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Teaching of Angle of Elevation by Integrating Angle Measurement in Ancient Egyptian Mathematics

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Abstract

The aim of this study is to stimulate reflections and determine the effects of teaching mathematics by integrating its history through Lesson Study. Lesson Study was utilized as a process to delve into the possible outcomes of incorporating history of mathematics in teaching angle of elevation to 15 freshmen college students taking BS Air Transportation. The researchers followed the three steps in conducting a Lesson Study; planning, implementing, and conducting the post-lesson discussion. The implementation of the lesson and the post-lesson discussion were video and audio recorded which later on transcribed. Three issues in attaining the objectives of the lesson were identified: (1) Being Able to Set-up the Condition and Being Clear with the Instructions (2) Being Realistic with Examples, and (3) Importance of Processing Methodologies, which greatly play an important share on maximizing the learning process and students' success. Furthermore, feedbacks from the observers and students suggests that using history of mathematics enhanced students' curiosity about the significance of the lesson in real life. This study may contribute to the advancements of innovative teaching strategies to the rest of educators and researchers.

Keywords: *Ancient Egyptian; Angle of elevation; Angle measurement device; History of mathematics; Lesson study.*

1. Introduction

Today, students require stronger mathematical knowledge, skills and values to pursue higher education, to compete and be part of the technologically oriented workforce and to be an informed citizen [1]. As students in the 21st century become more diverse in terms of students' educational needs [2], teachers are expected to respond to these needs as their learning approach is different to those of the previous generations [3].

Lesson study is considered as one of the effective tools in improving students' achievement in both basic and higher education. It is a professional development model in which teachers aim to improve their teaching methods by working with other teachers to examine and critique ones' teaching technique and strategy [4, 5].

Teaching mathematics by integrating its history have many benefits for the teachers and students. According to Fauvel [6] and Swetz [7], integrating the history of mathematics can motivate students in learning mathematics. It can also improve students' problem- solving skills, ensure that students learn mathematical concepts meaningfully, and help them connect mathematics to other discipline [8]. Most importantly, it allows students to relate mathematics with the society [9].

In the mathematics framework proposed by SEI-DOST and MATHTED [1] of the Philippines, one of its goals is for students appreciate the cultural value of mathematics and origin, history, and how mathematics continue to evolve. In the current curriculum of the tertiary level, one course that students need to take is Mathematics in the Modern World. It revolves around the nature of mathematics and using math as tool, in which a glimpse of the history of mathematics was included. Hence, the aim of this paper is to provide students a different perspective in learning angles of elevation in relation to its history. Moreover, this paper intends to stimulate reflections on how to teach angle of elevation by using its history as tool and a goal.

2. Background of the Study

Diversity of learners has been one of the major challenges in the Philippine educational system. With this being said, there were attempts to improve the kind of learning system that we have. According to Gisela and David Slavit [10], in order to combat the diversities inside the Filipino classroom, a facilitator must consider the students' perspectives, understandings, and practices in mathematics towards the lesson. This is very similar to the root of lesson study in Japan. Japanese lesson study has been a popular professional development approach in recent years. This object lesson method is based on Pestalozzian theory, according to which teaching should start from observation of objects that helps students recognize the concepts through their intuition. Meanwhile, a study by Wilson and Chauvot [9] revealed that history of mathematics has much to contribute in the improvement of teaching and learning of mathematics. Hence, it can become a tool for effective teaching. This is why the researchers came up with the idea of combining these two findings towards innovating the teaching-learning process of Mathematics education.

3. Methodology

The researchers, together with their professor, cooperated in planning, designing, and implementing the Lesson Study. The participants of this study were 15 first year college students taking up Bachelor of Science in Air Transportation at Philippine State College of Aeronautics. This study argues the importance and effectiveness of the integration of history of mathematics in teaching and

learning mathematics. The topic of the lesson is about angle measurement devices and angle of elevation which seemed to be significant for the participants since they are taking up aeronautics courses. The Lesson Study was observed by other graduate students taking up the same course as the researchers, Masters of Science in Teaching Mathematics, and by the course professor.

3.1. Planning for the research lesson

In the extent of the planning, the group considered the following: (1) use of history to the lesson, (2) relevance of the lesson, (3) main activity for the lesson, (4) intellectual capability of the students, (5) art of questioning, and (6) medium of instruction. The researchers decided that history will be used as a tool and as a goal [11] in teaching the lesson. Moreover, the researchers decided to focus on the following objectives of the lesson that will allow the students to: (1) describe and illustrate angle of elevation, (2) appreciate the history of angle measurement devices in ancient Egypt, and (3) make and use a clinometer in solving real-life problem involving the concept of angle elevation. One issue that occurred in the planning stage was whether to give the steps in making the clinometer or let the students discover the method in making one. The researchers then settled to give the steps in making a clinometer by attaching pieces of papers on the materials that will be given in the treasure hunting activity. Researchers also discussed the possible misconceptions of the students regarding the image of the pyramid in the main activity. Furthermore, the use of English and Filipino as a medium of instruction was also considered so that students will be more comfortable in asking questions and will elicit enough intellectual quality in recitation.

3.2. The research lesson

One member of the group was assigned to teach the lesson and the others were assigned in observing, documenting, and collecting data all throughout the implementation of the Lesson Study. The teacher started the lesson by giving warm-up activities such as hand exercises and tapping their classmates. This warm-up activity was meant to give a glimpse of the measuring units used by the ancient Egyptians, which are the hands (palms) and arms (cubits). Then, the students were grouped into three according to the colors of their name tags which was provided by the teacher. To recall and review the concepts of angle and common trigonometric ratios, the teacher gave a treasure hunting activity. There were six stations in this activity and each station contained questions about angles and common trigonometric ratios. Every correct answer corresponded to a material that will be used in the main activity of the lesson proper. Before going to the next station, one representative of the group must go to the teacher to show their answer and get the corresponding material. After the task, the teacher discussed some of the questions in the activity to review the whole class.

The teacher incorporated the concept of taking a selfie as a transition to introduce and define angle of elevation. After that, she posted the main problem and gave the scrolls to each group that contained the direction for the activity. There were two tasks in the activity: make their own clinometer and find the height of the pyramid as shown in Figure 1. However, instead of explaining the directions in front of the whole class, she went to each group to thoroughly explain the guidelines and instructions to follow. She facilitated the whole activity until all groups finished the tasks.

In synthesizing the lesson, the teacher asked the students to form a circle after the group activity. With this set-up, students are more comfortable to have a dialogue and ask questions to the teacher. The teacher let the students write and present their answers in the main activity on the board. After that, the teacher discussed and related the different angle measurement devices of ancient Egyptians to the present devices such as clinometer. Towards the end of the lesson, the students were able to answer the three important questions: (1) Why did the Ancient Egyptians come up with angle measurement devices? (2) How did the Ancient Egyptians develop the instruments? (3) What do you think is the importance/relevance of angle measurement devices in your course?

3.3. Post-lesson discussion

After the implementation of the lesson, the researchers, together with other graduate students, and their professor conducted a post-lesson discussion. The post-lesson discussion formally started as the teacher of the Lesson Study reflected about the expected and actual flow of discussion. She even shared her difficulties, lapses and decisions made in the lesson proper. The other members of the team also shared their experiences in planning and preparing for the research lesson. Then, observers gave their comments, suggestions, and criticisms about the integration of history to the content of the lesson, teaching strategies, questioning skills, relevance of the activities, classroom management, and the intellectual capability of the students. Observers were given the actual lesson plan and an evaluation sheet before the start of the lesson.

3.4. Data analysis

The data collected were analyzed and coded using phenomenological perspectives. All the comments and recommendations of the observers, responses and reflections of the students were summarized, encoded, and used for the data analysis.

4. Results and Discussions

Engaging in Lesson Study has allowed the authors to critically look deeper into usual practices in a classroom setting wherein teachers sometimes have taken for granted. As a result, three emerging issues were acknowledged such as (1) being able to set-up the condition and being clear with the instructions, (2) being realistic with examples, and the (3) importance of processing methodologies. In spite of these issues, the researchers were still able to integrate the history in teaching mathematics lesson in an effective way.

4.1. Being able to set-up the condition and being clear with the instructions

Before starting the treasure hunt activity, the teacher clearly stated that groups will move from one station to another. She also gave an envelope containing the rest of the instructions incorporated at the scroll. The teacher then assumed that students will read all the instructions given to them. However, it created a longer activity time from an ideal 10 minutes to 20 minutes. The teacher then decided to intervene from time to time and reminded the students to read the instructions carefully. It was essential that clear and complete instructions were laid down at the start of the activity – and should be done verbally, at all times. Below are some critical evidences during the activity:

Each team should assign roles on each member of the group based on the written instruction; (1) a pharaoh that will stand as the leader, (2) a nomarch that will help the pharaoh in decision making, (3) an intelligentsia that will be the brain of the group, (4) a labourer that will help the group, and (5) a military that will be responsible for the group's noise. A representative of the group was supposed to give their answer to the teacher before going to the next station. However, students went to different stations and tried to answer all the questions before going to the teacher.

(Green team is done with Question 1&2 – still sitting and waiting for teacher's verbal instructions)
Teacher: *Are you done?*
Students: *Yes, Ma'am.*
Teacher: *Then come over here. Make sure you also read the instructions at the bottom.*
Students: *Now? (started re-reading the scroll)*
Oh yeah! Pharaoh, get in front quickly!

Same concern emerged during the main activity in the lesson proper. Problem on getting the height of the Pyramid of Giza was imposed on the board before all three groups start constructing their own measuring device. Each group received another envelope with instructions and now need to utilize the materials they received during the recall activity.

One member from the Orange team kept saying:

"I know this. We used this in high school. I just don't remember how it looks like."

Hence, other groups were still struggling:

"Hold that still (referring to the string that needs to be attached at the tip of the pyramid), then I will attach the other end to the protractor."

"Perhaps we need to insert the protractor on the straw. Or attach it on the side?"

Tension between groups was really evident. Instead of the teacher announcing some instructions to the whole class, she decided to handle facilitation by group.

Teacher: *Are you guys following the instructions on the scroll?*

Students: *Yes, Ma'am.*

Teacher: *Are you sure that should be the position of the protractor? And what do you think is the use of the straw? What is specifically in there?*

Fig. 1: Students trying to make a clinometer

Students: *Hole. There is a hole! I told you so. Protractor should be the opposite way around.*

Teacher: *How about the string? Do you think you need a string that long? Or do you think it can just be attached to the protractor? How do you find its equilibrium?*

Students: *That's why there is a clip! It will serve as a weight.*

Teacher: *Are you sure you are following the instructions attached on the materials?*

Students: *Oh no. We didn't know there are still written instructions. Ma'am, I think we lost ours.*

Again, clear and complete instructions before the start of the activity could have prevented such issues and could have saved time. The good thing about the students is that they are creative and have different strategies in constructing their own devices, and looking at different perspective. String, as one of the materials that each team has, should be used solely as the equilibrium in the clinometer, as the researchers have thought of. However, the students were able to utilize it as an easier way in measuring longer lengths of cubits and palms. There was also a consensus with the observers regarding introducing clearer instructions:

"Activity took a lot of time and it seems like the instructions were not clear. The instructions in making the clinometer weren't that emphasized."

Another thing that they have observed was the added height of the pyramid at the lower part of the image which added confusion to the students when they were about to solve for the total height of the pyramid.

"Ma'am, is 247 cubits the partial height of the pyramid? We're quite confused with the tape because it isn't attached to the edge of the wall."

On the other hand, observers also took note of the performance of the students:

"It was good because you can see that students are actively involved; challenged but persevering. Students were actually good and they were able to answer the questions even if the delivery of instructions was not clear."

4.2. Being realistic with examples

The researchers decided to utilize a pyramid to add a personal touch on Ancient Egypt. Nuances, however, are quite evident, which lead to confusion of the students, and observers as well. The course professor also suggested that a 3D image could have been better provided in the activity, perhaps a tent, so as to make it more realistic. One observer also suggested some modification on the big problem. Though he considered that the image was constructed on the board after the problem was introduced to the class, correct choice of words could have been observed through the discussion. The highlighted part of the big problem below could have been changed to *"the distance from the center perpendicular to the edge of the base".*

"*Thales, the founder of the great Ancient Greek tradition of Mathematics, travelled widely in the Mediterranean region, including Egypt. He then asked the Egyptian priests about the height of the Great Pyramid but they did not disclose the measurement at all. The only thing they mentioned is that* ***the measurement from the center to the edge of the base*** *is 10 Egyptian royal cubits. Find the height of the Great Pyramid in cubits."*

Being realistic with examples is a critical condition which should always be observed in a class. This aids in the clarity of the topic and allows students to visualize what is really asked, creatively think of how to solve the problem, and accurately perform the solution.

4.3. Importance of processing methodologies

All throughout the class, the teacher paid attention as to how the students feel on each activity they have experienced and performed. It is important that it is highlighted because one of the main objectives of the study is allowing them to determine the angle measurement devices of Ancient Egypt through discovery and immersion. The observers also noted that:

"It's good that the teacher asks her students regarding their experience in doing the activity. She used non-threatening approach, soft-spoken, and calm in nature. The students can relate, are engaged, and have good insights. They are quick in terms of responses."

The observers also appreciated the transition from motivation (recall) to introduction (selfie example). After the recall, the teacher then happily announced to the students that they are ready to go to Egypt. She then asked the class, *"Whenever you go to a place you've never been to before, what is the very first thing that you do?"* It was a quick response and everyone answered in chorus that they take a selfie. The observers also agreed that: *"The incorporation of the concept of selfie is a good way to introduce angle of elevation."*

However, one observer noted that at some point, the teacher is giving away some concepts. There was a time that the teacher asked a volunteer in front to demonstrate on how to take a selfie. It was the perfect time to discuss the angle of elevation, and perhaps develop the concept from the students themselves. Line of sight could have been mentioned by a student through some guide questions. It is also important to clarify which line of sight they are talking about.

Addressing the responses of the students is also an opportunity for improvement. When students were asked on how they define an angle, one of their answers is that it is measured in degrees. One student also answered in radians; however, it was not discussed anymore.

In the interest of time, the teacher opted not to discuss the whole process on how the students were able to solve the height of the pyramid. The teacher also realized that it will consume more time and will create more confusion as there were a lot of issues that occurred during the activity proper. Hence, they just focused on which trigonometric function students used. Everyone agreed that the appropriate function to be used is tangent since they are looking for the opposite side of the angle (height). As a result, the teacher then decided to ask each group to tabulate their data on the board and revealed that the actual height is 280 royal cubits. Below is a comment from an observer about the processing of the activity:

"The solution process in determining the height of the pyramid was not discussed. Were cubits converted to palms in computation? There are different measurements for the height of the pyramid? Why?"

Table 1: Students' tabulated data on the height of the pyramid

	Green	Orange	Blue
height (cubits)	3.7	3.5	3.07
distance (palms)	65.8	86	25
angle of elevation	10°	10°	65°
trigonometric function	Tan	tan	tan
height of the pyramid	256.67	262	297.54

The observers appreciated the synthesis through circle discussion. One observer mentioned that it may be really applicable to a class since it will encourage an open conversation. It will also allow the teacher to see clearly whether the students are listening and on task. During the circle time, one student also clarified,

"Ancient Egypt is using palms and cubits as their unit of measurement. But what if other persons have longer arms and hands?"

It was then discussed that Egyptians have a standard in palms and cubits. However, it was not clearly emphasized that the standard used in the ancient times was the measurement of their Pharaoh. During processing, the teacher must always be prepared in answering these kinds of questions still. Moreover, other parts of the lesson plan were also not followed due to lack of simulation. It was also then observed by the authors' classmates and professor.

Proper simulation plays a vital role on the success of the lesson study. It will allow the team to foresee whatever issue may arise during the execution. And also, it will let the teacher to be fully equipped in responding to the students.

4.4. History integration in teaching mathematics lesson

The main objective of this study is to integrate history of mathematics in teaching practice. In this study, the authors utilized history as a tool and as a goal at the same time. The lesson proper started by giving trivia about the measuring units of ancient Egyptians, followed by the problem in the main activity that involved the history about finding the height of the pyramid, and lastly was the discussion of the ancient Egyptians' angle measuring devices and its relation to the modern devices like clinometer The students were also able to see the connection and relevance of the angle measuring devices from the past to present. Hence, the integration of history was evident in the whole discussion of the lesson. Some of the observers were asked if the integration of the history was effective in teaching the lesson. Some of their comments were as follows:

"The lesson study group was able to integrate history in teaching the angle of elevation. They were able to include the history of measurements from using body parts to creating standard measurements. Having said that, it could have been better if the question regarding the difference between the old and new devices were made clearer."

"It was nice for the students to have a problem solving question related to a known fact regarding a very significant historical structure, the pyramid of Egypt."

Utilizing history of mathematics in teaching the lesson is indeed an effective strategy in the development of the lesson. However, the relevance and the connection of the history to the lesson must be made clearer at the end of the discussion. Moreover, the students were also asked for a feedback about the effectiveness of integrating of history in learning the lesson. Some of their comments were as follows:

"The articulation of the history of mathematics has a significant value since calculators and other mathematical materials were not yet invented."

"Having known the basic principles of the topic step by step, gave us an idea on how clinometer develops thus making it easier for us to understand. Also introducing us the history of mathematics in Egypt made the topic more interesting."

"The integration of history helped us by giving us cultural background which shaped our perception of ancient societal context. Understanding history paved way in answering the why's and how's of technological innovation in mathematics. Overall, making the students know the story of a concept helped us remember lessons efficiently in a long term run."

This only shows that the history of mathematics made the lesson more interesting and challenging for the students. The Lesson Study led the students to appreciate the history of mathematics and to become more engaging in the lesson.

5. Conclusion

Through this study, the researchers were able to gain additional insights and suggestions to improve the whole process of instruction. In summary of the different recommendations focused on appraisals brought about by the study, the researchers were able to point out four vital aspects that need further focus in conducting this Lesson Study. The first aspect is being clear in giving instructions. Giving clear instructions is essential especially if an activity is part of the learning process so that students will be able to comprehend the desired output of the teacher without being misled into doing something unnecessary. Another aspect is by setting up classroom conditions so that the teacher would be able to execute the whole lesson with ease and without wasting too much time in preparing the props and visual aids. Moreover, by giving realistic examples from which the students can relate with and apply in real life situations. According to John Dewey's [12] learning by doing, learning should be relevant and practical not just passive and theoretical. The last vital aspect is the processing of the methodologies wherein the teacher takes into action of the whole plot of the lesson carrying his/her principles and methods. Having anticipated outcomes of the discussion helps the teacher to have a smooth flow of the Lesson Study.

References

[1] SEI-DOST & MATHTED, Mathematics framework for Philippine basic education. Manila: SEI-DOST & MATHTED. 2011.

[2] Tassel-Baska JV & Stambaugh T, "Challenges and possibilities for serving gifted learners in the regular classroom", *Theory Into Practice*, Vol.44, No.3, (2005), pp.211–217.

[3] Brunsell E & Horejsi M, "Flipping your classroom in one "take" ", *The Science Teacher*, Vol.80, No.3, (2013), p.8.

[4] Elipane EL, "Introducing lesson study as a professional development model in the islands of the Philippines", *Advanced Science Letters*, Vol.23, No.2, (2017), pp.116-1129.

[5] Elipane LE, "Towards the embodiment of competency standards: Incorporating the elements of Lesson Study in the pre-service mathematics teacher education in the Philippines", *The Asia-Pacific Education Researcher*, Vol.21, No.2, (2012), pp.365-374.

[6] Fauvel J & Maanen JV, "The role of the history of mathematics in the teaching and learning of mathematics discussion document for an ICMI study (1997-2000)", *ZDM*, Vol.29, No.4, (1997), pp.138-140.

[7] Swetz F, *Learning Activities from the History of Mathematics*, The United States of America:J.Weston Walch, Publisher, (1994).

[8] Yildiz C, Cabakcor B, Ozdogan Z & Arslan S, "The views of the teacher and students in regards to the use of the history of mathematics in the teaching of fractal subject", *Procedia Social and Behavioral Sciences*, Vol.15, (2011), pp. 868-872.
[9] Wilson PS & Chauvot JB, "Who? How? What? A strategy for using history to teach mathematics", *Mathematics Teacher*, Vol.93, No.8, (2000), pp. 642-645.
[10] Slavit GE & Slavit D, "Educational Reform, Mathematics, & Diverse Learners: Meeting the Needs of All Students", *Multicultural Education*, Vol.14, No. 4, (2007), pp.20-27.
[11] Jankvist UT, "A categorization of the "whys" and "hows" of using the history in mathematics education", *Educational Studies in Mathematics*, Vol.71, No.3, (2009), pp.235-261.
[12] Dewey J, *Experience and education*, New York: Collier, (1938).