  $x_2$  and  $x_1$  in  $X$  with  $Clo(U_{y_i}) \supset f(V_{x_i})$  for  $i = 1, 2, 3, \dots$ . Therefore  $\varphi = (V_{x_2}) \cap (V_{x_1})$ . Hence  $X$  is  $gab$  -<sup>H</sup>space.

**Corollary:** With the condition that  $f$  is  $gab$  - contra  $gab$  -<sup>H</sup>space.

**Proof:** Consider the  $x_1$  and  $x_2$  two different points in  $X$ . Through the hypothesis,  $gab$  -  $^a$ function in  $X$  into a Uryshon space  $Y$  such that  $f(x_2) \neq f(x_1)$ , because  $f$  is injective. Hence by theorem,  $X$  is  $gab$  -<sup>H</sup>space.

## 5. Conclusion

The classes of contra summed  $ab$  - set<sup>c</sup> frame that lies between the class of the class of  $^a$ function map and  $gb$  -  $^a$ function map.

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