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Research paper

The Cyclic Decomposition of the Group ($Q_{2m}\times C_4$) When m= $2^h.p_1^{r_1}.p_2^{r_2}.\ldots.p_n^{r_n}$,h, r∈ Z^+ and p is Prime Number

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Abstract

The main purpose of this paper is to find The Cyclic decomposition of the group ($Q_{2m}\times C_4$) when m= $2^h.p_1^{r_1}.p_2^{r_2}.\ldots.p_n^{r_n}$ h, r∈ $Z^+$and p is prime number, which is denoted by AC ($Q_{2m}\times C_4$) where Q_{2m} is the Quaternion group and C_4 is the cyclic group of order 4 . We have also found the general form of Artin's characters table of Ar($Q_{2m}\times C_4$) when m= $2^h.p_1^{r_1}.p_2^{r_2}.\ldots.p_n^{r_n}$,h,r∈ $Z^+$and p is prime number.

***Keywords**: Quaternion group, the cyclic group, Artin's characters, Artin's characters table, the cyclic decomposition.*

1. Introduction

Thesquare matrix whose rows correspond to Artin's characters and columns correspond to the Γ- classes of G is called Artin's characters table . this matrix is very important to find the cyclic decomposition of the factor group AC(G) and Artin's exponent A(G). In 1967 T.Y. lam [11] studied A(G) extensively for many groups In 1981 C.Curits and I. Reiner[3] studied Methods of Representation Theory with Application to Finite Groups. In 2009 S.J. Mahmood [10] studied the general from of Artin's characters table Ar(Q_{2m}) when m is an even number.

The aim of this paper is to find the general from of The Cyclic decomposition and the Artin's characters table of the group ($Q_{2m}\times C_4$)when m= $2^h.p_1^{r_1}.p_2^{r_2}.\ldots.p_n^{r_n}$,h,r∈ $Z^+$and p is prime number.

2. Preliminaries

This section introduce some important definitions and basic concepts the Artin characters, the Artin characters table, the factor group AC(G) of a group G and the matrix M(G), M(Q_{2m}),P(Q_{2m}) and W(Q_{2m}).

Theorem1:[4]Let T_1 : $G_1\rightarrow GL(n,F)$ and T_2: $G_2\rightarrow GL(m,F)$ be two irreducible representations of the groups G_1 and G_2 with characters $\chi_1$and $\chi_2$respectively then :

$T_1\otimes T_2$is irreducible representation of the group $G_1\times G_2$ with the character $\chi_1\cdot\chi_2$.

Proposition 1:[9]The rational valued characters table of the group ($Q_{2m}\times C_4$) when m is an even number is equal to the tensor product of the rational valued characters table of Q_{2m} when m is an even number and the rational valued characters table of C_4 that is:

$\overset{*}{\equiv}(Q_{2m}\times C_4)=\overset{*}{\equiv}(Q_{2m})\otimes\overset{*}{\equiv}(C_4)$.

Theorem 2: [6]Let H be a cyclic subgroup of G and $h_1, h_2, \ldots,$ h_m are chosen as representative for m-conjugate classes of H contained in CL(g) in G, then :

1. $\varphi'(g)=\dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i\in H\cap CL(g)$
2. $\varphi'(g)=0$ if $H\cap CL(g)=\phi$.

Definition 1:[11]Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called **Artin's characters of G.**

In theorem 2 , if φ is the principal character , then $\varphi(h_i)=\varphi(1)=1$, where $h_i\in H$

Proposition 2:[3]The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G.Furthermore, Artin's characters are constant on each Γ-classes.

Definition 2: [2]Artin's characters of finite group G can be displayed in a table **called Artin's characters table of G** which is denoted by Ar (G).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest rows contain the values of Artin's characters.

Table 1

Γ- classes	Γ- classes of C_{2m}					
	[1]	$[x^{2^h.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}}]$			[y]	[xy]
$\lvert CL_\alpha\rvert$	1	1	2 2 ...	2	$2^h.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$	$2^h.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$
$\lvert C_{Q_{2^{h+1}.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}}}(CL_\alpha)\rvert$	$2^{h+2}.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$	$2^{h+2}.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$	$2^{h+1}.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$ ...	$2^{h+1}.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$	4	4
Φ_1	$2Ar(C_{2^{h+1}.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}})$				0	0
Φ_2					0	0
⋮					⋮	⋮
Φ_l					0	0
Φ_{l+1}	$2^h.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$	$2^h.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$	0... 0	0	2	0
Φ_{l+2}	$2^h.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$	$2^h.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}$	0 ... 0	0	0	2

Proposition 3: [10]The Artin's characters table of the Quaternion group $Q_{2m}$when $m=2^h.p_1^{r_1}.p_2^{r_2}\ldots.p_n^{r_n}$,$h,r\in Z^+$and p is prime number is given as follows:

$Ar(Q_{2^{h+1}.p_1^{r_1}.p_2^{r_2}\ldots p_n^{r_n}})=$ where l is the number of Γ- classes of C_{2m} and Φ_j ; $1 \le j \le l+2$ are the Artin characters of the Quaternion group Q_{2m} when $m=2^h.p_1^{r_1}.p_2^{r_2}\ldots.p_n^{r_n}$,$h,r\in Z^+$ and p is prime number .

Definition 3:[8] Let T(G) be the subgroup of $\overline{R}$(G) generated by Artin's characters .T (G) is normal subgroup of $\overline{R}$(G) and denotes the factor abelian group $\overline{R}$(G)/T(G) by AC(G) which is called **Artin cokernel of** G.

Definition 4:[7]Let M be a matrix with entries in a principal domain R. A **k-minor of M** is the determinant of k×k sub matrix preserving row and column order.

Definition 5:[7]A k-th determinant divisor of M is the greatest common divisor (g.c.d) of all the k-minors of M.This is denoted by D_k (M)

Lemma 1:[7]Let M, P and W be matrices with entries in a principal ideal domain R, let P and W be invertible matrices ,Then D_k (P M W)= D_k (M) module the group of unites of R.

Theorem 3:[7]Let M be an n × n matrix with entries in principal ideal domain R, then there exist two matrices P and W such that:

1. P and W are invertible.
2. P M W = D.
3. D is diagonal matrix.
4. if we denote D_{ii} by d_i then there exists a natural number m ; $0 \le m \le n$ such that $j > m$ implies $d^j = 0$ and $j \le m$ implies $d^j \neq 0$ and $1 \le j \le m$ implies $d^j | d^{j+1}$.

Definition 6:[7]Let M be matrix with entries in a principal domain R, be equivalent to a matrix D = diag $\{d_1, d_2, \ldots, d_m, 0, 0, \ldots, 0\}$ such that $d_j | d_{j+1}$ for $1 \le j < m$

We call D **the invariant factor matrix of** M and $d_1, d_2, \ldots, d_m$ the invariant factors of M.

Theorem 4:[7] Let K be a finitely generated module over a principal domain R, then K is the direct sum of cyclic sub module with an annihilating ideal$\langle d_1\rangle, \langle d_2\rangle, \ldots, \langle d_m\rangle$, $d_j | d_{j+1}$ for $j = 1, 2, \ldots, K-1$.

Proposition 4:[8]AC(G) is a finitely generated Z- module .Let m be the number of all distinct Γ-classes then Ar(G) and ≡*(G) are of the rank l. There exists an invertible matrix M(G) with entries in rational number suchThat: ≡*(G)=M^{-1}(G).Ar(G) and this implies M(G)=Ar(G).(≡*(G))$^{-1}$

Theorem 5:[5]AC(G) = $\bigoplus_{i=1}^{l} C_{d_i}$ where $d_i = \pm D_i(G) / D_{i-1}(G)$ where l is the number of all distinct Γ-classes.

Corollary 1:[8] $|AC(G)|$ =| det (M(G))|

Lemma 2:[8]If A and B are two matrices of degree m and t respectively, then:

det (A⊗B) = (det (A))^t . (det (B))^m .

Lemma 3:[8]Let A and B be two non-singular matrices of rank l and m respectively, over a principal domain R and let:

$P_1 AW_1 = D(A) = \text{diag}\{d_1(A), d_2(A), \ldots, d_l(A)\}$ and $P_2 AW_2 = D(B) = \text{diag}\{d_1(B), d_2(B), \ldots, d_m(B)\}$

The invariant factor matrices of A and B then:

$(P_1 \otimes P_2)(A \otimes B)(W_1 \otimes W_2) = D(A) \otimes D(B)$

and from this the invariant factor matrices of A ⊗ B can be obtained.

Proposition 5:[5]Let H_1 and H_2 be $p_1$and p_2 - groups respectively where p_1 and p_2 are distinct primes and if M_1 is the matrix from all cyclic subgroups of $\overline{R}$ (H_1) basis and M_2 is the matrix which expresses the T(H_2) basis terms of $\overline{R}$ (H_2) basis then the matrix which expresses the T ($H_1 \times H_2$) basis of $\overline{R}$ ($H_1 \times H_2$) basis is $M_1 \otimes M_2$.

Remarks 1: [1]In general if m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots.p_n^{r_n}$ such that p_i are prime numbers $p_i \neq 2$ and g.c.d (p_i , p_j)=1 ,h and r_i are any positive integer numbers for all i=1,2,…,n then we can write C_m as the from :

$$C_m = C_{2^h} \times C_{p_1^{r_1}} \times C_{p_2^{r_2}} \times \cdots \times C_{p_n^{r_n}}$$

(i) By the proposition 5 we get

$M(C_m)=$

$$M(C_{2^h}) \otimes M(C_{p_1^{r_1}}) \otimes M(C_{p_2^{r_2}}) \otimes \cdots \otimes M(C_{p_n^{r_n}})$$

We can write M(C_m) in the form:

$$M(C_m)=$$

$$\begin{bmatrix} & & & & \left\{\begin{matrix}1 \\ \vdots \\ 1\end{matrix}\right. & \begin{matrix}1 \\ 1 \\ \\ \end{matrix} \\ & & & h.times & 0 & \\ & R_2(C_m) & & h.times & \left\{\begin{matrix}1 \\ \vdots \\ 1\end{matrix}\right. & \vdots \\ & & & & 0 & \\ & & & & \vdots & \\ & & & h.times & \left\{\begin{matrix}1 \\ \vdots \\ 1\end{matrix}\right. & \\ 0 & 0 & & & & 1 \\ 0 & 0 & \cdots & & 0 & 1 \end{bmatrix}$$

which is (r_1+1) ……(r_n+1)(h+1)×(r_1+1)……(r_n+1)(h+1) square matrix,

$R_2(C_m)$ is the matrix obtained by omitting the last two rows $\{0,0,\cdots,1,1\}$ and $\{0,0,\cdots,0,1\}$ and the last two columns $\{1,\cdots,1,0,1,\cdots,1,0,\cdots,1,0\}$ and $\{1,1,\cdots,1\}$ from the tensor product.

$$M(C_{2^h}) \otimes M(C_{p_1^{r_1}}) \otimes M(C_{p_2^{r_2}}) \otimes \cdots \otimes M(C_{p_n^{r_n}})$$

(ii) By lemma 3 we have :

1- $P(C_m)=$

$$P(C_{2^h}) \otimes P(C_{p_1^{r_1}}) P \otimes (C_{p_2^{r_2}}) \otimes \cdots \otimes P(C_{p_n^{r_n}})$$

2- $W(C_m)=$

$$W(C_{2^h})\otimes W(C_{p_1^{r_1}})\otimes W(C_{p_2^{r_2}})\otimes\cdots\otimes W(C_{p_n^{r_n}})$$

Proposition 6:[10]If $m=2^h\cdot p_1^{r_1}\cdot p_2^{r_2}\cdot\cdots\cdot p_n^{r_n}$, such that p_i 's are all distinct primes, $p_i\neq 2$ and g.c.d $(p_i,p_j)=1$ for all i=1,2,⋯,n , h and r_i any positive integers then

$$M(Q_{2m})=\left[\begin{array}{cccccccccccccc}
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & \vdots & \vdots & \vdots & \vdots & h+1.times \\
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & 2.R_2(C_{2m}) & & & & & 0 & 1 & 0 & 1 & \\
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & \vdots & \vdots & \vdots & \vdots & h+1.times \\
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & 0 & 1 & 0 & 1 & \\
 & & & & & & & & \vdots & \vdots & \vdots & \vdots & \\
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & \vdots & \vdots & \vdots & \vdots & h+1.times \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 1 & 1 & \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 1 & \\
0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 0 & 0 & 1 & 1 & \\
0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 1 & 0 & 0 & 1 &
\end{array}\right]$$

which is

$[(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+2]\times[(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+2]$

square matrix .

$R_2(C_{2m})$ is similar to the matrix in the remark 1

Proposition 7:[10]If $m=2^h\cdot p_1^{r_1}\cdot p_2^{r_2}\cdot\cdots\cdot p_n^{r_n}$, such that p_i 's are all distinct primes, $p_i\neq 2$ and g.c.d $(p_i,p_j)=1$ for all $i=1,2,\cdots,n$, h and r_i any positive integers then the matrices $P(Q_{2m})$ and $W(Q_{2m})$ are taking the forms :

$$P(Q_{2m})=\left[\begin{array}{ccccccc}
 & & & & & 0 & 0 \\
 & & & & & 0 & 0 \\
 & & P(C_{2m}) & & & \vdots & \vdots \\
 & & & & & \vdots & \vdots \\
 & & & & & 0 & 0 \\
 & & & & & -1 & 1 \\
 & & & & 0 & 0 & -1 \\
0 & 0 & \cdots & \cdots & 0 & 1 & -1 \\
0 & 0 & \cdots & \cdots & 0 & 0 & 1
\end{array}\right] \quad and$$

$$W(Q_{2m})=\left[\begin{array}{ccccccccccccc}
 & & & & & & & & & & 0 & 0 & 0 \\
 & & & & & & & & & & 0 & 0 & 0 \\
 & & & I_k & & & & & & & \vdots & \vdots & \vdots \\
 & & & & & & & & & & \vdots & \vdots & \vdots \\
 & & & & & & & & & & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & \cdots & 0 & -1 & -1 & \cdots & -1 & -1 & 0 & 0 & 1
\end{array}\right]$$

$k=[(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)]-1$ and I_k is the identity matrix of the order k ,they are $[(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+2]\times[(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+2]$ square matrix .

Theorem 6:[10]If $m=2^h\cdot p_1^{r_1}\cdot p_2^{r_2}\cdot\cdots\cdot p_n^{r_n}$, such that p_i 's are all distinct primes, $p_i\neq 2$ and g.c.d $(p_i,p_j)=1$ for all $i=1,2,\cdots,n$, h and r_i any positive integers then the cyclic decomposition of $AC(Q_{2m})$ is :

$$AC(Q_{2m})=\bigoplus_{i=1}^{(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)-1} C_2$$

3. Results

In this section we find the general form of The Cyclic decomposition and the Artin's characters table of the group ($Q_{2m}\times C_4$)when m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,h,r∈ $Z^+$and p is prime number.

Proposition 8: The general from of the Artin's characters table of the group ($Q_{2m}\times C_4$)when m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,h,r∈ $Z^+$and p is prime number is give as follows:

$Ar(Q\,2^{h+1}.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}\times C_4)=$

Table 2

Γ- classes of $(Q_{2m})\times\{I\}$							Γ- classes of $(Q_{2m})\times\{z^2\}$						Γ- classes of $(Q_{2m})\times\{z\}$					
Γ-classes	$[1,I]$	$[x^m,I]$	..	$[x,I]$	$[y,I]$	$[xy,I]$	$[I,z^2]$	$[x^m,z^2]$	⋯	$[x,z^2]$	$[y,z^2]$	$[xy,z^2]$	$[I,z]$	$[x^m,z]$	⋯	$[x,z]$	$[y,z]$	$[xy,z]$
$\lvert CL_\alpha\rvert$	1	1	..	2	m	m	1	1	..	2	m	m	1	1	⋯	2	m	m
$\lvert C_{Q_{2m}\times C_4}(CL_\alpha)\rvert$	16m	16m	..	8m	16	16	16m	16m	..	8m	16	16	16m	16m	⋯	8m	16	16
$\Phi_{(1,1)}$	4Ar(Q_{2m})						0						0					
$\Phi_{(2,1)}$																		
⋮																		
$\Phi_{(l,1)}$																		
$\Phi_{(l+1,1)}$																		
$\Phi_{(l+2,1)}$																		
$\Phi_{(1,2)}$	2Ar(Q_{2m})						2Ar(Q_{2m})						0					
$\Phi_{(2,2)}$																		
⋮																		
$\Phi_{(l,2)}$																		
$\Phi_{(l+1,2)}$																		
$\Phi_{(l+2,2)}$																		
$\Phi_{(1,3)}$	Ar(Q_{2m})						Ar(Q_{2m})						Ar(Q_{2m})					
$\Phi_{(2,3)}$																		
⋮																		
$\Phi_{(l,3)}$																		
$\Phi_{(l+1,3)}$																		
$\Phi_{(l+2,3)}$																		

Proof : Let g∈($Q_{2m}\times C_4$) ;g=(q,I) or g=(q,z) or g=(q,z^2)or g=(q,z^3) q∈Q_{2m} ,I,z,z^2,z^3 ∈ C_4

Case (I):

If H is a cyclic subgroup of $Q_{2m}\times\{I\}$,then:

1.H=⟨(x, I)⟩2. H=⟨(y, I)⟩

3. H=⟨(xy, I)⟩

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem 2

1. H=⟨(x, I)⟩

(i) If g=(1,I) and g∈H

$$\Phi_{(j,1)}((1,I))=\frac{\lvert C_{Q_{2m}\times C_4}(g)\rvert}{\lvert C_H(g)\rvert}\cdot\varphi(g)$$

$$=\frac{16m}{\lvert C_H(I,1)\rvert}.1=\frac{4.4m}{\lvert C_H(I,1)\rvert}.1=\frac{4\lvert C_{Q_{2m}}(1)\rvert}{\lvert C_{\langle x\rangle}(1)\rvert}.\varphi(1)=4.\Phi_j(1)$$

since H∩CL(1,I)={(1,I)}

(ii) if g= (x^m,I) and g∈H

$$\Phi_{(j,1)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{16m}{\left|C_H(g)\right|}.1=\frac{4.4m}{\left|C_H(g)\right|}.1=\frac{4\left|C_{Q_{2m}}(x^m)\right|}{\left|C_{\langle x\rangle}(x^m)\right|}.\varphi(g)=4.\Phi_j(x^m)$$

since H∩CL(g)={g},φ(g)=1

(iii) if g= $(x^i,I), i\neq m$ and $i\neq 2m$ and g∈H

$$\Phi_{(j,1)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{\left|C_H(g)\right|}(1+1)=$$

$$\frac{4.2m}{\left|C_H(g)\right|}.(1+1)=\frac{4\left|C_{Q_{2m}}(q)\right|}{\left|C_{\langle x\rangle}(q)\right|}.(\varphi(g)+\varphi(g^{-1}))=4.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_{2m} and q≠ x^m ,q≠1.

(iv) if g ∉ H Since H∩CL(g)= ϕ 2. H= ⟨(y, I)⟩ ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{16m}{4}.1=4m=4.\Phi_{l+1}(1)$$

(ii) If g= (x^m,I)=(y^2,I) and g∈ H

$$\Phi_{(l+1,1)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{16m}{4}.1=4m=4.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) and g∈H

$$\Phi_{(l+1,1)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{4}.(1+1)=4.2=4.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+1,1)}(g)=0$$ since H∩CL(g)=∅

3- H=⟨(xy, I)⟩ ={(1,I),(xy,I),(($(xy)^2$,I),(($(xy)^3$,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+2,1)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{16m}{4}.1=4m=4.\Phi_{l+2}(1)$$

(ii) If g= (x^m,I)=(($(xy)^2$,I)=(y^2,I) and g∈ H

$$\Phi_{(l+2,1)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{16m}{4}.1=4m=4.\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g=(($(xy)^3$,I) =(xy^3,I) and g∈H

$$\Phi_{(l+2,1)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{4}.(1+1)=4.2=4.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+2,1)}(g)=0$ since H∩CL(g)=∅

Case (II):

If H is a cyclic subgroup of $Q_{2m}\times\{z^2\}$,then:

1.H=⟨(x, I)⟩= $\langle(x,z^2\rangle$ 2. H=⟨(y, I)⟩ = $\langle(y,z^2\rangle$ ⟩

3. H=⟨(xy, I)⟩ = $\langle(xy,z^2\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\leq j\leq l+2$ then by using Theorem 2

1. H=⟨(x, I)⟩ = $\langle(x,z^2\rangle$

(i) If g=(1,I) or g=(1,z^2) and g∈H

$$\Phi_{(j,2)}((1,I))=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{16m}{\left|C_H(I,1)\right|}.1=\frac{4.4m}{\left|C_H(I,1)\right|}.1=\frac{4\left|C_{Q_{2m}}(1)\right|}{2\left|C_{\langle x\rangle}(1)\right|}.\varphi(1)=2.\Phi_j(1)$$

since H∩CL(1,I)={(1,I),(1,z^2)}

(ii) if g= (x^m,I) and g∈H

$$\Phi_{(j,2)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{16m}{\left|C_H(g)\right|}.1=\frac{4.4m}{\left|C_H(g)\right|}.1=\frac{4\left|C_{Q_{2m}}(x^m)\right|}{2\left|C_{\langle x\rangle}(x^m)\right|}.\varphi(g)=2.\Phi_j(x^m)$$

since H∩CL(g)={g},φ(g)=1

(iii) if g= $(x^i,I), i\neq m$ and $i\neq 2m$ and g∈H

$$\Phi_{(j,2)}(g)=\frac{\left|C_{Q_{2m}\times C_4}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{\left|C_H(g)\right|}(1+1)=$$

$$\frac{4.2m}{\left|C_H(g)\right|}.(1+1)=\frac{4\left|C_{Q_{2m}}(q)\right|}{2\left|C_{\langle x\rangle}(q)\right|}.(\varphi(g)+\varphi(g^{-1}))=2.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_{2m} and $q\neq x^m, q\neq 1$

(iv) if g ∉ H Since H∩CL(g)= ϕ

2. H= ⟨(y, I)⟩ ={(1,I),(y,I),(y^2,I),(y^3,I), (1, z^2),(y, z^2),(y^2, z^2),(y^3,z^2)}

(i) If g=(1,I) or g=(1,z^2) H∩CL(1,I)={(1,I), (1,z^2)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$\Phi_{(j,2)}(g)=2.0=2.\Phi_j(q)=\frac{16m}{8}.1=2m=2.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) or g=(y^2,z^2) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{8}.1=2m=2.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) or g=(y,z^2) or g=(y^3, z^2) and g∈H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{8}.(1+1)=2.2=2.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+1,2)}(g)=0 \quad \text{since } H\cap CL(g)=\emptyset$$

3- H= ⟨(xy, I)⟩ ={(1,I),(xy,I),($(xy)^2$,I),($(xy)^3$,I), (1, z^2),(xy, z^2),($(xy)^2$, z^2),($(xy)^3$,z^2)}

(i) Ifg=(1,I)org=(1,z^2) H∩CL(1,I)={(1,I),(1,z^2)}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{8}.1=2m=2.\Phi_{l+2}(1)$$

(ii) If g= (x^m, I)=($(xy)^2$,I)=(y^2,I) or

g= (x^m, z^2)=($(xy)^2$,z^2)=(y^2,z^2) and g∈ H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{8}.1=2m=2.\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g=($(xy)^3$,I) =(xy^3,I) or g=(xy,z^2) or g=($(xy)^3$, z^2) =(xy^3, z^2) and g∈H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{8}.(1+1)=2.2=2.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+2,2)}(g)=0 \text{ since } H\cap CL(g)=\emptyset$$

Case (III):

If H is a cyclic subgroup of (Q_{2m}×{z}),then:

1.H=⟨(x,z)⟩= $\langle(x,z^2)\rangle=\langle(x,z^3)\rangle$

2. H=⟨(y,z)⟩ = $\langle(y,z^2)\rangle$ = $\langle(y,z^3)\rangle$

3. H=⟨(xy,z)⟩ = $\langle(xy,z^2)\rangle$ = $\langle(xy,z^3)\rangle$

And φ the principal character of H, Φ_j Artin characters of $Q_{2m}$where $1\le j\le l+2$ then by using Theorem 2

1.H=⟨(x,z)⟩

(i) If g=(1,I) or g=(1,z)or g=(1,z^2) or g=(1,z^3) and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(1,I)|}\cdot\varphi(g)$$

$$=\frac{16m}{|C_H(1,I)|}.1=\frac{4.4m}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H∩CL(g)={(1,I),(1,z),(1,z^2),(1,z^3)}

(ii) If g=(1,I) or g=(x^m,I) or g=(x^m,z) or g=(1,z)or g=(x^m,z^2) or g=(1,z^2)or g=(1,z^3) or g=(x^m,z^3) and g∈H

(a) if g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3) and g∈H.

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H∩CL(g)={g},φ(g)=1

(b)If g=(x^m,I) or g=(x^m,z) or g=(x^m,z^2) or g=(x^m,z^3) and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q_{2m}}(x^m)|}{4|C_{\langle x\rangle}(x^m)|}.\varphi(x^m)=\Phi_j(x^m)$$

since H∩CL(g)={g},φ(g)=1

(iii) If g$=\{(x^i,I),(x^i,z),(x^i,z^2),(x^i,z^3)\}$,*i≠m,i≠2m* and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{|C_H(g)|}(1+1)$$

$$\frac{4.2m}{|C_H(g)|}.(1+1)=\frac{4|C_{Q_{2m}}(q)|}{4|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,z)=(q,z^3),q∈ Q_{2m} and q≠ x^m ,q≠1

(iv) if g∉H Since H∩CL(g)= ϕ $\Phi_{(j,)}(g)=0$

2. H= $\langle(y,z)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z), (1,z^2),(y,z^2),(y^2,z^2),(y^3,z^2), (1,z^3),(y,z^3),(y^2,z^3),(y^3,z^3)}

(i) If g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3)and g∈H H∩CL(g)={(1,I),(1,z),(1,z^2) ,(1,z^3)}

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{16}.1=m=.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y²,I) or g= (y²,z) or g=(y²,z²) or g=(y²,z³) and g∈ H

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{16m}{16}.1 = m = \Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I)or g= (y,z)or g=(y,z²) or g=(y,z³) or g=(y³,I₂)or g= (y³,z) or g= (y³,z²) or g= (y³,z³) and g∈H

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$= \frac{16}{16}.(1+1) = 2 = \Phi_{l+1}(y)$$

since H∩CL(g)={g,g⁻¹} and φ(g)=φ(g⁻¹)=1

Otherwise

$\Phi_{(l+1,3)}(g) = 0$ since H∩CL(g)=∅ 3.H= $\langle (xy, z)\rangle$ ={(1,I),(xy,I),((xy)²,I)=(y²,I),((xy)³,I)=(xy³,I),(1,z),(xy,z), ((xy)²,z)),((xy)³,z),(1,z²),(xy,z²),((xy)²,z²)),((xy)³,z²),(1,z³)(xy,z³) ,((xy)²,z³)),((xy)³,z³)}

(i) If g=(1,I) or g=(1,z) or g=(1,z²) or g=(1,z³) H∩CL(g)={g}

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{16m}{16}.1 = m = \Phi_{l+2}(1)$$

(ii) If g= (x^m, I)=((xy)²,I)=(y²,I)

or g=((xy)²,z)=(y²,z)or g=((xy)²,z²)=(y²,z²)

or g=((xy)²,z³)=(y²,z³) and g∈ H

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{16m}{16}.1 = m = \Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g= ((xy)³,I) or g= (xy,z) or g= ((xy)³,z) or g= (xy,z²) or

(iv) g= ((xy)³,z²) or g= (xy,z³) or g= ((xy)³,z³) and g∈H

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$= \frac{16}{16}.(1+1) = 2 = \Phi_{l+2}(xy)$$

since H∩CL(g)={g,g⁻¹} and φ(g)=φ(g⁻¹)=1

Otherwise

$\Phi_{(l+2,3)}(g) = 0$ since H∩CL(g)=∅

Proposition 9: If $m = 2^h\cdot p_1^{r_1}\cdot p_2^{r_2}\cdots\cdot p_n^{r_n}$, such that p_i 's are all distinct primes, $p_i \neq 2$ and g.c.d $(p_i, p_j) = 1$ for all i=1,2,⋯,n , *h* and r_i any positive integers then:

$$M(Q_{2m}\times C_4) = \left[\begin{array}{c|c|c} M(Q_{2m}) & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & 0 & M(Q_{2m}) \end{array}\right]$$

Which is

[3(r₁+1)(r₂+1)⋯(rₙ+1)(h+2)+6]×[3(r₁+1)(r₂+1)⋯(rₙ+1)(h+2)+6] square matrix M (Q_{2m}) is similar to the matrix of the proposition 6.

Proof : By Proposition 8 we obtain the Artin's characters Table Ar($Q_{2m}\times C_4$) of the group ($Q_{2m}\times C_4$) when m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,*h,r*∈ $Z^+$and p is prime number and from the Proposition8 we get the rational valued characters table $(\overset{*}{\equiv}(Q_{2m}\times C_4))$ of the group ($Q_{2m}\times C_4$) when m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,*h,r*∈ $Z^+$and p is prime number.

Thus, by definition of M(G) we can find the matrix M($Q_{2m}\times C_4$) when $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,*h,r*∈ $Z^+$and p is prime number.

$$M(Q_{2m}\times C_4) = Ar(Q_{2m}\times C_4)\cdot(\overset{*}{\equiv}(Q_{2m}\times C_4))^{-1}$$

$$= \left[\begin{array}{c|c|c} M(Q_{2m}) & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & 0 & M(Q_{2m}) \end{array}\right] = M(Q_{2m}\times C_4)$$

Example 1:

Consider the group ($Q_{48}\times C_4$), we can find the matrix M($Q_{48}\times C_4$) by using:

$$M(Q_{48}\times C_4)=M(Q_{3.2^4}\times C_4)=Ar(Q_{3.2^4}\times C_4)\cdot(\overset{*}{\equiv}(Q_{3.2^4}\times C_4))^{-1}$$

$$M(Q_{3.2^4}\times C_4)=\left[\begin{array}{cccccccccccccccccccccccccccccccccccc}
2&2&2&2&2&2&2&2&1&1&1&1&2&2&2&2&2&2&2&2&1&1&1&1&2&2&2&2&2&2&2&2&1&1&1&1\\
0&2&2&2&2&0&2&2&1&1&1&1&0&2&2&2&2&0&2&2&1&1&1&1&0&2&2&2&2&0&2&2&1&1&1&1\\
0&0&2&2&2&0&0&2&1&1&1&1&0&0&2&2&2&0&0&2&1&1&1&1&0&0&2&2&2&0&0&2&1&1&1&1\\
0&0&0&2&2&0&0&0&1&1&1&1&0&0&0&2&2&0&0&0&1&1&1&1&0&0&0&2&2&0&0&0&1&1&1&1\\
0&0&0&0&2&0&0&0&0&1&0&1&0&0&0&0&2&0&0&0&0&1&0&1&0&0&0&0&2&0&0&0&0&1&0&1\\
0&0&0&0&0&2&2&2&1&1&1&1&0&0&0&0&0&2&2&2&1&1&1&1&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&2&2&1&1&1&1&0&0&0&0&0&0&2&2&1&1&1&1&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&2&1&1&1&1&0&0&0&0&0&0&0&2&1&1&1&1&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&1&1&1&1&1&1&0&0&1&1&0&0&1&1&1&1&1&1&0&0&1&1&0&0&1&1&1&1&1&1&0&0&1&1\\
0&0&1&1&1&1&1&1&1&0&0&1&0&0&1&1&1&1&1&1&1&0&0&1&0&0&1&1&1&1&1&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&2&2&2&2&1&1&1&1&2&2&2&2&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&0&2&2&1&1&1&1&0&2&2&2&2&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&0&0&2&1&1&1&1&0&0&2&2&2&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0&0&1&1&1&1&0&0&0&2&2&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&1&0&1&0&0&0&0&2&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&1&1&1&1&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&1&1&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&1&1&0&0&1&1&1&1&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&0&0&1&0&0&1&1&1&1&1&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&0&0&1
\end{array}\right]$$

Proposition 10: If $m=2^h\cdot p_1^{r_1}\cdot p_2^{r_2}\cdot\cdots\cdot p_n^{r_n}$, such that p_i 's are all distinct primes, $p_i\neq 2$ and g.c.d $(p_i,p_j)=1$ for all i=1,2,⋯,n , *h* and r_i any positive integers then the matrices P($Q_{2m}\times C_4$) and W($Q_{2m}\times C_4$) are taking the forms :

$$P(Q_{2m}\times C_4)=\left[\begin{array}{c|c|c} P(Q_{2m}) & -P(Q_{2m}) & 0\\ \hline 0 & P(Q_{2m}) & -P(Q_{2m})\\ \hline 0 & 0 & P(Q_{2m})\end{array}\right]$$

Which is
[3(r_1+1)(r_2+1)⋯(r_n+1)(h+2)+6]×[3(r_1+1)(r_2+1)⋯(r_n+1)(h+2)+6] square matrix .
And

$$W(Q_{2m}\times C_4)=\left[\begin{array}{c|c|c} W(Q_{2m}) & 0 & 0\\ \hline 0 & W(Q_{2m}) & 0\\ \hline 0 & 0 & W(Q_{2m})\end{array}\right]$$

which is
[3(r_1+1)(r_2+1)⋯(r_n+1)(h+2)+6]×[3(r_1+1)(r_2+1)⋯(r_n+1)(h+2)+6] square matrix .

Proof :

By using the proposition 9 taking the matrix M($Q_{2m}\times C_4$) and the above forms P($Q_{2m}\times C_4$) and W($Q_{2m}\times C_4$) then we have :

P($Q_{2m}\times C_4$). M($Q_{2m}\times C_4$).W($Q_{2m}\times C_4$) =

$$\text{diag }\{\underbrace{2,2,2,2,\cdots,2}_{[3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)-3]-time},1,1,1,1,1,1,1,1,1\}$$

=D($Q_{2m}\times C_4$)

which is
[3(r_1+1)(r_2+1)⋯(r_n+1)(h+2)+6]×[3(r_1+1)(r_2+1)⋯(r_n+1)(h+2)+6] square matrix .

Example 2:

To find the matrices P($Q_{48}\times C_4$) and W($Q_{48}\times C_4$) by the proposition 7 to find P(Q_{48}) and W(Q_{48}) :

$$P(Q_{3.2^4})=\begin{bmatrix}
1&-1&0&0&0&0&0&0&0&0&0&0\\
0&1&-1&0&0&0&0&0&0&0&0&0\\
0&0&1&-1&0&0&0&0&0&0&0&0\\
0&0&0&1&-1&0&0&0&0&0&0&0\\
0&0&0&0&1&-1&0&0&0&0&0&0\\
0&0&0&0&0&1&-1&0&0&0&0&0\\
0&0&0&0&0&0&1&-1&0&0&0&0\\
0&0&0&0&0&0&0&1&-1&0&0&0\\
0&0&0&0&0&0&0&0&1&-1&-1&1\\
0&0&0&0&0&0&0&0&0&0&0&-1\\
0&0&0&0&0&0&0&0&0&0&1&-1\\
0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}$$

$$and\quad W(Q_{3.2^4})=\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&1&1&1&1&1&1&1&1&0&0\\
0&0&0&0&0&0&0&0&1&0&1&0\\
0&0&-1&-1&-1&-1&-1&-1&-1&0&0&1
\end{bmatrix}$$

And by the proposition 9 then:

$$P(Q_{48}\times C_4)=\left[\begin{array}{c|c|c} P(Q_{48}) & -P(Q_{48}) & 0\\ \hline 0 & P(Q_{48}) & -P(Q_{48})\\ \hline 0 & 0 & P(Q_{48})\end{array}\right]$$

$$and\quad W(Q_{48}\times C_4)=\left[\begin{array}{c|c|c} W(Q_{48}) & 0 & 0\\ \hline 0 & W(Q_{48}) & 0\\ \hline 0 & 0 & W(Q_{48})\end{array}\right]$$

Example 3: To find D($Q_{48}\times C_4$) and the cyclic decomposition of the factor group

We find the matrices P($Q_{48}\times C_4$) and W($Q_{48}\times C_4$) as in example 2 and M($Q_{48}\times C_4$) as in example 1,then :

P($Q_{48}\times C_4$).M($Q_{48}\times C_4$).W($Q_{48}\times C_4$)=

diag{2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,1,1,1,1,1,1,1,1,1}=D($Q_{48}\times C_4$)

Then by Theorem 5 we have

$$\text{AC(D(}Q_{48}\times C_4\text{))}=\bigoplus_{i=1}^{27}C_2$$

The following theorem gives the cyclic decomposition of the factor group AC(D($Q_{2m}\times C_4$)) when m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,*h*,*r*∈ $Z^+$and p is prime number .

Theorem 7: If m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,*h*,*r*∈ $Z^+$and p is prime number then the cyclic decomposition of AC($Q_{2m}\times C_4$) is :

$$\text{AC(D(}Q_{2m}\times C_4\text{))}=\bigoplus_{i=1}^{3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)-3}C_2$$

Proof : By using the proposition 9,we can find matrix M(Q_{2m}×C_4) and by the proposition 10,we find P(Q_{2m}×C_4) and W(Q_{2m}×C_4) when m= $2^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$,$h,r\in Z^+$and p is prime number:

P(Q_{2m}×C_4). M(Q_{2m}×C_4). W(Q_{2m}×C_4)= diag{2,2,2,2,2,2,…,2,2,2,1,1,1,1,1,1,1,1,1}

Then ,by the theorem 6 we have :

$$AC(D(Q_{2m}\times C_4))=\bigoplus_{i=1}^{3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)-3} C_2$$

Example 4: Consider the groups ($Q_{7087500}$×C_4) ,(Q_{98000}×C_4),then :

1. $AC(Q_{7087500}\times C_4)=AC(Q_{2^2.3^4.7.5^5}\times C_4)=\bigoplus_{i=1}^{537}C_2$

2. $AC(Q_{98000}\times C_4)=AC(Q_{2^4.7^2.5^3}\times C_4)=\bigoplus_{i=1}^{141}C_2$

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