

Mixed Convection Flow in a Double Lid-Driven Oblique Cavity Filled With Ferro Fluids in the Presence of an External Magnetic Field

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Abstract

Steady laminar mixed convection and MHD effect inside a double lid-driven oblique cavity filled with a ferrofluid is studied numerically using the finite element method. An isothermal heater is placed on the left inclined wall of the cavity while the right wall is maintained at a constant cold temperature. The horizontal top a bottom moving walls are kept adiabatic. The oblique cavity is filled with a mixture of kerosene-cobalt ferrofluids. The numerical computations are obtained for various parametric values of the inclination angle of the sloping walls, Hartmann number and the volume fraction of the ferromagnetic particles. It is shown that the transfer rate is clearly enhanced with the augmentation of the ferromagnetic particles volume fraction under the influence of a relative magnetic field.

Keywords: Magneto-hydrodynamics; Mixed convection; Double lid-driven flow; Ferrofluid; Oblique cavity.

1. Introduction

Mixed or combined convection flow and heat transfer in cavities are a significant phenomenon in science and engineering systems due to its wide applications in the operation of solar collectors, heat exchangers, drying technologies, home ventilation, high-performance building insulation and lubrication technologies. Mixed convection is considered more complicated than other convections due to the coupling between the buoyancy force by temperature difference (natural convection) and the shear force by the movement of wall (forced convection). The study of Papanicolaou and Jaluria [1] performed a numerical investigation of laminar mixed convection in a rectangular cavity with an isolated thermal source. The study of Khanafer and Chamkha [2] considered the mixed convection flow in a lid-driven cavity filled with a Darcian fluid-saturated porous medium. Abu-Nada and Chamkha [3] considered the mixed convection flow in a lid-driven inclined square enclosure filled with a nanofluid. Muneer et al. [4] investigated numerically the mixed convection inside a lid-driven square cavity filled with pure fluid.

A ferrofluid, also known as a magnetic colloid, is a magnetic colloidal suspension consisting of a base liquid and magnetic nanoparticles with a size range of 5-15 nm in diameter coated with a surfactant layer. A magnetic field significantly enhances the convection heat transfer phenomenon in engineering systems due to the physical properties of the magnetic nanoparticles which tend to make ferrofluids a valuable material in many technical applications such as micro-electronic devices, coolers of nuclear reactors, actuators, rotating or linear electrical drives, purification of molten

metals, fine positioning systems, lubricated systems and adaptive bearing and dampers [5, 6]. There is a good number of excellent books and review papers that have a clear description for ferrofluids [7-10]. Jue [11] numerically considered the laminar mixed convection in a square cavity using the semi-implicit finite element method. The vertical walls having different temperature and the horizontal walls were kept adiabatic with and a magnet next to the bottom wall of the cavity. They found that the heat transfer rate decreased with increasing the strength of the magnetic field.

Sheikholeslami and Bandpy [12] numerical investigated the natural convection of ferrofluid in a cavity heated from below in presence of external magnetic field by using Lattice Boltzmann method. Kefayati [13] conducted a numerical study on the ferrofluid free convection flow in a cavity with linearly temperature distribution by using Lattice Boltzmann method. The results found that the heat transfer decreased the nanoscale ferromagnetic particle volume fraction increment for various Rayleigh numbers. Recently, Rahman [14] considered the unsteady magnetohydrodynamic convection in a semi-circular cavity filled with cobalt-kerosene ferrofluid using numerical and statistical techniques. They found that higher ferrofluid solid volume fraction increased the heat transfer rate. Louaraychi et al. [15] made an investigation in the problem of mixed convection heat transfer in a uniformly heated double lid driven shallow rectangular cavity. Nevertheless, the study of MHD mixed convective heat transfer in a double lid-driven oblique cavity filled with ferrofluids has not yet been investigated. Therefore, this is the aim of this study with the ferrofluid being a mixture of cobalt and kerosene.

2. Mathematical Modeling

The steady two-dimensional natural convection problem in a double lid-driven oblique with side L and inclination angle φ filled with a cobalt-kerosene ferrofluid, is illustrated in Fig. 1. The ferrofluid used in the analysis is assumed to be incompressible and the flow is laminar, and the base fluid (kerosene) and the solid nanoparticles (cobalt) are in thermal equilibrium. The left inclined wall of the cavity is maintained at constant hot temperature T_h and the right inclined wall is maintained at constant cold temperature T_c , while the horizontal walls are kept adiabatic. The top and bottom horizontal walls are assumed to move with constant velocity U_0 . The ferrofluid used in the analysis is assumed to be incompressible and the flow is laminar, and the base fluid (kerosene) and the solid nanoparticles (cobalt) are in thermal equilibrium. The thermo-physical properties of the kerosene and cobalt are given in Table 1. The thermo-physical properties of the ferrofluid are assumed constant except for the density variation which is determined based on the Boussinesq approximation. By considering these assumptions, the continuity, momentum and energy equations for the steady state mixed convection can be written in dimensionless form as follows:

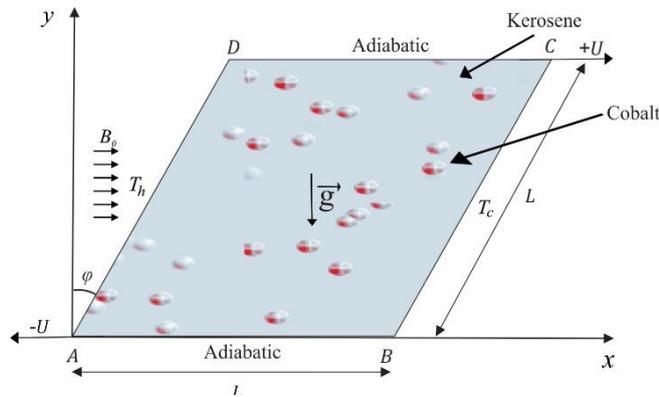


Fig. 1: Physical model of convection in an oblique cavity together with the coordinate system.

$$\sin x + \cos x = a.$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\text{Re}} \left(\frac{\nu_{ff}}{\nu_f} \right) \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \left(\frac{\nu_{ff}}{\nu_f} \right) \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \text{Ri} \frac{(\rho\beta)_{ff}}{\rho_{ff} \cdot \beta_f} \theta + \left(\frac{\rho_f}{\rho_{ff}} \right) \left(\frac{\sigma_{ff}}{\sigma_f} \right) \frac{Ha^2}{\text{Re}} V, \quad (3)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left(\frac{1}{\text{Pr Re}} \right) \frac{\alpha_{ff}}{\alpha_f} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right), \quad (4)$$

where,

$$\text{Pr} = \frac{\nu_f}{\alpha_f}, \quad \text{Re} = \frac{U_0 H}{\nu_f},$$

$$\text{Gr} = \frac{g \beta_f H^3 (T_h - T_c)}{\nu_f^2}, \quad \text{Ha} = B_0 H \sqrt{\frac{\sigma_f}{\mu_f}}$$

The governing equations of Navier Stokes equations (1)–(3), the energy equation (4) are transformed into dimensionless forms using the following dimensionless variables:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0}, \quad P = \frac{p}{\rho_{ff} U_0^2}, \quad (5)$$

$$\theta = \frac{T - T_0}{T_h - T_c}, \quad T_0 = \frac{T_h - T_c}{2}, \quad \text{Ri} = \frac{Gr}{\text{Re}^2}.$$

The dimensionless boundary conditions for Eqs. (1)–(4) are as follows:

$$\text{On the left inclined wall (AD): } U = V = 0, \quad \theta = 1, \quad (6)$$

$$\text{On the right inclined wall (BC): } U = V = 0, \quad \theta = 0, \quad (7)$$

On the adiabatic top horizontal moving wall (DC):

$$U = 1, \quad V = 0, \quad \frac{\partial \theta}{\partial Y} = 0, \quad (8)$$

On the adiabatic bottom horizontal moving wall (AB):

$$U = -1, \quad V = 0, \quad \frac{\partial \theta}{\partial Y} = 0. \quad (9)$$

The local Nusselt number at the left hot boundary may now be defined as follows:

$$\text{Nu}_{nf} = \frac{k_{ff}}{k_f} \left[-\tan \varphi \frac{\partial \theta}{\partial \eta} + \frac{1}{\cos \varphi} \frac{\partial \theta}{\partial \xi} \right]_{\xi=0}. \quad (10)$$

Finally, the average Nusselt number evaluated at the hot left wall which is given by:

$$\overline{\text{Nu}}_{nf} = \int_0^1 \text{Nu}(\eta) d\eta. \quad (11)$$

The governing dimensionless equations (1)–(4) subject to the boundary conditions (7)–(9) are solved with Galerkin weighted residual finite element method. The computational domain is discretized into triangular elements.

3. Results and discussion

This section presents numerical results for the streamlines, isotherms, local Nusselt number and the average Nusselt number for various values of Hartmann number ($0 \leq Ha \leq 100$), inclination angle of the sloping walls ($-60 \leq \varphi \leq 60$), volume fraction ($0 \leq \phi \leq 0.05$) where the other parameters are fixed at Richardson number ($\text{Ri} = 1$), Prandtl number ($\text{Pr} = 6.2$) and Grashof number ($\text{Gr} = 10^4$ and 10^5).

Figures 2 and 3 present the effects of various Hartmann numbers (Ha) on the streamlines and isotherms for $\text{Gr} = 10^4$. The flow within the cavity is characterized by a singular streamlines cell in the absence of the magnetic field ($Ha = 0$). The intensity of the isotherm patterns increases next to the non-adiabatic sloping walls while in the middle of the cavity the patterns tend to take an almost horizontal lines pattern. Applying a magnetic field ($Ha = 20$) slows down the movement of the fluid inside the cavity as a result, the intensity of the streamlines is decreased. A primary cell appears close to the lower segment of the cavity while a weak secondary cell occurs in the upper segment of the cavity next to the top wall. This happens due to the fact that the magnetic field force crushes the fluid movement. The intensity of the isotherm patterns decreases with the Hartmann number increment

due to the reduction of the gradient of the boundary layer. Increasing the Hartmann number to a high value ($Ha = 50$) clearly affects the flow structure and the temperature distribution. Next to the horizontal adiabatic walls, the intensity of the streamlines increases and most of the flow moves to the upper and lower segments of the cavity.

Figure 4 shows the effects of various Hartmann numbers on the local Nusselt number and along the heated inclined left wall for $\varphi = 30^\circ$, $Gr = 10^4$ and $\phi = 0.03$. Increasing the Hartmann number tends to decrease the gradient of the boundary layer and as a result, the local heat transfer decreases. Higher heat transfer enhancement is obtained in the absence of the magnetic field force which leads to the maximum local Nusselt number.

Figure 5 shows the effects of the Hartmann number on the average Nusselt number with the inclination angle of the sloping walls (φ) for $Gr = 10^5$ and $\phi = 0.03$. The heat transfer rate is clearly enhanced with increasing the inclination angle of the sloping walls. The values of average Nusselt number denote that the heat transfer rate increases and then decreases with the increment of the inclination angle of the sloping walls from the negative direction to the positive direction. Due to the various amounts of the flow velocity for different types of cavity. However, the average Nusselt number shows a decreasing trend in the heat transfer for higher values of the Hartmann number ($Ha = 100$). This is due to the reduction of the gradient of the boundary layer due to the strong magnetic field force. Figure 6 illustrates the effects of the Hartmann number on the average Nusselt number with the ferromagnetic particles volume fraction (ϕ) for $Gr = 10^5$ and $\varphi = 30^\circ$. As observed previously, the heat transfer rate significantly increases as the volume fraction of nanoparticles increases. However, the average Nusselt number tends to decrease with the increasing of Hartmann number. The best enhancement on the heat transfer rate is obtained in the absence of the magnetic field ($Ha = 0$).

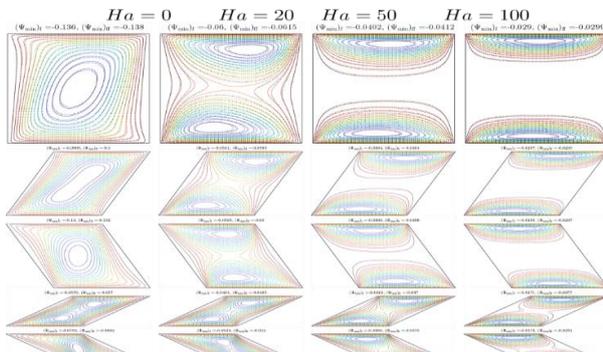


Fig. 2: Variations of the isotherms evolution by Hartmann number Ha left to the right and inclination angle of the sloping wall φ top to the bottom for $Gr = 10^4$, $\phi = 0$ (solid lines) and $\phi = 0.03$ (dashed lines).

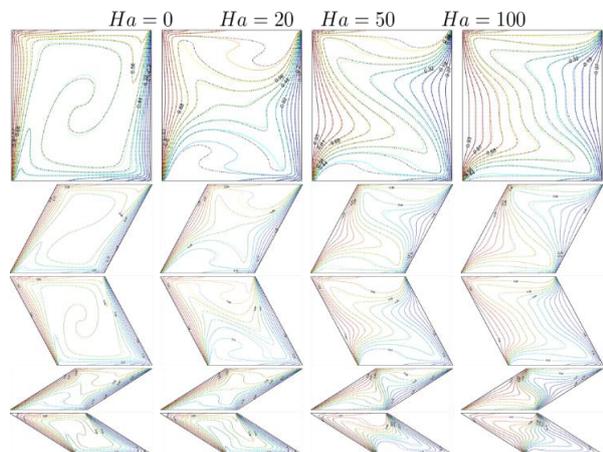


Fig. 3: Variations of the isotherms evolution by Hartmann number Ha left to the right and inclination angle of the sloping wall φ top to the bottom for $Gr = 10^4$, $\phi = 0$ (solid lines) and $\phi = 0.03$ (dashed lines).

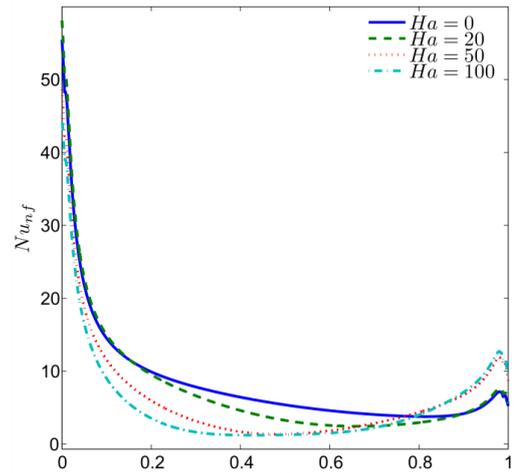


Fig. 4: Variations of local Nusselt numbers interface with Y for different Ha at $\varphi = 30^\circ$, $Gr = 10^4$ and $\phi = 0.03$.

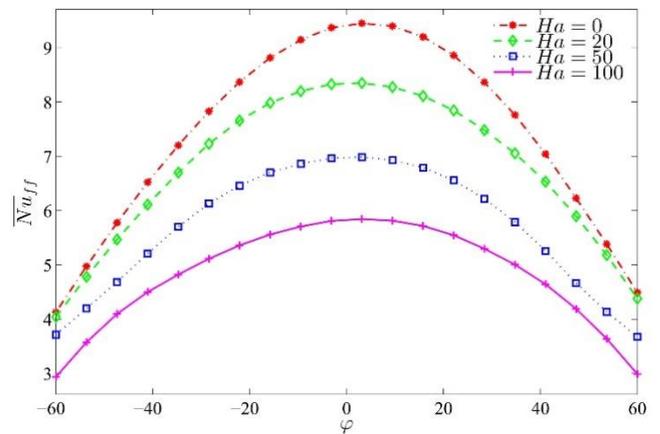


Fig. 5: Variations of average Nusselt number interface with φ for different Ha at $Gr = 10^5$ and $\phi = 0.03$.

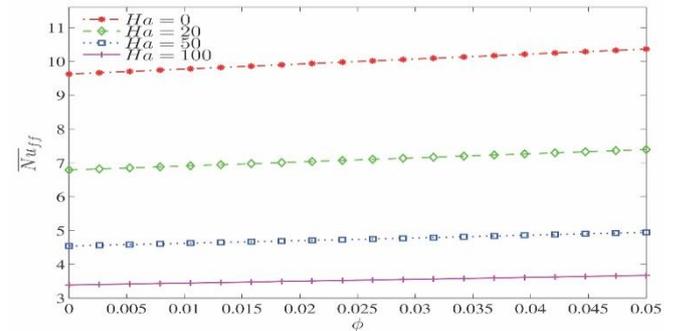


Fig. 6: Variations of average Nusselt number interface with ϕ for different Ha at $Gr = 10^5$ and $\varphi = 30^\circ$.

4. Conclusion

The present study considered MHD mixed convective heat transfer in a double lid-driven oblique cavity filled with ferrofluids. The governing equations with the boundary conditions were solved numerically using the finite element method. It shown that the intensity of the streamlines and the isotherm patterns enhanced with an increment of the inclination angle of the sloping walls. The local heat transfer rate decreased with an increment in Hartmann number tends to decrease the gradient of the boundary layer and as a result, the local heat transfer decreases the heat source length. This behaviour was due to the decreasing of the gradient of the boundary layer. The heat transfer rate was increased with the augmentation of the ferromagnetic particles volume fraction under the influence of a relative magnetic field.

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