



# **Mathematical Model of Vacuum Evaporation Using Apparatus with Mechanical Water Vapor Recompression**

**L.A. Shirkin<sup>1\*</sup>, O.G. Selivanov<sup>1</sup>, T.A. Trifonova<sup>1</sup>, S.I. Roshchina<sup>1</sup>, M.E. Ilina<sup>1</sup>**

<sup>1</sup> *Vladimir State University, Vladimir, Russia*

\*Corresponding author E-mail: lshirkin@yandex.ru

## **Abstract**

This article presents mathematical model of evaporation of aqueous solutions of mineral salts on apparatus with mechanical recompression of water vapor and process intensification upon vacuuming. The developed mathematical model is an engineering prediction where all calculations are associated with technological flowchart of evaporator. Predictions according to the mathematical model for evaporator are exemplified for the case of 2% aqueous solution of sodium chloride evaporated to 15 wt % at distillation capacity of 30 kg/h. Engineering predictions were performed consecutively in 5 stages including material calculation of evaporator; estimation of evaporator temperature mode; hydrodynamic and thermal predictions; estimation of heat transfer coefficient. The developed mathematical model was applied to stationary direct flow evaporation using Mathcad Prime software. In order to perform high quality analysis and to verify adequacy of the mathematical model, 12 criteria were selected characterizing evaporation efficiency in evaporator. Advantages of the developed mathematical model were highlighted and substantiated, practical recommendations on its application were given upon efficiency estimation of vacuum evaporation with mechanical recompression of water vapor.

**Keywords:** Mathematical Model, Vacuum Evaporator, Mechanical Recompression of Vapor, Aqueous Solutions Of Mineral Salts, Film Evaporation, Heat Transfer Coefficient, Model Adequacy.

## **1. Introduction**

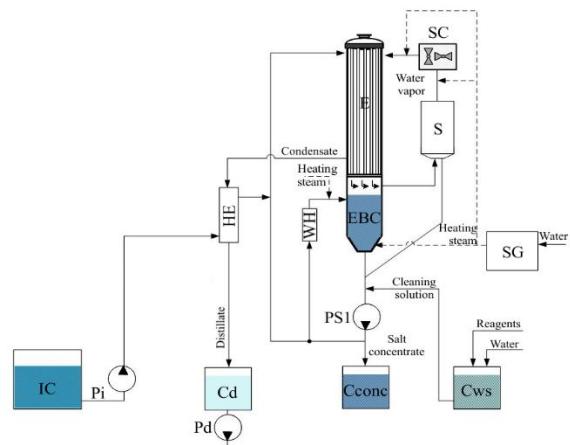
Decrease in energy consumption per unit of evaporated amount is an urgent issue during development of evaporators [1], hence, improvement of energy efficiency of evaporators is of highest importance requiring for innovative approaches to evaporation. Such approaches include development of apparatuses with mechanical water vapor recompression (MVR) based on vacuum film evaporator with free draining film. Such apparatuses can be applied for salt removal and concentrating of various aqueous solutions of mineral salts [2-4], pharmaceutical organic fluids, purification of working solutions and rinsing waters of various industries. However, models for efficiency prediction and analysis of developing and operating evaporators with regard to industrial conditions are unavailable. Moreover, criteria characterizing industrial evaporation efficiency are not determined. Hence, this work is aimed at analytical estimation of evaporation efficiency at preset parameters of capacity in terms of distillate and concentration of dissolved matter in initial and final solution.

## **2. Methods**

The subject matter of mathematical simulation is evaporation using pilot apparatus on the basis of shell-and-tube vacuum film evaporator with free draining film and recompression of water vapor developed at OOO BMT (Vladimir, Russia) (Fig. 1). Schematic view of the evaporator is illustrated in Fig. 2.



**Fig. 1:** Pilot apparatus on the basis of shell-and-tube vacuum film evaporator with free draining film and recompression of water vapor.



**Fig. 2:** Schematic view of evaporator with MVR

\*E – film evaporator; SC – blower for steam compression; EBC – evaporator boiling chamber; WH – tankless water heating; HE – recuperative heat exchanger, S – separator; IC – initial reservoir; Cd, Cconc



Copyright © 2018 Authors. This is an open access article distributed under the [Creative Commons Attribution License](#), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

– receptacles; **Cws** – reservoir for washing solution; **Pi**, **Pd**, **PC1** – pumps, **SG** – electric steam generator.

The evaporator with MVR has the following specifications:

– rarefaction in the system (residual pressure)	0.7–0.8 atm
– evaporation temperature	89–92°C
– salt content in distillate, not less than	100 mg/l
– distillate temperature	35–40°C
– temperature of concentrated product, not higher than	95°C
– capacity in terms of distillate (evaporated moisture)	0.03 m <sup>3</sup> /h
– rated capacity, not more than	7.0 kW
– power consumed for evaporation, not more than	3.0 kW
– pressure drop provided by air blower	0.3–0.35 atm (30–35 kPa)
– specific energy consumption for evaporation	40–60 W·h/l
– evaporator heat exchange surface area, not more than	1.6 m <sup>2</sup>
- number of pipes in body, n	13–15
- pipe diameter, d	25 × 2 mm
– evaporator dimensions, including: evaporator (H×D.), not more than	1.7×0.16 m
evaporator boiling chamber (H×D.), not more than	0.95×0.22 m

Evaporation in this evaporator is performed as follows: initial solution with salt content up to 40 g/l from the reservoir **IC** by the pump **Pi** is supplied for preheating to 60–70°C in the recovering heat plate exchanger **HE**. Heat carrier for heating of initial solution in the **HE** is steam condensate after the evaporator **E** supplied to the heat exchanger at 97–100°C in counter-flow to initial solution.

Heated solution after the heat exchanger **HE** is fed to the circulation pipeline: the boiling chamber **EBC** - the circulation pump **PC1** - the evaporator **E** – the boiling chamber **EBC**.

Solution from the evaporator bottom part is fed by the circulation pump **PC1** to flow distributor in the upper part of the evaporator **E**, and then in the form of thin film is drained by internal walls of the evaporator **E**. Under residual pressure of 0.7–0.8 atm (rarefaction from –0.3 to –0.2) created by the thermocompressor (air blower) **SC** in the evaporator **E** the solution is evaporated; boiling point in the boiling chamber is 90–92°C. The vapor liquid mixture at the output of the evaporator **E** is separated into liquid and vapor phases.

Water vapor from the evaporator **E** is fed to the separator **S** where condensed moisture is separated. The steam is sucked by the thermocompressor (air blower) **SC**, compressed to 1.0–1.01 atm, herewith the steam temperature increases from 89–90°C to 100°C. Adiabatically compressed water vapor is fed to shell side of the evaporator **E** where it is condensed releasing the condensation heat via pipe walls to initial evaporating solution.

Water vapor condensate at 97–100°C is supplied to the heat exchanger **HE** for heating of initial solution and then, at 45°C, is drained to the reservoir **Cd** from which it is continuously pumped by the pump **Pd**. The amount of the pumped distillate (30 l/h) is measured by flow meter, its readings are used for adjustment of flow rate of initial solution by the pump **Pi**.

A portion of saturated salt solution (6–10 kg/h) from the bottom part of the evaporator **E** is taken as ready product to the reservoir **Cconc**.

The acquired amount is adjusted automatically according to the level in the bottom part of the evaporator **E**. Compositions of distillate and concentrated product are controlled by electric conductance and pH meter of the medium.

In order to heat the solution to boiling point during startup, onset of operation mode and compensation of heat loss, the heater **WH** is used across which the initial solution from the boiling chamber **EBC** circulates.

Operation expenses of evaporator are mainly determined by energy consumption. In steady operation mode there should be the balance between energy input and output. Hence, the evaporation efficiency in such apparatuses is determined by 5 main parameters: solution boiling point upon rarefaction in the system; average residence time of the solution in heating area with accounting for recirculation; power consumption for evaporation; specific energy consumptions for evaporation of 1 kg of distillate; heat transfer coefficient in the evaporator.

Mathematical model for evaporator has been developed for the case study of 2% solution of sodium chloride evaporated to 15 wt % at distillation capacity of 30 kg/h.

The predictions have been based on data for 43 variables arranged in 6 categories:

1. Evaporator design variables.
2. Specifications of evaporation.
3. Thermal and physical properties of water.
4. Thermal and physical properties of saturated water vapor.
5. Thermal and physical properties of solution.
6. Reference values, constants.

Methods and procedures applied at the stage of initial data acquisition and preparation are as follows:

1. Numerical values of coefficients in functions of thermal and physical properties of solutions are determined by means of algorithms of multivariate regression analysis based on the least square method and optimization in Mathcad `using automatic enumeration of Interior-Point and Active-Set algorithms. Solution accuracy is controlled by coefficient of determination.

2. The functions of thermal and physical properties of solution are preset using spreadsheet without exact forms of dependencies on the basis of bilinear interpolation.

Mathematical and computer simulation is stipulated by requirements of engineering design of evaporator:

1. Material analysis of evaporator is based on the methods of material balance, theory of hydrodynamic similarity, on iterations, since the mathematical model initially considers for interrelation between solution recirculation and hydrodynamic conditions of draining film.

2. Estimation of temperature mode of evaporator is required for hydrodynamic and thermal predictions, it is based on logarithmic mean temperature difference applied for development of shell-and-tube heat exchangers.

3. Hydrodynamic prediction is based on criterion equations of hydrodynamic similarity for draining film and, in addition to minimum and operation levels of linear density of water concentration, provides information about thickness, draining rate, and average residence time of solution in heating area.

4. Thermal prediction of evaporator is based on heat balance which provides information of thermal efficiency of evaporator and makes it possible to consider with sufficient details for heat loss into ambient environment both with output flows and through external surface of evaporator with accounting for applied thermal insulation.

5. Heat exchange coefficient in evaporator has been estimated using the model of cylindrical wall based on the theory of thermal similarity, it includes consecutive calculation of heat exchange coefficients using criteria of thermal similarity depending on hydrodynamic conditions of film flow of heat transfer fluids, herewith, contamination of heat transfer surfaces is taken into account and evaporation mode in the apparatus is analyzed for bubble boiling.

Technological prediction by the developed mathematical model is implemented for stationary direct flow evaporation mode in Mathcad Prime.

The model adequacy is verified by approximate methods and procedures.

### 3. Results and discussion

The proposed mathematical models of film evaporators are aimed at engineering prediction and selection of required equipment. The applied equations are based on semi-empirical criterion dependencies of hydrodynamic and thermal similarity [5-7].

The following should be determined in formulation of the simulation problem for vacuum evaporator with MVR:

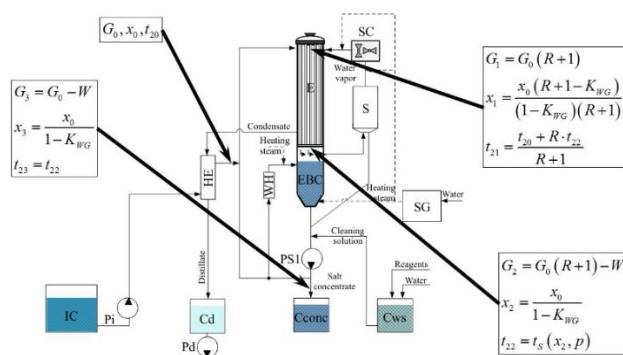
- the solution recirculation rate which should be maintained in evaporator;
- the temperature distribution in flows and temperature head (driving force) in evaporator;
- the water concentration rate which should be maintained in evaporator in order to provide steady hydrodynamic mode of film draining;
- the expected heat loss with consideration for applied thermal insulation in evaporator;
- the range of heat transfer coefficient in evaporator, including accounting for contamination of heat exchange surfaces in evaporator.

On the basis of the formulation of the problem, the developed mathematical model should be comprised of engineering prediction of evaporator, since the calculations are interrelated with the facility layout. Engineering prediction in general consists of a set of calculations directly interrelated with the type, features, and operation variables of evaporation.

Mathematical formulation of the simulation problem was comprised of engineering prediction performed consecutively in 5 stages:

- 1) the material prediction of evaporator;
- 2) the estimation of evaporator temperature mode;
- 3) the hydrodynamic prediction;
- 4) the thermal prediction of evaporator;
- 5) the estimation of heat transfer coefficient.

At the first stage the material prediction was obtained interrelated with the flowchart of pilot evaporator (Fig. 3).



$$K = \frac{1}{\frac{\delta_{paint}}{\lambda_{paint}} + \frac{\delta_{ins}}{\lambda_{ins}}}$$

where  $\delta_{paint}$ ,  $\delta_{ins}$  were the thicknesses of paint and insulation, respectively, m;  $\lambda_{paint}$ ,  $\lambda_{ins}$  were the thermal conductance of paint and insulation, respectively, W/(m·K).

Heat loss through evaporator external surface was, W:

$$Q_1 = q_1 \cdot S$$

where  $q_1$  was the specific heat flow through evaporator external surface, W/(m<sup>2</sup>·K);  $S$  was the external surface area of evaporator external, m<sup>2</sup>.

Heat consumption with evaporated solution at outlet of the boiling chamber (EBC) was determined as follows, W:

$$Q_{22} = [G_0 \cdot (R+1) - W] \cdot c_s(x_2, t_{22}) \cdot t_{22}$$

Heat loss with output flow of evaporated solution was, W:

$$Q_{23} = G_3 \cdot c_s(x_3, t_{23}) \cdot t_{23}$$

Heat consumption in recirculation circuit was, W:

$$Q_{24} = Q_{22} - Q_{23}$$

Heat consumption with distillate vapors at outlet of the evaporator (E) was, W:

$$Q_{12} = W \cdot h_{sw}(t_{22}, P)$$

where  $h_{sw}$  was the vapor enthalpy as a function of temperature and pressure, J/kg.

Heat consumption with distillate vapors after passing through the separator (S) and the gas blower was, W:

$$Q_{11} = W \cdot h_{sw}(t_{11}, P_0)$$

Cumulative heat loss to ambient environment was, W:

$$\Delta Q = Q_{14} + Q_{23} - Q_0 + Q_1$$

Cumulative heat consumption in the evaporator (E) for heating and evaporation of draining film and for compensation of loss into ambient environment was, W:

$$Q = W \cdot \left[ r_{sw}(t_{22}) + \left( \frac{R+1}{K_{wg}} - 1 \right) \cdot c_s(x_2, t_{22}) \cdot (t_{22} - t_{21}) \right] + \Delta Q$$

Compensation of heat loss in the water heater (WH) was, W:

$$Q_{25} = Q - (Q_{11} - Q_{13})$$

Coefficient of efficiency of the apparatus was:  $\eta_0 = \frac{Q - \Delta Q}{Q}$

Operation heat load was, W/m<sup>2</sup>:  $q = \frac{Q}{F}$

Specific energy consumption for evaporation of 1 kg of water in steady direct flow mode was, J/kg:  $w_Q = \frac{Q_{25}}{W}$

At the 5th stage the coefficient of heat transfer from heated wall to liquid film was estimated, W/(m<sup>2</sup>·K):

$$\alpha_2 = \begin{cases} \frac{Re_2^{0.2} \cdot Pr_2 \cdot \lambda_s(x_2, t_{avg2})}{\left( 5 \cdot Pr_2 + 2,9 \cdot Re_2^{0.2} \cdot Pr_2^{\frac{1}{3}} \right) \cdot \sqrt[3]{\frac{V_s(x_2, t_{avg2})^2}{g}}}, & 60 < Re < 500 \\ \frac{0,023 \cdot \sqrt{4 \cdot Re_2} \cdot \sqrt{Pr_2} \cdot \lambda_s(x_2, t_{avg2})}{\sqrt[3]{\frac{V_s(x_2, t_{avg2})^2}{g}}}, & Re > 500 \end{cases}$$

$$\text{where } Pr_2 = \frac{c_s(x_2, t_{avg2}) \cdot \mu_s(x_2, t_{avg2})}{\lambda_s(x_2, t_{avg2})}$$

The utilization rate of ultimate heat load was determined as

$$w_T = \frac{q}{q_T}$$

$$\text{where } q_T = \left[ 0,75 \cdot A_1 + \left( 0,56 \cdot A_1^2 + 130 \cdot A_2 \right)^{0.5} \right]^2$$

$$A_1 = \alpha_2 \cdot \sqrt{\frac{\sigma_s(x_2, t_{avg2}) \cdot t_{22}}{\rho_{sw}(t_{22}, P) \cdot r_{sw}(t_{22}) \cdot \lambda_s(x_2, t_{avg2})}}$$

$$A_2 = \begin{cases} \frac{\alpha_2 \cdot \rho_{sw}(t_{22}, P) \cdot r_{sw}(t_{22})}{c_s(x_2, t_{avg2}) \cdot \rho_s(x_2, t_{avg2}) \cdot Re_2^{0.4}}, & 60 < Re < 500 \\ \frac{2,6 \cdot \alpha_2 \cdot \rho_{sw}(t_{22}, P) \cdot r_{sw}(t_{22})}{c_s(x_2, t_{avg2}) \cdot \rho_s(x_2, t_{avg2}) \cdot Re_2^{0.55}}, & Re \geq 500 \end{cases}$$

where  $r_{sw}$  was the specific heat of vapor generation as a function of temperature, J/kg;  $t_{avg}$  was the average temperature of the second heat transfer fluid (solution) in evaporator, °C.

Estimated coefficient of heat transfer from condensing vapor to wall was, W/(m<sup>2</sup>·K):  $\alpha_1 = 2,04 \cdot \sqrt{\frac{\lambda_w(t_K)^3 \cdot \rho_w(t_K)^2 \cdot r_{sw}(t_K)}{\mu_w(t_K) \cdot l \cdot (t_K - t_{wall})}}$

where  $t_K = t_w(P_0)$  was the condensation temperature of water

vapors;  $t_{wall} = t_{avg2} + \frac{q}{\alpha_2} + \frac{q \cdot \delta_{wall}}{\lambda_{wall}}$  was the wall average temperature, °C.

Heat transfer coefficient in the evaporator (E) with noncontaminated heat exchange surfaces was, W/(m<sup>2</sup>·K):

$$K_{max} = \frac{1}{d_{avg} \cdot \left( \frac{1}{\alpha_1 \cdot d_1} + \frac{1}{2 \cdot \lambda_{wall}} \cdot \left( \frac{d_1}{d_2} \right) + \frac{1}{\alpha_2 \cdot d_2} \right)}$$

Heat transfer coefficient in the evaporator (E) with contaminated heat exchange surfaces was, W/(m<sup>2</sup>·K):

$$K_{max} = \frac{1}{d_{avg} \cdot \left( \frac{1}{\alpha_1 \cdot d_1} + \frac{1}{2 \cdot \lambda_{wall}} \cdot \left( \frac{d_1}{d_2} \right) + \frac{1}{\alpha_2 \cdot d_2} \right) + R_1 + R_2}$$

where  $d_{avg} = \frac{d_1 + d_2}{2}$  was the average tube diameter, m;  $R_1$  was

the thermal resistance of contaminations at external surface of tube walls upon contact with pure water vapor, m<sup>2</sup>·K/W;  $R_2$  was the thermal resistance of contaminations at external surface of tube walls upon contact with evaporated solution, m<sup>2</sup>·K/W.

Output data of the mathematical model were comprised of 31 parameters classified according to 4 categories: specifications and elements of evaporation material balance; properties of evaporation hydrodynamic mode; properties of evaporation thermal mode; calculated parameters determining efficiency of heat transfer and evaporation.

Adequacy of the evaporator model was verified for the case of 2% solution of sodium chloride evaporated to 15 wt % at distillate capacity of 30 kg/h; analysis was performed with regard to 12 selected criteria characterizing evaporation efficiency (Table 1).

**Table 1:** Verification of adequacy of mathematical model

No.	Criterion	Units of measurement	Adequacy condition	Calculations according to the mathematical model
1	Solution boiling point upon rarefaction in system	°C	< 100	93.4
2	Capacity in terms of initial flow	kg/h	≤ 40	34.6
3	Capacity in terms of salt concentrated product	kg/h	≤ 10	4.6

4	Heat consumption of solution heating in recovering heat exchanger	kW	$\leq 2.3$	2.2
5	Operation heat load in evaporator	$\text{kW/m}^2$	$\leq 40$	14
6	Utilization rate of ultimate heat load in evaporator	%	$\leq 100$	56.1
7	Power consumption for evaporation	kW	$\leq 3.0$	1.9
8	Specific energy consumption for evaporation of 1 kg of water	$\text{kW}\cdot\text{h/kg}$	0.04–0.06	0.06
9	Temperature head	$^{\circ}\text{C}$	$\leq 10$	5.2
10	Coefficient of heat release from condensing vapor to wall (film condensation of water vapors)	$\text{W}/(\text{m}^2\cdot\text{K})$	5,000–15,000	8,959
11	Coefficient of heat release from heated wall to fluid film (boiling of water and aqueous solutions)	$\text{W}/(\text{m}^2\cdot\text{K})$	1,000–10,000	6,333
12	Heat transfer coefficient (from condensing vapor to water)	$\text{W}/(\text{m}^2\cdot\text{K})$	800–3,500	1,790–3,137

The simulation results demonstrate that the mathematical model of evaporation using shell-and-tube vacuum film evaporator with free draining film and MVR is relevant and in the presented form provides qualitative description of evaporation according to all 12 criteria, which makes it possible to apply this model for estimation of evaporation efficiency.

The obtained mathematical model has constraints: it cannot be used for analysis of concentrating of nonaqueous solutions; it does not account for solidification of substances in highly concentrated solutions; it cannot predict evaporation modes described as bubble boiling; it cannot be applied for analysis of nonstationary (unsteady) evaporation modes.

The mathematical model is characterized by productiveness, since it provides possibility to obtain reliable initial data where all thermal physical properties of water, vapor, and evaporated aqueous solution are predefined as analytical functions of temperature and pressure.

#### 4. Conclusion

Evaporator efficiency is mainly determined by optimum selection of recirculation rate in film apparatus which provides: a) achievement of high concentrating rate of solutions; b) decrease in specific energy consumption for evaporation; c) decrease in heating surface incrustation. According to the simulation data, the optimum recirculation rate of sodium chloride solution is 26.18.

The developed model can be applied in practice to study the properties and features of behavior of vacuum evaporator with MVR in stationary mode at various combinations of initial data as a tool of numerical experiments aiming at development of

optimization models, and as a tool of prediction and estimation of evaporation efficiency under industrial conditions.

#### Acknowledgments

This work was supported by the Ministry of Education and Science of the Russian Federation (Agreement No. 75/Б/4526/15/П218 dated July 10, 2015).

#### References

- [1] Povorov AA, Pavlova VF, Kornilova NV, Shinenkova NA (2015), Isparitel'nye apparaty s rekompressiей vodyanogo para – energosberegayushchaya tekhnologiya i oborudovanie (ESVA) [Evaporators with water vapor recompression – power saving technology and equipment (ESVA)]. Mir gal'vaniki, 2(30), 24–27.
- [2] Trifonova TA, Povorov AA, Shirkin LA, Selivanov OG, Il'ina ME (2015), Kompleksnaya tekhnologiya ochistki fil'tratsionnykh vod poligonov tverdykh bytovykh otkhodov [Integrated purification of filtration waters of domestic solid wastes landfills]. Ekologiya i promyshlennost' Rossii, 19(11), 4–9.
- [3] Trifonova TA, Roschina SI, Shirkin LA, Selivanov OG, Il'ina ME (2015), An integrated innovative technology for the treatment of municipal solid waste landfill leachate. Biosciences Biotechnology Research Asia, 12(3), 2481–2488.
- [4] Trifonova TA, Shirkin LA, Selivanov OG, Il'ina ME, Podolets AA (2016), Otsenka effektivnosti protsessa kontsentrirovaniya vodnykh rastvorov mineral'nykh solei i organicheskikh sred metodom vyparivaniya v rotorno-plenochnom isparitele pri ochistke prirodnnykh i stochnykh vod [Efficiency estimation of concentrating of aqueous solutions of mineral salts and organic mediums by evaporation in rotating film evaporator upon purification of natural and effluent waters]. Fundamental'nye issledovaniya, 9, 511–515.
- [5] Lezhneva NV, Elizarov VI, Getman VV (2011), Modelirovaniye i diagnostika protsesssa distillyatsii v rotorno-plenochnom isparitele s sharnirno zakreplennymi lopastyami s tsel'yu povysheniya ego effektivnosti [Simulation and diagnostics of distillation in rotating film evaporator with pivotally mounted blades aiming at efficiency improvement]. Vestnik Kazanskogo tekhnologicheskogo universiteta, 18, 185–192.
- [6] Ronkin VM (2012), Opredelenie ratsional'noi konstruktsii isparitel'nogo apparausa s padayushchey plenkoi s tsel'yu uvelicheniya effektivnosti raboty [Determination of rational evaporator design with free draining film aiming at efficiency improvement]. Sfera. Neft' i Gaz, 2, 90–92.
- [7] Tokarev SM (2013), Matematicheskoe modelirovaniye termicheskoi distillyatsii vod pri plenochnom techenii v vakuum [Mathematical simulation of water thermal distillation upon film flow in vacuum]. Komp'yuternye issledovaniya i modelirovaniye, 5(2), 205–211.
- [8] Udyoma PG, (1985) Plenochnye ispariteli [Film evaporators]. Moscow, Russia: Moscow Energy Institute.
- [9] Protsessy i apparaty khimicheskikh tekhnologii [Processes and apparatuses of chemical technologies. Reference book of chemist and technologist. Part II]. (2006). St. Petersburg, Russia: NPO Professional.
- [10] Petrenko VP, Ryabchuk AN (2014), Rezhimy ukhudshennoi teplootdachi v plenochnykh isparitel'nykh apparatakh [Degraded heat exchange in film evaporators]. Sakhhar, 8, 40–47.
- [11] Filippov VV (2014), Teploobmen v khimicheskoi tekhnologii. Teoriya. Osnovy proektirovaniya [Heat exchange in chemical industry. Theory. Designing backgrounds]. Samara, Russia: Samara State Technical University.