International Journal of Engineering & Technology, 7 (4.10) (2018) 892-895

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

An Intuitionistic Fuzzy Graph's Signless Laplacian Energy

Obbu Ramesh[1] , S. Sharief Basha[2]

[1]*PhD Scholar, SAS, Vellore Institute of Technology, Vellore 632 014, Tamilnadu, India*
.[2]Applied Algebra Division, SAS, VIT University, Vellore 632 014, Tamilnadu, India.
Corresponding author E mail: shariefbasha.s@vit.ac.in

Abstract

We are extending concept into the Intuitionistic fuzzy graph' Signless Laplacian energy instead of the Signless Laplacian energy of fuzzy graph. Now we demarcated an Intuitionistic fuzzy graph's Signless adjacency matrix and also an Intuitionistic fuzzy graph's Signless Laplacian energy. Here we find the Signless Laplacian energy 's Intuitionistic fuzzy graphs above and below boundaries of an with suitable examples.

Keywords: *Fuzzy graphs, Intuitionistic Fuzzy graphs, Intuitionistic fuzzy graph 's Energy, Laplacian energy*.

1. Introduction

Formal Modeling, reasoning and computing which of our traditional tools are cnsp, deterministic and precise in character. Limitations of a model characterize exactly either our awareness of the sensation demonstrated or the landscapes of the real system that has been demonstrated are assumes to be Precision. Lotfi. A. Zadeh suggested the mathematical frame work to describe these phenomena in his seminal paper entitled "Fuzzy Sets" [13]. The crisp set is demarcated in such a way as to distinguish the personalities in some creation of dissertation in to two collections: members and non -members, whose logic relies entirely on the classical Aristotlian one, "A or not A".Based on Zadeh's fuzzy relations [13]Kaufmann in 1973 defined the first definition of a fuzzy graph. But in 1975,Azriel Rosenfeld [11] measured fuzzy dealings on fuzzy sets and established the concept of fuzzy graphs.
In this article, we familiarize the concept of an intuitionistic fuzzy graph's Signless Laplacian's energy. Similarly , in Section 2, we comprises of some beginnings and description of Signless Laplacian's energy of fuzzy graph and also in section 3,we existing thean intuitionistic fuzzy graph's Signless Laplacian's energy . The Signless Laplacian energy's upper and lower bounds of a fuzzy intuitionistic graph are also imitativein Section4. Finally We present the decision in last Sector.

2. Intiation

2.a.Signless Laplacian Energy of Fuzzy Graph

Let $\tilde{G}$ (V,,E,,σ,μ) be a p number vertices and qquantity of edges having vertex set V($\tilde{G}$)={$p_1, p_2, \dots \dots p_n$} and also edge set E($\tilde{G}$)={$q_1, q_2, \dots \dots q_n$}. The adjacency matrix A = [a_{ij}] of $\tilde{G}$ is fuzzy square matrix of order n whose (i,j) entry is equal to$\mu(u_i, u_j)$ if i≠j , otherwise zero.

Let the diagonal matrix is D ($\tilde{G}$) = diag ($d_1, d_2, \dots \dots d_n$) and associated to G where $d_i = \deg(u_i)$ for all i=1, 2, 3 ...n. The matrices $L(\tilde{G}) = D(\tilde{G}) - A(\tilde{G})$ and $Q(\tilde{G}) = D(\tilde{G}) + A(\tilde{G})$ are said to be Laplacian and Signless Laplacian fuzzy spectrum (Q-spectrum) respectively of the real symmetric and positive semi-definite. Now we let 0 = $\mu_n \le \mu_{n-1} \le \cdots . \le \mu_1$ and 0 =$q_n \le q_{n-1} \le \cdots \le q_1$ to be the L-spectrum and Q-spectrum of a fuzzy graph $\tilde{G}$ respectively.

3. Signless Laplacian Intuitionistic Fuzzy Graph's Energy.

In third sector, we are presenting the definition of a fuzzy Intuitionistic graph' s Signless Laplacian's energy where the graph is no loops .

Definition 3.1:-let Intuitionistic fuzzy graph is G(V,E, μ, δ) with vertices set 'V', and edges set 'E' and membership function is 'μ'' of fuzzy system defined on V×V, 'γ'' is a non- membership function of fuzzy system , then we define $\tilde{\mu}(v_i, v_j)$ by $\tilde{\mu}_{ij}$ and $\tilde{\gamma}_{ij}$ in $\tilde{\gamma}(v_i, v_j)$ then

$$(i) 0 \le \tilde{\mu}_{ij} + \tilde{\gamma}_{ij} \le 1$$

$$(ii) 0 \le \tilde{\mu}_{ij}, \quad \tilde{\gamma}_{ij}, \pi_{ij} \le 1 \ \ where\ \pi_{ij} = 1 - \tilde{\mu}_{ij} - \tilde{\gamma}_{ij}$$

Example 3.2:-

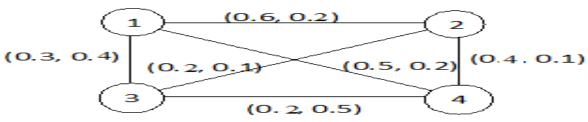

Figure $\tilde{G}_3$:

Definition 3.3 :- A Fuzzy intuitionistic graph $\tilde{G}$=(V,E,μ,γ) with an matrix adjacency of a fuzzy intuitionistic graph is demarcated by A(IG)=[a_{ij}] where $a_{ij} = [\tilde{\mu}_{ij}, \tilde{\gamma}_{ij}]$ .Reminder that $\tilde{\mu}_{ij}$ denotes the potency of the membership among v_i and v_j and γ_{ij} the strength of non-membership between v_i and v_j .

Example 3.4:-For a Fuzzy intuitionistic graph of figure III, so the matrix adjacency as

$$A = \begin{bmatrix} 0 & (0.6,0.2) & (0.3,0.4) & (0.5,0.2) \\ (0.6,0.2) & 0 & (0.2,0.1) & (0.4,0.1) \\ (0.3,0.4) & (0.2,0.1) & 0 & (0.2,0.5) \\ (0.5,0.2) & (0.4,0.1) & (0.2,0.5) & 0 \end{bmatrix}$$

Definition 3.5:-We use to write the of a Fuzzy intuitionistic graph's adjacency matrix can be inscribed into two matrices, first one involving the passes as relationship values, second one containing non-relationship values i.e. A (I FG) = $\left[\left(\mu_{ij}\right),\left(\gamma_{ij}\right)\right]$, where

$$A(\mu_{ij}) = \begin{bmatrix} 0 & 0.6 & 0.3 & 0.5 \\ 0.6 & 0 & 0.2 & 0.4 \\ 0.3 & 0.2 & 0 & 0.2 \\ 0.5 & 0.4 & 0.2 & 0 \end{bmatrix}$$

$$A(\gamma_{ij}) = \begin{bmatrix} 0 & 0.2 & 0.4 & 0.2 \\ 0.2 & 0 & 0.1 & 0.1 \\ 0.4 & 0.1 & 0 & 0.5 \\ 0.2 & 0.1 & 0.5 & 0 \end{bmatrix}$$

Definition 3.6:-The set (X,Y) is a fuzzy Intuitionistic graph 's adjacency matrix A (IFG)Eigen values ,Where Eigen values set X of A [μ_{ij}] and also the Eigen values set Y of A [γ_{ij}].

Definition3.7:-The matrix $\tilde{Q}(IFG) = D(IFG) + A(IFG)$ is demarcated as Signless Laplacian matrix intuitionistic fuzzy of IG.

In this place, we are interested to write as two matrices where one holding the passes as values of membership function and other holding the values of non- membership function of Signless Laplacian matrix of a Fuzzy intuitionistic graph.

i.e. $Q(IG) = \left[\left(Q\left(\mu_{ij}\right)\right),\left(Q\left(\gamma_{ij}\right)\right)\right]$

Example 3.8:- The values of membership and non-membership of Fuzzy intuitionistic graph's Signless Laplacian matrix $\tilde{G}_3$ is given by

$$Q(\mu_{ij}) = \begin{bmatrix} 1.4 & 0.6 & 0.3 & 0.5 \\ 0.6 & 1.2 & 0.2 & 0.4 \\ 0.3 & 0.2 & 0.7 & 0.2 \\ 0.5 & 0.4 & 0.2 & 1.1 \end{bmatrix} \text{ and}$$

$$Q[\gamma_{ij}] = \begin{bmatrix} 0.8 & 0.2 & 0.4 & 0.2 \\ 0.2 & 0.4 & 0.1 & 0.1 \\ 0.4 & 0.1 & 1.0 & 0.5 \\ 0.2 & 0.1 & 0.5 & 0.8 \end{bmatrix}$$

Now here, the Signless Intuitionistic Fuzzy graph's Laplacian energy is demarcated as IFG .

Theorem 3.9:- Let $\tilde{G} = (V, E, \sigma, \mu)$be a intuitionistic fuzzy graph with set $|V| = n$ vertices and the membership values are $\tilde{\mu}_1 \geq \tilde{\mu}_2 \geq \ldots \geq \tilde{\mu}_n$ of Signless Laplacian eigen values ofintuitionistic fuzzy graph, hence we have

(a) $$\sum_{i=1}^{n} \tilde{\lambda}_i^2 = 2 \sum_{1 \leq i \leq j \leq n} \tilde{\mu}_{ij}$$ (b)

$$\sum_{i=1}^{n} \lambda_i^2 = 2 \sum_{1 \leq i \leq j \leq n} \tilde{\mu}_{ij}^2 + \sum_{i=1}^{n} \deg^2_{\mu_{ij}(G)}(u_i)$$

Proof:- (a) Since a symmetric matrix is $Q\left[\tilde{\mu}_{ij}(IFG)\right]$ and those Eigen values of Signless Laplacian are positive such that

$$\operatorname{tr}\left(Q\left(\tilde{\mu}_{ij}\left(IFG\right)\right)\right) = \sum_{i=1}^{n} \deg_{\tilde{\mu}_{ij}(IFG)}(u_i) = 2 \sum_{1 \leq i \leq j \leq n} \tilde{\mu}_{ij}$$

(b) Rendering to explanation of matrix of Signless Laplacian, then we prove that

$$\begin{bmatrix} d_{\tilde{\mu}_{ij}(G)}(u_1) & \tilde{\mu}(u_1,u_2) & \ldots & \tilde{\mu}(u_1,u_n) \\ \tilde{\mu}(u_2,u_1) & d_{\tilde{\mu}_{ij}(G)}(u_2) & \ldots & \tilde{\mu}(u_2,u_n) \\ . & . & . & . \\ \tilde{\mu}(u_n,u_1) & \tilde{\mu}(u_n,u_2) & \ldots & d_{\tilde{\mu}_{ij}(G)}(u_n) \end{bmatrix}$$

Then obtain $\operatorname{tr}\left(L_{i,i}^2\right) = \sum_{i=1}^{n} \tilde{\mu}_i^2$

Where

$$tr(L_{ii}^2) = [\deg^2_{\mu_{ij}(G)}(u_1) + \tilde{\mu}^2(u_1,u_2) + \ldots + \tilde{\mu}^2(u_1,u_n)] +$$

$$[\tilde{\mu}^2(u_2,u_1) + d^2_{\mu_{ij}(G)}(u_2) + \ldots + \tilde{\mu}^2(u_2,u_n)] + \ldots\ldots + [\tilde{\mu}^2(u_n,u_1) + \tilde{\mu}^2(u_n,u_2) + \ldots + \deg^2_{\mu_{ij}(G)}(u_n)]$$

$$=2 \sum_{1 \leq i \leq j \leq n} \tilde{\mu}_{ij}^2 + \sum_{I=1}^{N} \deg^2_{\mu_{ij}(G)}(u_i)$$

Likewise, the Intuitionistic fuzzy Signless Laplacian matrix's non-membership function can be proved as

(a) $$\sum_{i=1}^{n} \tilde{\delta}_i = 2 \sum_{1 \leq i \leq j \leq n} \tilde{\gamma}_{ij}$$

(b) $$\sum_{i=1}^{n} \tilde{\delta}_i^2 = 2 \sum_{1 \leq i \leq j \leq n} \tilde{\gamma}_{ij}^2 + \sum_{i=1}^{n} \deg^2_{r_{ij}(IFG)}(u_i)$$

Where $\delta_1 \geq \delta_2 \geq \ldots \geq \delta_n$ is the non-membership values of Intuitionistic fuzzy graph Signless Laplacian Eigen value .

Result 3.10:- Let the membership function is $\tilde{G}$=(V,E,σ, μ) of an intuitionistic fuzzy graph with $|V|$ =n is set of vertices and $\lambda_1 \geq \lambda_2 \geq .. \geq \lambda_n$ is the Eigen values of Signless Laplacian's membership function of a Fuzzy intuitionistic graph, where $\tilde{\xi}_i = \tilde{\lambda}_i - \frac{2 \sum_{1 \leq i \leq j \leq n} \tilde{\mu}_{ij}}{n}$ so we get

(a) $\sum_{i=1}^{n}\xi_i = 0$ (b) $\sum_{i=1}^{n}\xi_i^2 = 2M$

Where

M= $\sum_{1\le i\le j\le n}\tilde{\mu}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\left(d_{\tilde{\mu}_{ij}(G)}(u_i) - \frac{2\sum_{1\le i\le j\le n}\tilde{\mu}_{ij}}{n}\right)^2$

Likewise, the Signless intuitionistic fuzzy Laplacian matrix's non-membership function can be proved as

$\tilde{\chi}_i = \delta_i - \frac{2\sum_{1\le i\le j\le n}\gamma_{ij}}{n}$, then we have

(a) $\sum_{i=1}^{n}\chi_i = 0$ (b) $\sum_{i=1}^{n}\chi_i^2 = 2N$

Where N

= $\sum_{1\le i\le j\le n}\tilde{\gamma}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\left(d_{\tilde{\gamma}_{ij}(G)}(u_i) - \frac{2\sum_{1\le i\le j\le n}\tilde{\gamma}_{ij}}{n}\right)^2$

Definition 3.11:-Let a Fuzzy intuitionistic graph with set $|V|$=n Vertices be $\tilde{G} = (V, E, \sigma, \mu)$ and $\tilde{\lambda}_1 \ge \tilde{\lambda}_2 \ge .. \ge \tilde{\lambda}_n$ is the Eigen values Signless Laplacian as membership function of IFG. The intuitionistic fuzzy graph's Signless Laplacian energy is demarcated as,

$$QE\left(\tilde{\mu}_{ij}(IFG)\right) = \left|\tilde{\lambda}_i - \frac{2\sum_{1\le i\le j\le n}\tilde{\mu}(u_i, u_j)}{n}\right|$$

The energy of Signless Laplacian's Fuzzy intuitionistic graph $\tilde{G}$= (V, E, μ, γ) is defined as $\left[QE\left(\tilde{\mu}_{ij}(IFG)\right), QE\left(\tilde{\gamma}_{ij}(IFG)\right)\right]$

Example 3.12:-In Fig 3 , For an fuzzy intuitionistic graph , we have the following grades

spect$\left(Q\left(\tilde{\mu}_{ij}(IFG)\right)\right)$={0.5851,0.7,0.7524,,2.3625}

spect$\left(Q\left(\tilde{\gamma}_{ij}(IFG)\right)\right)$={0.0,0.5161,1.0087,1.4752}

QE$\left(\mu_{ij}(G_3)\right)$=

$$\left|\left(0.5851 - \frac{(4.4)}{4}\right)\right| + \left|\left(0.7 - \frac{(4.4)}{4}\right)\right| + \left|\left(0.7524 - \frac{(4.4)}{4}\right)\right| + \left|\left(2.3625 - \frac{(4.4)}{4}\right)\right|$$

= 2.5251

$QE\left(\tilde{\gamma}_{ij}(G_3)\right)$ =

$$\left|\left(0 - \frac{(3)}{4}\right)\right| + \left|\left(0.5161 - \frac{(3)}{4}\right)\right| + \left|\left(1.0087 - \frac{(3)}{4}\right)\right| + \left|\left(1.4752 - \frac{(3)}{4}\right)\right|$$

=1.9677

Here $\sum_{1\le i\le j\le n}\tilde{\mu}(u_i, u_j)$=4.4 and $\sum_{1\le i\le j\le n}\gamma(u_i, u_j) = 3.0$

∴ The Signless Laplacian energy of an intuitionistic fuzzy graph G_3 is[2.5251,1.9677]

4. Results

Theorem4.1:-Let $\tilde{G}$ be a fuzzy intuitionistic graph having no loops with $|V|$=n is set of vertices and $|E|$ =m is set of edges , a fuzzy intuitionistic matrix adjacency is (IFG)=[$[\mu_{ij}, \gamma_{ij}]$ and a fuzzy intuitionistic Laplacian matrix of $\tilde{G}$ is L(IFG)=$\left(L(\mu_{ij}), L(\gamma_{ij})\right)$ then

(i) $QE\left(\mu_{ij}(G)\right) \le \sqrt{\left(2\sum_{1\le i\le j\le n}\mu_{ij}^2 + \sum_{i=1}^{n}\left(\deg_{\mu_{ij}(G)}(u_i) - \frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)n}$

(ii) $QE\left(\gamma_{ij}(G)\right) \le \sqrt{\left(2\sum_{1\le i\le j\le n}\gamma_{ij}^2 + \sum_{i=1}^{n}\left(\deg_{\gamma_{ij}(G)}(u_i) - \frac{2\sum_{1\le i\le j\le n}\gamma_{ij}}{n}\right)^2\right)n}$

Proof: From the inequality of Cauchy's –Schwarz to (1,..., 1) and ($|\tilde{\xi}_1|, |\tilde{\xi}_2|, \ldots, |\tilde{\xi}_n|$),we get

$$\left|\sum_{i=1}^{n}\tilde{\xi}_i\right|^2 \le n\sum_{i=1}^{n}\left|\tilde{\xi}_i\right|^2$$

Where $QE\left(\tilde{\mu}_{ij}(IFG)\right) \le \sqrt{n\sum_{i=1}^{n}\left|\tilde{\xi}_i\right|^2} = \sqrt{2Mn}$

Since

M= $\sum_{1\le i\le j\le n}\mu_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\left(\deg_{\mu_{ij}(G)}(u_i) - \frac{2\sum_{1\le i\le j\le n}\mu_{ij}}{n}\right)^2$ we have

$$QE\left(\mu_{ij}(\tilde{G})\right) \le \sqrt{\left(2\sum_{1\le i\le j\le n}\tilde{\mu}_{ij}^2 + \sum_{i=1}^{n}\left(\deg_{\tilde{\mu}_{ij}(IFG)}(u_i) - \frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)n}$$

Likewise, we can be proved as

$$QE\left(\tilde{\gamma}_{ij}(G)\right) \le \sqrt{\left(2\sum_{1\le i\le j\le n}\tilde{\gamma}_{ij}^2 + \sum_{i=1}^{n}\left(d_{\tilde{\gamma}_{ij}(IFG)}(u_i) - \frac{2\sum_{1\le i\le j\le n}v_{ij}}{n}\right)^2\right)n}$$

TheoremIV.2:- Let Fuzzy intuitionistic graph is $\tilde{G}$ having no loops with $|V|$=n is set of vertices and $|E|$ =m is set of edges and a Fuzzy intuitionistic adjacency matrix is (IFG)=$\left((\mu_{ij}), (\gamma_{ij})\right)$ and L(IFG)=$\left(L(\mu_{ij}), L(\gamma_{ij})\right)$ be a intuitionistic fuzzy Laplacian matrix of $\tilde{G}$ then

(i) $QE\left(\mu_{ij}(G)\right) \ge 2\sqrt{\left(\sum_{1\le i\le j\le n}\tilde{\mu}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\left(\deg_{\tilde{\mu}_{ij}(IFG)}(u_i) - \frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)}$

(ii)

$$QE\left(\gamma_{ij}(G)\right) \ge 2\sqrt{\left(\sum_{1\le i\le j\le n}\tilde{\gamma}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\left(\deg_{\tilde{\gamma}_{ij}(IFG)}(u_i) - \frac{2\sum_{1\le i\le j\le n}v_{ij}}{n}\right)^2\right)}$$

Proof: From the definition 3.7, we have

$$\left(LE\left(\mu_{ij}\left(\tilde{G}\right)\right)\right)^2 = \left(\sum_{i=1}^{n}\left|\tilde{\xi}_i\right|\right)^2 =$$

$$\sum_{i=1}^{n}\left|\tilde{\xi}_i\right|^2 + 2\sum_{1\le i\le j\prec n}\left|\tilde{\xi}_i\right|\left|\tilde{\xi}_j\right| \ge 4M$$

We get

$$LE\left(\mu_{ij}\left(\tilde{G}\right)\right) \le 2\sqrt{\left(\sum_{1\le i\le j\le n}\tilde{\mu}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\left(\deg_{\tilde{\mu}_{ij}(IFG)}\left(u_i\right) - \frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)}$$

Hence the result.
Similarly

$$LE\left(\gamma_{ij}\left(\tilde{G}\right)\right) \le 2\sqrt{\left(\sum_{1\le i\le j\le n}\gamma_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\left(d_{\gamma_{ij}(G)}\left(u_i\right) - \frac{2\sum_{1\le i\le j\le n}\gamma_{ij}}{n}\right)^2\right)}$$

5. Decision

In this paper we demarcated the Signless Laplacian matrix for a Fuzzy Intuitionistic graph. Some outcomes on the spectra of energy of Signless Laplacian of an Intuitionistic fuzzy graph may divulge more equivalent outcomes of these kinds and will be deliberated in the forthcoming investigating research articles.

Acknowledgement

This is a text of acknowledgements. Do not forget people who have assisted you on your work. Do not exaggerate with thanks. If your work has been paid by a Grant, mention the Grant name and number here.

References

[1] Anjali N , Mathew S , Energy of a Fuzzy Graph , Annals Fuzzy Math. Inf.2013; 6: 455-465.
[2] AtanassovK, Intutionistic fuzzy sets , Fuzzy Sets and Fuzzy Systems, 1986; 20(1): 87-96.
[3] Atanassov.K., Intutionistic fuzzy Sets Theory and Applications, Springer-Verlag, Heidelberg, 1999.
[4] Balakrishnan R , The Energy of a Graph , Linear algebra Appl., 2004; 387-: 287-295.
[5] Gutman I, Zhou B, Laplacian Energy of a Graph , Linear Algebra Appl.,2006;414: 29-37.
[6] Lee K H, First Course on Fuzzy Theory and Applications, Springer-Verlag, Berlin, 2005.
[7] Morderson J N, Nair P S, Fuzzy Graphs and Fuzzy Hypergraphs , Springer- Verlag,2000.
[8] Muhammad Akram,ShaistaHabib, Imran Javed, Intuitionistic Fuzzy Logic Control for Washing Machines, Indian Journal of Science and Technology,2014May;7(5):654-661.
[9] Parvathi R, Karumbigai M G , Intutionistic Fuzzy graphs, Computational Intelligence, Theory and Applications, 139-150.
[10] Praba B ,Chandrasekharan V M, Deepa G , Energy an Intuitionistic Fuzzy Graphs ,talian Journal of Pure and Applied Mathematics , 2014; 32:431-444.
[11] Rosenfeld A , Fuzzy Graphs in:L.A.Zadeh, K.S.Fu and M.Shimura (Eds) Fuzzy sets and their Applications, Academic Press, New York, 1975, 77-95.
[12] SadeghRahimiSharbaf ,FatmehFayazi , Laplacian Energy of a fuzzy Graph, Iranian Journal of Mathematical Chemistry, 2014; 5(32): 31-40.
[13] Zadeh. L.A., Fuzzy sets , Information and Control, 1965; 8:338-353.