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Research paper

Magneto Metric Resistivitysoundingover Binomially Overburden

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Abstract

In this paper, inverse problem with the use of optimization technique is proposed. Mathematicalmodel of steady state magnetic field response is formulated. It is accomplished by using analytical method to solve a boundary value problem in the wave number domain and then transforming back to the special domain. One dimensional geometric model of a two layered earth is considered. Magnetic field response is computed numerically to see their behavior against source-receiver spacing. Unfortunately, there are no fluctuation of the curve to show relations between magnetic field response and conductivity parameters or overburden thicknesssignificantly.However, inversion process with the use of conventional conjugate gradient can beapplied to investigate overburden thickness accurately.

Keywords: *Magnetic, Hankel Transforms, Conjugate gradient*

1. Introduction

Magnetometric Resistivity Method was developed for more than forty decades. Following the most popular article "On the theory of magnetometric resistivity (MMR) methods" conducted by Edwards and et al.[4,5], Magnetometric Resistivity (MMR) method is based on the measurement of low-level, low-frequency magnetic fields associated with non-inductive current flow underground surface. There are a few quantitative interpretational schemes for deriving resistivity from MMR data.The magnetic fields produced by the current in wire between the two electrodes effect to the ground surface and effect to any conductivity boundaries in the ground. Historically, for a surface MMR survey, the wire connecting the two current electrodes is typically plugged into ground surface and data measurements are made somewhere in between the electrode spread. Information concerning the conductivity distribution beneath the surface and the layered thickness are then extracted withthe aid of optimization techniques. In our study, two layered earth, one dimensional conductivity ground profile with the use ofconventional conjugate gradient are proposed to investigate overburden thickness.

2. Response Ofmagnetic Field Dueto a Semi-Infinite Source of Two Layered Earth Model

A semi-infinite vertical wire DC source carries an exciting current I is located on the ground. The electrode C is placed deliberately at the interface $z = h$ of overburden and host. Overburden has conductivity as a function of depth, $\sigma(z)$, with thickness h as shown in Figure 1. The Maxwell's equations can be usedto determine the magnetic field intensity $\vec{H}$ as[2, 9]

$$\nabla \times \vec{E} = \vec{0} \tag{1}$$

and

$$\nabla \times \vec{H} = \sigma \vec{E} \tag{2}$$

where $\vec{E}$ is electric field intensity, $\vec{H}$ is magnetic field intensity, σ is conductivity of medium, ∇ is Gradient operator. Using equations (1) and (2) yield

$$\nabla \times \frac{1}{\sigma} \nabla \times \vec{H} = \vec{0}. \tag{3}$$

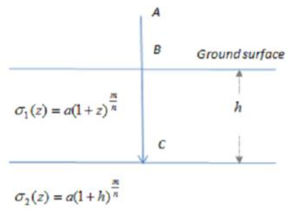

Figure 1: Geometric model of two-layered earth structure

In cylindrical coordinates (r, φ, z), the equation (3) can be expressed in terms ofthree unit vectors $\vec{e}_r$, $\vec{e}_\varphi$ and $\vec{e}_z$ as[6, 7]

$$\left\{ \frac{1}{r}\frac{\partial}{\partial \varphi}\frac{1}{\sigma}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(rH_\varphi\right) - \frac{1}{r}\frac{\partial H_r}{\partial r}\right] - \frac{\partial}{\partial z}\frac{1}{\sigma}\left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r}\right)\right\}\vec{e}_r +$$
$$\left\{ \frac{\partial}{\partial z}\frac{1}{\sigma}\left(\frac{1}{r}\frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z}\right) - \frac{\partial}{\partial r}\frac{1}{\sigma}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(rH_\varphi\right) - \frac{1}{r}\frac{\partial H_r}{\partial \varphi}\right]\right\}\vec{e}_\varphi +$$
$$\left\{ \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{1}{\sigma}\left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r}\right)\right] - \frac{1}{r}\frac{\partial}{\partial \varphi}\frac{1}{\sigma}\left(\frac{1}{r}\frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z}\right)\right\}\vec{e}_z = \vec{0},$$

where H_r, H_ϕ and H_z are magnetic field components in $\bar{e}_r$, $\bar{e}_\varphi$ and $\bar{e}_z$, respectively. Since the problem is axi-symmetric, and $\vec{H}$ has only an azimuthal component incylindrical coordinate,for simply, we use H to represent H_ϕ, thus, we obtain

$$\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial H}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial}{\partial r}(rH)\right)=0. \tag{4}$$

For simply, we denote σ as a function of depth z only, thus, the equation(4)becomes

$$\frac{\partial^2 H}{\partial z^2}+\sigma\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right)\left(\frac{\partial H}{\partial z}\right)+\frac{\partial^2 H}{\partial r^2}+\frac{1}{r}\frac{\partial H}{\partial r}-\frac{1}{r^2}H=0. \tag{5}$$

We introduce the Hankel transforms pair [1], as

$$\tilde{H}(\lambda,z)=\int_0^\infty rH(r,z)J_1(\lambda r)dr \tag{6}$$

And

$$H(r,z)=\int_0^\infty \lambda\tilde{H}(\lambda,z)J_1(\lambda r)d\lambda, \tag{7}$$

where J_1 is the Bessel function of the first kind of order one. Applying equation (6) to equation(5), we obtain

$$\frac{\partial^2\tilde{H}}{\partial z^2}+\sigma\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\frac{\partial\tilde{H}}{\partial z}-\lambda^2\tilde{H}=0. \tag{8}$$

3. Mathematical Formulationof Magnetic Field Response from Transitional Ground Profile

In many locations of the Earth, ground has various structures. A commontype is that usually can be found is an overburden located on host rock. The conductivity of an overburden can be denoted by $\sigma(z)=a(1+z)^{\frac{m}{n}}$, $0\le z\le h, 0<a\in R, m,n\in Z, n\ne 0$, where R is the set of real number and Z is the set of integer. Host rock has high resistivity and the small positive value of constant conductivity can be approximately used. Substituting $\sigma(z)=a(1+z)^{\frac{m}{n}}$ into equation (8), we obtain

$$\frac{\partial^2\tilde{H}}{\partial z^2}-\frac{m}{n(1+z)}\frac{\partial\tilde{H}}{\partial z}-\lambda^2\tilde{H}=0. \tag{9}$$

The solution to equation (9) is given by

$$\tilde{H}(\lambda,z)=(1+z)^{\frac{m+n}{2n}}\left[A_1 I_{\frac{m+n}{2n}}\left(\lambda(1+z)\right)+B_1 K_{\frac{m+n}{2n}}\left(\lambda(1+z)\right)\right], \tag{10}$$

where A_1 and B_1 are arbitrary constants. $I_{\frac{m+n}{2n}}(\ )$ and $K_{\frac{m+n}{2n}}(\ )$ are the modified Bessel functions of the first and second kind of order $\frac{m+n}{2n}$,respectively.

4. Mathematical Formulationof Magnetic Field Response from Homogeneous Ground Profile

At host region under overburden, the conductivity is constant and denoted by $\sigma(z)=a(1+h)^{\frac{m}{n}}$, $z>h$. The equation (8) can be simplified to be

$$\frac{\partial^2\tilde{H}}{\partial z^2}-\lambda^2\tilde{H}=0,$$

andthe solution is denoted by

$$\tilde{H}(\lambda,z)=A_2 e^{\lambda(z-h)}+B_2 e^{-\lambda(z-h)}, \tag{11}$$

where A_2 and B_2 are arbitrary constants. In our study, we consider for two-layered earth model as shown in Figure 1.We design the conductivity of ground for overburden and host rock, respectively, as

$$\sigma_{over}(z)=a(1+z)^{\frac{m}{n}},\ 0\le z\le h,$$

$$\sigma_{host}(z)=a(1+h)^{\frac{m}{n}},\ z>h.$$

For the first layer, magnetic field consists of two parts caused by ground and probe source. The first part of magnetic field is responded from overburden and given by.

$$\tilde{H}(\lambda,z)=(1+z)^{\frac{m+n}{2n}}\left[A_1 I_{\frac{m+n}{2n}}\left(\lambda(1+z)\right)+B_1 K_{\frac{m+n}{2n}}\left(\lambda(1+z)\right)\right].$$

The second part, magnetic field due to the probe source[8], which is defined by Ampere's law

$$H(r,z)=\frac{I}{2\pi r}.$$

By using the Hankel Transforms[1] as in equation (6), we obtain

$$\tilde{H}(\lambda,z)=\frac{I}{2\pi\lambda}.$$

Therefore, the solution in the first layer can be denoted by

$$\tilde{H}_{over}(\lambda,z)=\frac{I}{2\pi\lambda}+(1+z)^{\frac{m+n}{2n}}\left[A_1 I_{\frac{m+n}{2n}}\left(\lambda(1+z)\right)+B_1 K_{\frac{m+n}{2n}}\left(\lambda(1+z)\right)\right].$$

For the second layer, the magnetic field solution can be written by

$$\tilde{H}_{host}(\lambda,z)=A_2 e^{\lambda(z-h)}+B_2 e^{-\lambda(z-h)}.$$

5. Boundary Conditions and Particular Solutions

The arbitrary constants in magnetic field solutions obtained from equation (9) can be found by using the following boundary conditions [8]:
(1) The magnetic field is continuous at the interface of each layer $H_{over}(r,z)|_{z=h^-} = H_{host}(r,z)|_{z=h^+}$.
(2) The radial component of electric field is continuous at the interface of each layer
$\lim_{z\to h^-} E^r_{over}(r,z) = \lim_{z\to h^+} E^r_{host}(r,z)$,
where $\bar{E}^r_{over}$ and $\bar{E}^r_{host}$ are radial electric fields in overburden and host rock, respectively.
(3) As the depth z tends to infinity, the magnetic field tends to zero.
(4) Since no current across the Air-Earth interface, then $\sigma_{over}(z)E^z_{over}(r,z)|_{z=0} = 0$, where E^z_{over} is an electric field in vertical direction in overburden.
Applying the above boundary conditions and taking inverse Hankel Transforms to equation (7), with $m=1, n=2,$ we obtain the magnetic field solutions as

$$H_{over}(r,z) = \int_0^\infty \lambda\left[\frac{I}{2\pi\lambda}\right]\left[1+\left(\frac{1+z}{1+h}\right)^{\frac{3}{4}}\left\{\frac{K_{\frac{3}{4}}(\lambda)I_{\frac{3}{4}}(\lambda(1+z)) - I_{\frac{3}{4}}(\lambda)K_{\frac{3}{4}}(\lambda(1+z))}{Q_1 I_{\frac{3}{4}}(\lambda) - Q_2 K_{\frac{3}{4}}(\lambda)}\right\}\right]J_1(\lambda r)\,d\lambda, \quad (12)$$

$$H_{host}(r,z) = \int_0^\infty \lambda\left[\frac{I}{2\pi\lambda}\right]\left[e^{-\lambda(z-h)}\left\{\frac{-K_{\frac{3}{4}}(\lambda)I_{-\frac{1}{4}}(\lambda(1+h)) - I_{\frac{3}{4}}(\lambda)K_{-\frac{1}{4}}(\lambda(1+h))}{Q_1 I_{\frac{3}{4}}(\lambda) - Q_2 K_{\frac{3}{4}}(\lambda)}\right\}\right]J_1(\lambda r)\,d\lambda, \quad (13)$$

where $Q_1 = K_{\frac{3}{4}}(\lambda(1+h)) - K_{-\frac{1}{4}}(\lambda(1+h))$, and $Q_2 = I_{\frac{3}{4}}(\lambda(1+h)) + I_{-\frac{1}{4}}(\lambda(1+h))$.

6. Numerical Experiments

The magnetic field as described in equations (12) and (13) can be computed by using Chave's Algorithm[3]. In general, the behavior of magnetic field response from high conductive ground will be very strong. We hope to see some signal from graph to indicate the information underground surface such the thickness of overburden and conductivity parameters. In our numerical experiments, we will show many cases of conductivity parameter to support our mathematical model. For our initial case, we start with overburden thickness $h=5$ *meters*, the conductivity parameters $a=1\,S/m, m=1, n=2.$ Numerical results for magnetic field due to Direct Current source are performedas in Figure 2 and 3.

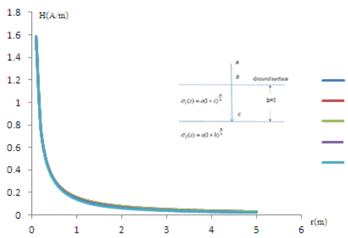

Figure 2: Graph of magnetic fields against source-receiver spacing using $I=1$ *Ampere* at various depths.

As shown in Figure 2, the curve of magnetic fields against source-receiver spacing(r) are plotted at various depths, $z=0,1,2,3,4$ meters with the used of electric current (I) equal to 1 *Ampere* in Figure 2 and 3 *Ampere* in Figure 3.

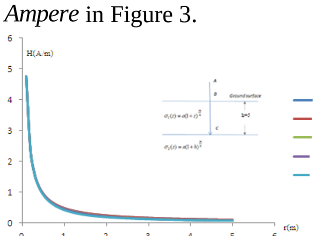

Figure 3: Graph of magnetic fields against source-receiver spacing using $I=3$ *Ampere* at various depths.

We can see that the magnetic field drop very fast to zero as the source-receiver spacing is increased. At higher values of electric current is injected into the ground, the stronger magnetic field will be released from ground and can be shown in Figure 4.

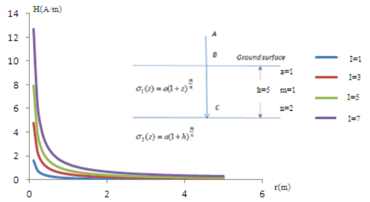

Figure 4: Graph of magnetic fields against source-receiver spacing on ground surface at various electric currents.

For our second case, we start with overburden thickness $h=1$ *meters*, the conductivity parameters $a=1\,S/m, m=1, n=2.$ Numerical results for magnetic field due to Direct Current source are performed in Figure 5.

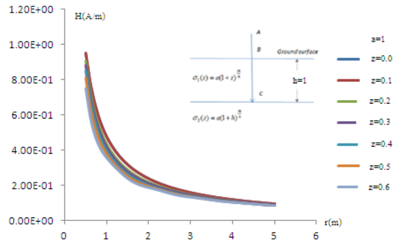

Figure 5: Graph of magnetic fields against source-receiver spacing using $I=1$ *Ampere* at various depths.

As shown in Figure 5, the curve of magnetic fields against source-receiver spacing(r) are plotted at various depths, $z=0.0, 0.1, 0.2, \ldots, 0.6$ *meters* with the used of electric current (I) equal to 1 *Ampere* . The graphs of magnetic field drop very fast to zero as the source-receiver spacing increases. At largedepth, the response of magnetic field is lightly.
As shown in above two cases,unfortunately, the curves of magnetic field do not give any fluctuation related to the conductivity profile of the ground or overburden thickness. Thus, in the next section, we are going to propose the inversion process.

The conjugate gradient will be used to investigate the overburden thickness.

7. Inversion Process

The most important problem in Archeology is an overburden thickness. Archaeologists often encounter problems in interpreting the genesis of layers and the context in which specific materials occur in excavations. In our context here, the calculation and measurement of magnetic fields are compared to find the thickness of overburden. The relative error of magnetic fields are used to terminate the iterative process. In our first example, synthesis data is formulated. Equations (12) and (13) are used to compute the synthesis data. Two percent of Gauss error is added to perturb our data as noise signal. Inversion process is started by using initial guess $h = 10m.$ Asshown in Table 1, conventional conjugate gradient is used only 6 iterations to get our solution accurately. To comfirm our mathematical model, the second example is proposed by using initial guess $h = 1m.$ The results are shown in Table 2 accurately with only 7 iterations.

Table 1: Inversion results of conventional conjugate gradient using initial guess h=10 m.

Overburden thickness	1st(initial guess) (h=10.000)	2nditeration (h=1.500)	3rditeration (h=3.810)	4thiteration (h=2.090)	5thiteration (h=2.980)	6thiteration (h=3.003)
Relative error	4.45E-04	6.36E-04	3.19E-04	1.48E-04	4.54E-06	2.26E-07

In our second example, we repeat as in the first example except for using $h = 1m.$ and result is shown in Table 2.

Table 2: Inversion results of conventional conjugate gradient using initial guess h=1 m.

Overburden thickness	1st(initial guess) (h=1.000)	2nditeration (h=2.577)	3rditeration (h=3.970)	4thiteration (h=2.722)	5thiteration (h=3.120)	6thiteration (h=2.891)	7thiteration (h=3.001)
Relative error	5.25E-04	3.56E-04	4.57E-04	1.22E-04	8.56E-06	3.14E-06	2.14E-07

8. Conclusions

In this paper, we derive analytical solution of steady state magnetic field for binomial conductivity ground profile. It is accomplished by solving a boundary value problem in the wave number domain and then transforming back to the spatial domain. We consider two layered Earth model in our study. The magnetic field can be computed to see the behavior by using Chave's algorithm [3]. Numerical results due to Direct Current source on the ground surface are shown in Figure 2 to Figure 5. With the use of electric current $I = 1, 2 \; and \; 3 \; Amperes$, the curves of magnetic field drop rapidly as we increase the source-receiver spacing (r) . With the use of three values of electric current $I = 1, 2 \; and \; 3 \; Amperes$, the curves of magnetic field drop in a similar manner. Unfortunately, the curves of magnetic field do not give any fluctuationrelated to the conductivity profile of the ground. There are very fewrelations between magnitude of magnetic fields and conductivity parameters which imply the conductive of ground as mention in above section.Surprisingly, equations (12) and (13) are independent from conductivity parameter a. At ground surface, $z = 0 \; meter$, the magnetic field response do not depend on both thickness of overburden and conductivity parameter a according to the mathematical term in braces of equation (12) disappear.

In our sounding for overburden thickness, conventional conjugate gradient is used. Two examples are performed to show very good convergence of the solutions at 6 and 7 iterations only.

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