



# Numerical Experiment for the Determination of the Stress-Strain Condition of the System “Basis – Vibroreinforced Soil-Cement Pile”

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## Abstract

The order of model creation and results of numerical finite element analysis on electronic computers of models of the vibroreinforced soil-cement piles is considered. The pile in the basis (soil) is in volumetric load-bearing conditions, therefore, among the features of creating such a model, the order of application of the load system and the laying of fastenings on its surface is described. Full-scale tests for vertical static load of investigated constructions have been carried out, as a result of which the vibroreinforced soil-cement pile has been destroyed. Surveys and investigations of the type and destruction mechanics were carried out. Revealed places that need to be strengthened. The results of numerical simulation are compared with the results of field tests. The analysis of the mechanics of destruction of the vibroreinforced soil-cement pile is carried out.

**Keywords:** *deep soil mixing method, soil-cement, vibroreinforced soil-cement pile, finite-element model, numerical experiment.*

## 1. Introduction

The development of modern construction of residential and public buildings, as well as artificial structures, aimed at increasing their height, span and cargo capacity. This results in increased load on the basis and foundation. In such conditions more rational use acquires pile foundations. More and more, the technologies of strengthening the soil of the foundation with cement, in particular the creation of soil-cement elements and piles with deep soil mixing technology, are becoming increasingly popular [4, 15-17].

Using numerical simulation of the finite elements of the stress-strain condition (SSC) of building structures and their individual elements, it is possible to reveal the nature of their destruction or the form of stability's loss. In the course of parallel numerical simulation and experimental study of central compressed full-scale vibroreinforced soil-cement piles, it is possible, firstly, to identify hazardous places of concentration of stresses on their surface (ie, the places of destruction) and, secondly, to study in more detail the work of reinforcing rods, invisible inside the soil-cement. [2]

## 2. Analysis of recent research and publications

The results of research on building constructions with the use of computer programs based on the finite element method (FEM) [1], especially taking into account the complete diagrams of the work of materials, are increasingly covered in scientific literature. In particular [7], a simplified modeling of nonlinear structure analysis was performed on the basis of Coiebert-Newton's method. A slight simplification reduces the numerical calculation and practi-

cally does not reduce the accuracy of the calculation results. Computer modeling allows us to conduct detailed research of rod elements in composite structures [5], as well as perform a complete analysis of three-tensioned stress-strain state of constructions [6, 15-25].

The interaction of the vibroreinforced soil-cement piles with a basis, arranged in different types of soil is still insufficient [3], so it is impossible to correctly set all the necessary data for calculation in geotechnical complexes (for example, PLAXIS). In the analyzed published scientific works, the important issues described above, which may be combined during the development of the principles of creating a numerical finite-element model of the vibroreinforced soil-cement piles, located in volumetric variables along the length of the structure under extremely loaded conditions, are disclosed. [15-16, 23]

## 3. Formulating the goals of the article

To improve the design scheme of the system "basis - vibroreinforced soil-cement pile" using the finite element method and the elastic model of the system for carrying out a numerical experiment with further evaluation of the stress-strain condition and comparison with the results by field tests. The modeling of the piles is decided to perform as a construction, and the impact of the surrounding soil to set the effective forces of the side pressure and friction resistance on the lateral surface.

#### 4. The main research material

Using numerical simulation of the finite elements of the stress-strain condition (SSC) of building structures and their individual elements, it is possible to reveal the nature of their destruction or the form of stability's loss. Thus, during the numerical simulation of identical experimentally investigated centrally-compressed full-scale the vibroreinforced soil-cement pile, firstly, there were found dangerous places of stress concentration on their surface (ie, the places of destruction) and, secondly, in more detail the work of reinforcing rods was investigated, invisible inside the soil-cement. [2, 8, 9]

Numerical modeling of the SSC was performed on NASTRAN (NAsa STRuctural ANalysis) Femap 10.1.1 SC 32bit / 64 bit. The training demo version of SDRc-FEMAP 8 / 1a S / N 000-00-00-DEMO-406F-00000000 was used. During the research, all the prerequisites of numerical simulation were met, and some assumptions were introduced that, according to the authors of the work, did not give a significant error on the results of the simulation. In particular, the physical and mechanical characteristics of steel and soil-cement were given as for completely homogeneous and isotropic materials. Also, an insignificant correction of the diagrams of their deformation was carried out due to the limitations of the used software complex (the seventh modulus of elasticity of any material cannot be greater than the initial elastic modulus). The actual weight of model designs is not taken into account due to its smallness in comparison with the external load.

Creation of a finite-element model, adequate to the full-scale the vibroreinforced soil-cement piles for the determination of their SSC, and analysis of these models were carried out in the order shown in table 1. Nearly, consider in more detail the individual stages of finite element analysis. [23]

**Table 1:** The order of finite element analysis of models of the vibroreinforced soil-cement piles

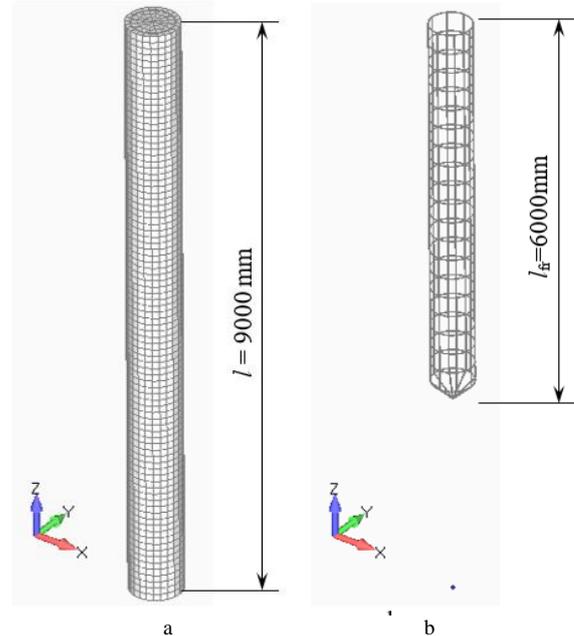
The order of conducting researches	The name of the stage	What is done
1	Creating model geometry	Creation of a spatial volumetric model in the Cartesian coordinate system, taking into account the presence of reinforcing rods
2	Introduction of physical and mechanical properties of materials	Specifying the values of the elastic modulus, the coefficient of transverse deformation $\nu$ and the deformation law ( $\sigma$ - $\epsilon$ ) according to the conducted field experiment
3	Choosing Finite Element Type (FE) and splitting the model into the FE	Completion of the created volumetric model of the FE type hexahedron, the size of which depends on the time of creation of a volumetric FE grid, the required disk space PC, accuracy and convergence of results
4	Setting of boundary conditions and formation of load system	The choice of the planes and the application of supporting reinforcements to them, as well as loads (vertical useful, forces of bending of the soil and forces of lateral friction) with indication of their magnitude
5	Verify the correctness of model	Quality control, symmetry, and the number of combinations of finite-element lattice-matching nodes
6	Choosing Finite Element Type of analyzing	Selection of parameters and conducting of nonlinear analysis taking into account given diagrams of materials
7	Analysis of the data	Formation of calculation results for practical use: schedules of distribution of stresses and deformations

In order to achieve the results of the calculation that would best match the results of a field experiment, full deformation diagrams of the source materials are not sufficient. The most accurately describe the working conditions of the pile in real conditions - the

conditions of compression of engineering-geological layers. That is, make an equal parity of the calculation scheme of the piles work. Firstly, it is necessary to set identical real conditions of fastening (boundary conditions) of the piles and, secondly, to form the appropriate system of loads, both useful and the loads of the surrounding soil compression - stage 4 of the order of finite element analysis (see. Fig. 1). [10]

Boundary conditions (Constraint Definition) were given uniformly-distributed to the upper and lower crop edges. Fixed was applied to the lower edge of the pile: it was forbidden to move translationally across all three axes (TX, TY and TZ) and angular displacements along the same axes (RX, RY and RZ), since the lower end of the pile was fixed in thicker soil. The upper trimming was pinned in a plane of the cross-section (Pinned - No Translation): prohibited movements in the TX and TY axes, as the upper end of the pile was secured only by the grillage, through which the payload was transmitted.

In figure 1, a ready-made model of the vibroreinforced soil-cement pile and a reinforcing frame is given.



**Fig. 1:** Model of a the vibroreinforced soil-cement pile (a) and a reinforcing frame (b), broken into finite elements of solid type in the form of hexahedrons Hex Mesh

Load on the vibroreinforced soil-cement pile models was applied as static evenly distributed over the corresponding areas and was divided into three groups: useful cargo, load of the lateral bending of the soil and lateral friction forces that appeared between the lateral surfaces of the pile and the soil during its compression. All these load groups and their application planes are shown in Fig 2. The value of the maximum payload was taken equal to the maximum load that was applied to the pile during a field experiment on a construction site - 105 tons.

Side load of compression of the pile by the soil and load of lateral friction is determined by calculation according to the conducted engineering-geological surveys on the construction site of conducting a full-scale experiment piles. Since the pile was divided in height by parts 300 mm high (for the purpose of arrangement of transverse reinforcing frame) and by a circle on 8 segments (for the arrangement of vertical working reinforcing bars), then the averaged values of forces with uniform distribution on these sites were asked. That is, the size of the sites to which the lateral loading was applied was at a height of 300 mm and a width of 295 mm (1/8 perimeter of the palace). Figure 3 shows the distributions of the lateral compression of the pile by the soil and the load of lateral friction.

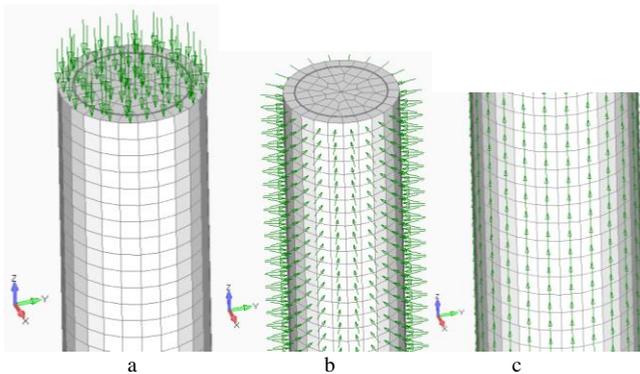


Fig. 2: Schemes of loads to the model of the pile: a) vertical useful to the pile head; b) lateral compression by the soil; c) the load of lateral friction

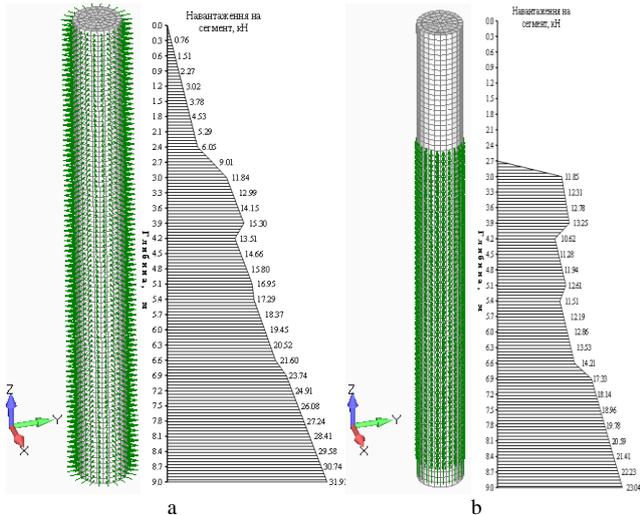


Fig. 3: The distribution of the height of the piles a) the side compression of the pile by the soil; b) frictional forces between the lateral surfaces of the pile and the soil during its compression

After creating a volumetric finite element model of the soil-cement pile, the verification of the correctness of the model was carried out - stage 5 of the order of the finite element analysis (see Table 1), namely the removal of superfluous boundary surfaces through which there could be irreparable errors in the calculations. It also controlled the quality and symmetry of the model's break-down into finite elements, the number of combinations of finite-element grid, the type of boundary conditions, the magnitudes of applied loads, and the surface of their propagation. [11]

To perform a numerical experiment, it was decided to use soil conditions and configuration of the piles exposed to static tests on the construction site. In particular, a pile was used which was tested by an exerting load during the construction of the building 8 on Esplanade district in Sumy, Ukraine. [13, 14, 12]

According to geological surveys on the site 17 engineering-geological elements have been identified within which the thickness is statically uniform in composition and properties.

The palace is located in 6 main engineering-geological elements: EGE Id - Alluvial Sands: dusty, brownish gray, gray, homogeneous, malachite for saturated moisture, medium density.

EGE II - Plant layer - loam with clay layers, dark gray, semi-solid, with layers of plant residues.

EGE IV - Clays black, dark gray, fluid-plasticized, with inclusions of plant residues, with peat layers.

EGE VI - Sands are dusty, gray, dark gray, homogeneous, saturated with damp, with layers of sandy loam, dense.

EGE VIa - Sands are dusty, gray, dark gray, homogeneous, moist to saturated with moisture, medium density.

EGE VIIa - Sands are shallow, gray, dark gray, homogeneous, saturated with water, with layers of sandy loam, dust of dust, rarely with the inclusion of organic substances.

EGE VIII - Sands of medium size, gray, greenish-gray, non-uniform, saturated with water, rarely with layers of sandstone, with inclusions of fragments of flasks up to 20%, dense. The position of the pile in the soil mass is shown in the engineering-geological section (Fig. 4)

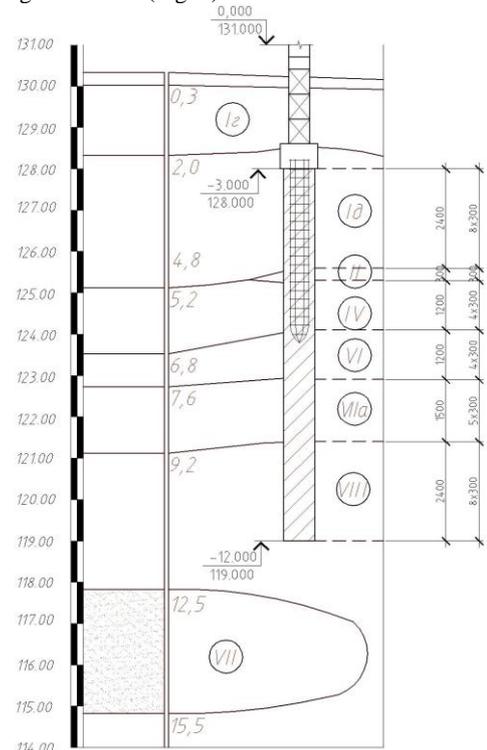


Fig. 4: The position of the vibroreinforced soil-cement pile on the engineering-geological section and its division into high-altitude sections

Table 2 shows the results of calculating the frictional force at the lateral surface of the pile, depending on the depth and type of soil. The calculation was carried out according to the following formula: [3, 8]

$$N_1 = \left( \sigma_{zg} \cdot \frac{v}{1-v} \right) \cdot A, \tag{1}$$

where  $N_1$  – is the calculated force. which is assigned to the area of the section of the pile, kN;

$\sigma_{zg}$  – value of pressure from the soil's own weight, kN;

$v$  – Poisson coefficient;

$A$  – the magnitude of the area of the segment to which the force is applied,  $m^2$ .

Table 3 shows the results of calculating the frictional force at the lateral surface of the pile, depending on the depth and type of soil. The calculation was carried out according to the following formula: [3, 8]

$$N_2 = \left( \left( \sigma_{zg} \cdot \frac{v}{1-v} \right) \cdot tg\varphi_{II} + C_{II} \right) \cdot A, \tag{2}$$

where  $N_2$  – is the calculated force, which is assigned to the area of piles section, kN;

$\sigma_{zg}$  – value of pressure from the soil's own weight, kN;

$v$  – Poisson coefficient;

$tg\varphi_{II}$  – the angle of internal friction;

$C_{II}$  – specific gravity of soil;

$A$  – the magnitude of the area of the segment to which the force is applied,  $m^2$ .

During field research, a series of tests of the vibroreinforced soil-cement piles by static compression was performed. Static tests of the vibrated reinforced cement piles were carried out on the construction site of a 10-storey residential building on the Esplanade

district, PrJSC "Sumbud" in Sumy. In general, the test consisted of four piles: № 1 - a soil-cement pile without reinforcement and №№ 2, 3 and 4 the vibroreinforced soil-cement pile. All piles had the following dimensions: length 9 m, diameter 750 mm. [23]

**Table 2:** Determination of the lateral pressure forces on the trunk of the pile (N1)

EGE	Depth, m	$\sigma_{zg}$ , kPa	$\xi$	$N_1$ , MN
Id	0.00	0.000	0.429	0.000
	0.30	2.496		0.00076
	0.60	4.992		0.00151
	0.90	7.488		0.00227
	1.20	9.984		0.00302
	1.50	12.480		0.00378
	1.80	14.976		0.00453
	2.10	17.472		0.00529
II	2.40	19.968	0.563	0.00605
	2.70	22.682		0.00901
IV	3.00	25.134	0.667	0.01184
	3.30	27.587		0.01299
	3.60	30.039		0.01415
	3.90	32.492		0.01530
VI	4.20	35.508	0.538	0.01351
	4.50	38.525		0.01466
	4.80	41.541		0.01580
VIIa	5.10	44.558	0.515	0.01695
	5.40	47.516		0.01729
	5.70	50.474		0.01837
	6.00	53.432		0.01945
	6.30	56.390		0.02052
VIII	6.60	59.348	0.538	0.02160
	6.90	62.414		0.02374
	7.20	65.480		0.02491
	7.50	68.546		0.02608
	7.80	71.612		0.02724
	8.10	74.678		0.02841
	8.40	77.744		0.02958
	8.70	80.810		0.03074
	9.00	83.876	0.03191	

**Table 3:** Determination of frictional forces on the lateral surface of the trunk of the pile

EGE	Depth, m	$f$ , kPa	$N_2$ , MN
Id	0.30	-	0.00
	0.60		0.00
	0.90		0.00
	1.20		0.00
	1.50		0.00
	1.80		0.00
	2.10		0.00
II	2.40	-	0.00
	2.70		0.00
IV	3.00	16.76942	0.01185
	3.30	17.42996	0.01231
	3.60	18.09050	0.01278
	3.90	18.75104	0.01325
VI	4.20	15.03206	0.01062
	4.50	15.96927	0.01128
	4.80	16.90647	0.01194
	5.10	17.84367	0.01261
VIIa	5.40	16.29855	0.01151
	5.70	17.25094	0.01219
	6.00	18.20332	0.01286
	6.30	19.15571	0.01353
	6.60	20.10810	0.01421
VIII	6.90	24.52509	0.01733
	7.20	25.68073	0.01814
	7.50	26.83638	0.01896
	7.80	27.99203	0.01978
	8.10	29.14767	0.02059
	8.40	30.30332	0.02141
	8.70	31.45897	0.02223
	9.00	32.61461	0.02304

According to the results of the tests for the period of hardening of 28 days, the bearing capacity of the pile number 1 was 65 tons, there was a collapse of piles on the material. Taking into account that the pile's base relies on small sands, the water-bearing capacity of the piles on the material is much smaller than the estimated bearing capacity of the pile on soil, which is 1368 kN.

The destruction of the material of the trunk of the pile is shown in Figure 5. Selected samples of soil-cement during tests on the press showed an average compressive strength of 1.48 MPa.



**Fig. 5:** Destruction of the unreinforced the vibroreinforced soil-cement pile on the material.

The piles were made by the RDK-250 drilling rig, immediately after the installation with the help of a deep high-frequency vibrator, a vibration dipping of the frame was carried out to a depth of weak soils - 6 m. The reinforcement of piles with a frame of 8 reinforcing bars A400s Ø12 mm, reinforced with reinforcement rings A240s Ø8 mm, was designed.

Under the pile's base there was sand of medium size, dense, saturated with water, the estimated bearing capacity of the pile on the soil was 1491 kN

The scheme and test equipment are shown in Figure 6. The load was transmitted through a cargo beam of two reinforced twin axes 60S2 by means of two jacks. Two anchor cement piles, reinforced for a length of 9 m, a reinforcement frame with 8 rods A400s Ø22 mm, were used for the main purpose.

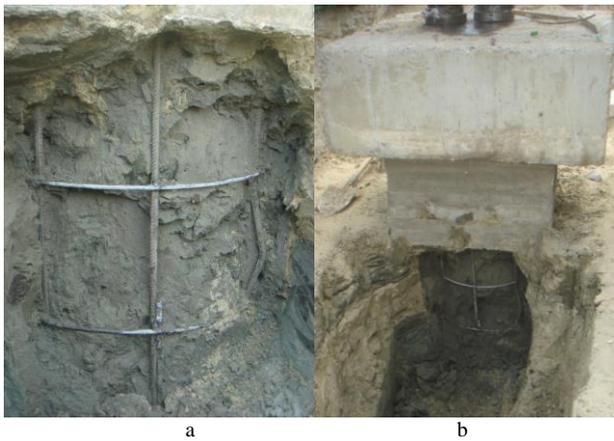


**Fig. 6:** Static pressure bench testing by means of jacks, cargo beam and two anchor piles.

According to the results of the tests, the pile of VRSCP №2 the term of 28 days was destroyed at a load of 85 t. At the same time, the total settling to the load of 65 tons was 1 mm.

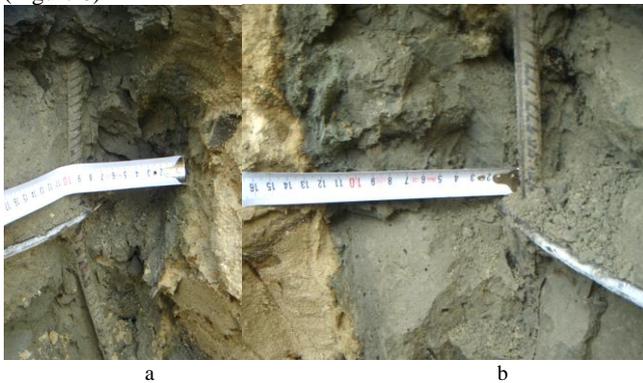
When excavating the piles, it was determined that the destruction was due to the local loss of stability of one of the reinforcing rods and the destruction of the protective layer of soil-cement. The destruction occurred on the material of the piles, at a depth of 45 cm. Examination of the tested piles is shown in Figure 7.

The test of the sample of samples of soil-cement showed an average compressive strength of 1 MPa.



**Fig. 7:** Reinforcement frame cleared from the protective layer of vibro-reinforced soil-cement pile after the test: a - study of the destruction of the frame after the test; б - the general view of piles at digging out.

Measurement of the protective layer of the soil-cement showed it was kept and uniform throughout the circumference of the pile. (Figure 8)



**Fig. 8:** Protective layer of soil-cement: a - from the side of destruction 10 cm; б - 12 cm perimeter

The vibroreinforced soil-cement pile VRSCP №2 and VRSCP №3 were tested with the term of 56 days.

Pile № 3 destruction on the material at a load of 105 tons (Fig. 9). Total settling up to 105 t was 8.33 mm. The destruction was based on material at a depth of 47 cm. The average strength of soil-cement samples taken from the vibroreinforced soil-cement pile number 3 was 1.7 MPa.

Figure 10 shows the reinforcing frame of the soil-cement pile №3, cleared from the protective layer of soil-cement, 10 cm in thickness.



**Fig. 9.** Investigation of the destruction vibroreinforced soil-cement pile №3

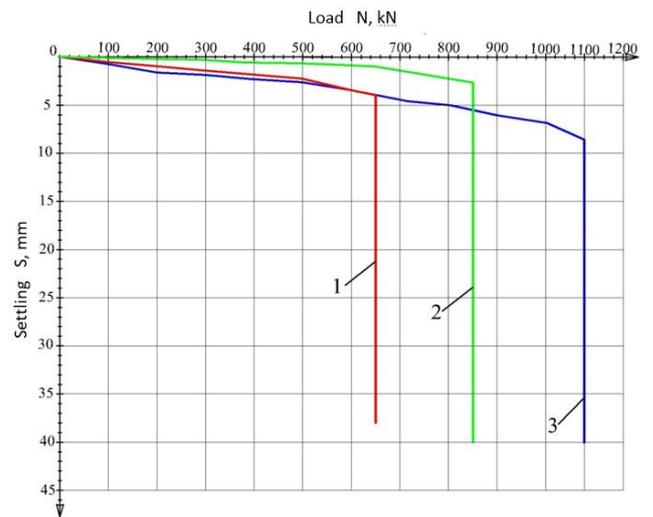
At a depth of 47 cm from the level of the pile's head, there was a loss of stability of the 3 rods of the reinforcing frame and the fragmentation of the part of the protective layer of the soil-cement, which also led to the destruction of the soil-cement pile. (Fig. 10)



**Fig. 10:** Loss of local resistance of three reinforcing bars of a frame during static testing of vibrated reinforced cement pile №3

Pile No.4 withstood a load of 110 tons at a total settling of 8.22 mm, the tests were suspended.

In fig. 11 shows a graph of settling of unreinforced soil-cement pile and the vibroreinforced soil-cement pile (VRSCP) №2 and №3.



**Fig. 11:** Charts of dependence of piles settling on vertical static loads: 1 - unreinforced soil-cement pile; 2 - VRSCP №2; 3 - VRSCP №3

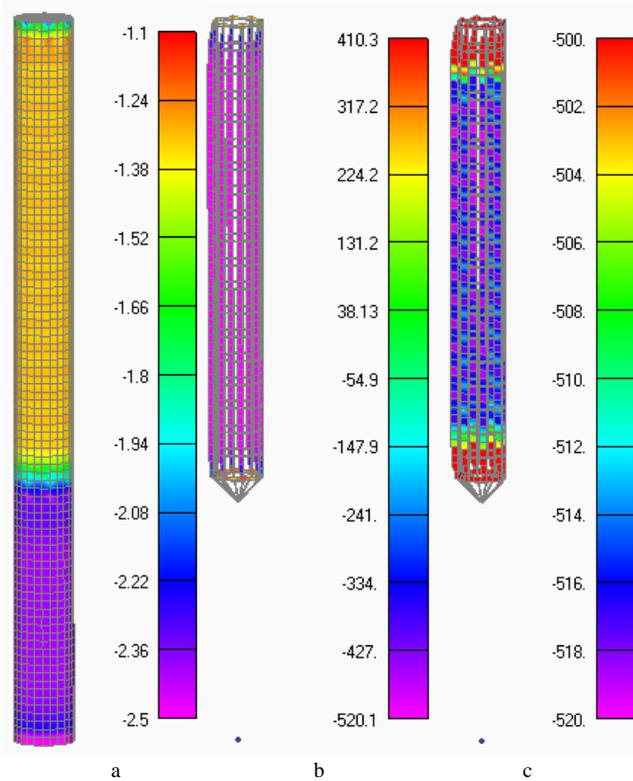
The strength of selected samples of soil-cement showed the reliability of the method for calculating the bearing capacity of a soil-cement pile on the material.

Static tests have shown that vibroreinforced soil-cement piles transfer the load to the bearing layer under the lower end of the pile. At the same time, the problem of resistance to the lateral surface of the pile remains incompletely studied.

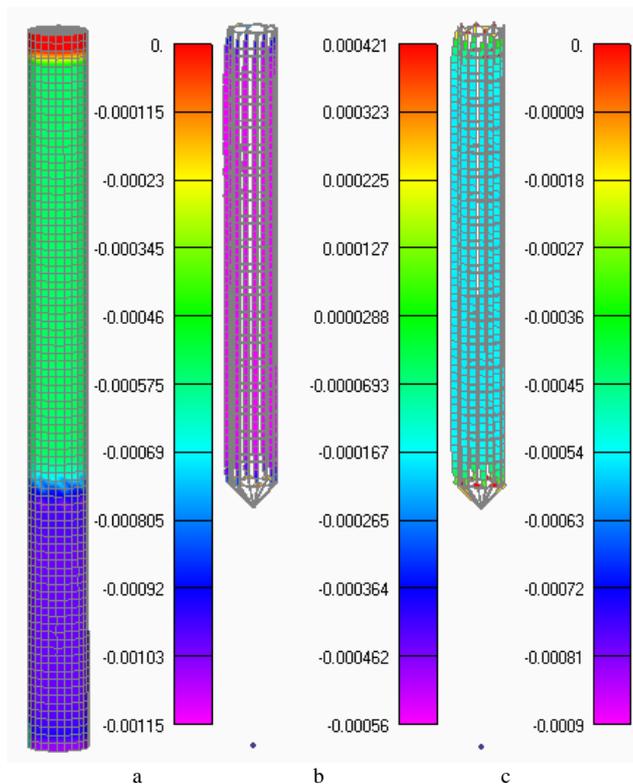
The result of finite-element nonlinear calculations is the diagrams of distribution of stresses and deformations in all three directions of coordinate axes on the surface, in cross sections and between contact finite elements of composite materials. Below are just the most characteristic and special charts.

Figure 12 shows graphs of distribution of stresses normal to the longitudinal axis on the surface of the vibroreinforced soil-cement pile and in the reinforcing bars of the frame at a load of 105 tons on the upper circumference of the pile model. Figure 13 shows the graphs of the distribution of relative longitudinal deformations on the surface of the soil-cement pile and in the reinforcing bars of the frame at a load of 105 tons on the upper circumference of the model of the pile.

Analyzing the graphs shown in Figures 12 and 13, it can be noted that the reinforcing frame and the soil-cement work in concert with the transfer of the payload to the upper trimming of the pile. This is evidenced by the leakage (increase) of stresses in the soil-cement at the level of the bottom of the reinforcing frame (at a distance of 6 m from the top of the pile).



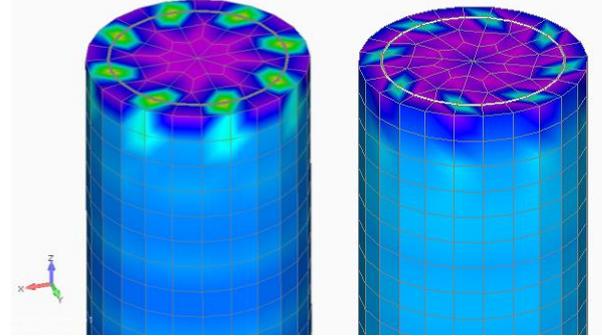
**Fig. 12:** Distribution of stresses normal to the longitudinal axis in MPa on the surface of the vibroreinforced soil-cement pile (a), in the reinforcing bar rods of the frame (b) and separately only the gripping (c)



**Fig. 13:** Distribution of relative longitudinal deformations on the surface of the vibroreinforced soil-cement pile (a) and in the reinforcing bar rods of the general (b) and separately compression (c)

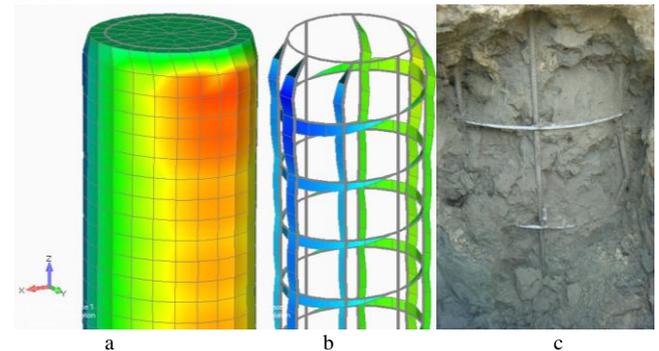
The joint work of reinforcing rods and soil-cement is evidenced by the same deformations at the same level as the height of the piles. Thus, at the main length of the piles (0.5 ... 5.5 m from the top of the pile), the relative deformations on the surface of the soil-cement and reinforcing rods are about  $50 \times 10^{-5}$ .

The inclusion of reinforcing rods in the joint operation of the soil-cement pile is evidenced by the redistribution of efforts in the soil-cement around the reinforcing rods in the section of the pile, as shown in Figure 14.



**Fig. 14:** Redistribution of effort between reinforcing rods and the vibroreinforced soil-cement pile in the section of the pile

The greatest value of the stresses and deformations according to the results of numerical studies was recorded at a distance of 0.2 ... 0.5 m from the top of the pile, as shown in Figure 15. A similar picture was obtained during the actual experiment of testing the static load of the vibroreinforced soil-cement pile at the construction site.



**Fig. 15:** Concentration of stresses near the head of the pile: a) on the surface of the model; b) in reinforcing rods; c) during a full-scale experiment

## 5. Conclusions

1. The accepted design models of vibroreinforced soil-cement pile in the software complex are adequate and correspond to the real tested constructions. When comparing the convergence of the results obtained in calculating the models with various volume finite elements (tetrahedra and hexahedra) with different sizes, it is decided to split the models of the studied piles on the hexade with a side of 100 mm, which is equal to about 1.1% of the total height of the piles.
2. Static tests of non-reinforced and vibroreinforced soil-cement piles have been performed. The results of the tests showed an effective joint operation of the reinforcing frame and soil-cement, which increased the bearing capacity of the vibroreinforced soil-cement pile.
3. Simulation of the stress-strain condition of piles allowed to confirm the results of full-scale tests of the vibroreinforced soil-cement pile on the construction site and allows to simulate the stress-strain condition of the same constructively solved piles with other standard sizes.
4. The results of simulation of the stress-strain condition of the models of piles have good convergence with the results of full-scale tests. The concentration determined by the results of numeri-

cal studies coincides with the place of destruction of experimentally explored piles.

## 6. Acknowledgement

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