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Research paper

Investigations of influence of rotor geometry on cogging torque in combined radial and axial flux permanent magnet synchronous motor

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Abstract

In this paper, the investigations of cogging torque in combined radial and axial flux permanent magnet synchronous motor (RAFPMSM) has been carried out using 3D FEM modelling. The influence of design parameters and optimizing techniques have been explored to minimize the cogging torque. In the base model of the machine the level of cogging torque, vibration and the noise are elevated due to the additive effect of both radial section and axial section. Both sections have been investigated separately by modelling in FEM. Slot opening, pole arc to pole pitch ratio and the rotor pole configuration are the major design parameters which directly influences the percentage of the cogging torque component in the torque profile of the machine. These design parameters have been targeted and the optimization attempt is made to achieve the goal of minimum Torque Ripple Factor (TRF). The results shown remarkable improvement in the performance from cogging torque view point. The main emphasis has been made on shape of magnets and skewing techniques in both the sections radial and axial. The techniques reflected noticeable improvement in TRF and discernible reduction in vibration. The performance features of high torque density at moderate speed of the machine has been examined and optimized model has been achieved.

Keywords: *AFPMSM; CRAFPMSM; Cogging Torque; RFPMSM.*

1. Introduction

There are two unwanted torque components in the torque profile of synchronous motor, one is ripple torque component caused by the input current and voltage waveform harmonics and second is the cogging torque caused by the attraction between stator slot teeth and rotor PM poles presented by Goga C et al. [1] and Lai Cet al.[2]. Both these components are major ripple components and makes the operation noisy accomplished with vigorous vibrations thereby deteriorating the performance of synchronous motor unfavourably. Minimization of cogging torque is becoming indispensable in view of the fact that the machine is finding its application in torque-ripple susceptible drives. Numerous techniques for cogging torque suppression are acknowledged in the literature. There are two types of parameters which can be used for optimization to control the cogging torque in PMSM, one is input parameters like current and voltage waveform and second is the design parameters. Investigations of the influence of PMSM parameter variations in optimal stator current design for torque ripple minimization was carried out by Lai C et al .[2] and achieved 20% reduction in ripple torque. Feng G et al. [3] given the analytical solution to optimal current design for torque ripple minimization in PMSM. Practical testing solutions for validating the results of analytical solutions for optimal stator harmonic current design of PMSM torque ripple minimization given by Feng G et al [4]. Ajay Kumar et al. [5] presented FEM based analysis of cogging torque reduction techniques in a permanent magnet wind turbine generator. The method was useful to analyze magnetic flux distribution in the entire body of both stator and rotor. Z.Q Zhu et al.[6] explored the influence of design parameters on the cogging torque in PMSM. PM pole geometry modification, skewing of rotor poles or stator slots, pole arc to pole pitch ratio and slot opening variation influenced the cogging torque. Rotor pole skewing in axial flux PMSM is the simplest and effective method of cogging torque reduction presented by Aydin M et al [7]. Goga C et at. [1] presented stator slot closure and magnet skewing method to minimize cogging torque of PM disc motor. Some techniques are applied in radial geometry and others in axial geometry [6], [8]. Zhou T et al. [9] proposed asymmetric flux barriers technique to reduce cogging torque and operation torque ripple in interior PMSM. Chetan Vasudeva et al. [10] proposed two-dimensional static analysis for magnetic flux density of PM linear electric motor. Similar model, based on 3D FEM provided better solution to diagnose magnetic flux stresses and flux leakage analysis of electromagnetic devices as presented by Ajay Kumar et al. [11]. The cogging torque tolerance analysis of brushless PMSM was efficiently computed by Bramerdorfer G [12]. The techniques mentioned in the literature to reduce the cogging torque in RFPMSM and AFPMSM separately were available [7], [13 - 16]. But less efforts were made for combined radial and axial flux permanent magnet synchronous motor (CRAFPMSM). Manna M et al.[17] presented the 3D FEM analysis of EM force on end winding structure for electrical rotating machines . In the CRAFPMSM there is no end winding wastage. In this paper cogging torque minimization of combined radial and axial flux permanent magnet synchronous motor has been examined which combines the feature of both radial and axial flux machines. The base model proposed by Singh G at el [18] and Ozeki M et al[19] of the combined machine suffers from higher level of cogging torque and higher THD of flux wave form, yielding increased vibrations and noise level of operation. The model has been investigated using Ansys. Software Maxwell version 15. The better torque profile at moderately high speed of the machine proved the machine to be best solution in electric vehicle's operation. In the present paper the CRAFPMSM with different pole shapes and skewing have been investigated for both radial and axial sections and optimization has been achieved by varying the input design parameters and enhancement in the performance was recorded to make the machine more suitable for electric vehicles operation.

2. Problem formulation

The torque profile of the combined radial and axial flux permanent magnet synchronous motor is very much corrupted due cogging torque and ripple torque contribution to large scale in the overall electromagnetic torque developed by the machine. The negative role of the cogging torque becomes intolerable, especially when the machine is used in the electric vehicles. There are two main appropriate solutions to reduce the cogging torque, one is the supplying harmonic free voltage and current input waveforms, and the second is the design parameters optimization. However the meeting out first conditions at the consumer end, is not in users hand and additionally causes economic implications. The paper will explore the possibility of design parameters optimization in order to minimize cogging torque.

3. Cogging torque equations

Cogging torque is the function of flux distribution, stator tooth geometry and rotor position. The complete analysis is categorized in two parts

1) Axial flux section (AFS) modification.
2) Radial flux section (RFS) modification.

Net cogging torque in the CRAFPMSM is calculated analytically using the results of both sections.

3.1. Axial section

Cogging torque in axial section of the proposed model is given by the equation [20].

$$T_{cog}(\theta_r) = -\frac{1}{2}\varphi_g^2 \frac{dR}{d\theta_r} \quad (1)$$

Where ϕg is air gap flux, R is airgap reluctance Ɵr Rotor angular position is given by the equation [20].

$$R(\theta_r) = \frac{\delta_g}{\mu_0[\overline{\Lambda}_0 + \sum_{k=1}^{z}(-1)^{k+1}\overline{\Lambda}_k \cos(kZ_s\theta_r)]} \quad (2)$$

δ_g is air gap length $\overline{\Lambda}_0$ is per unit mean airgap permeance, k is order of the stator slot permeance $\overline{\Lambda}_k$ is per unit value of the kth slotting permeance and Zs is number of stator slots.

3.2. Radial section

The cogging torque due to single slot in the radial section is odd function of rotor position and is given by the equation [14]

$$Tcslot\ (\theta) = \sum_{n=1}^{\infty} T_{csn}\mathrm{Sin}(np\theta) \quad (3)$$

In the equation p is magnet pole number, T_{csn} peak value corresponding to nth harmonics and θ is rotor angular position with respect to the middle of the slot.
The cogging torque due to all the slots is calculated by using the equation [14]

$$Tcg = \sum_{n=1}^{\infty}\sum_{k=0}^{s-1} \mathrm{Cos}\left(\frac{2knp\pi}{s}\right) T_{csn}\mathrm{Sin}(np\theta) \quad (4)$$

Where s is stator slot number

4. Geometry of proposed model

The proposed model is designed with inner single stator and outer compound rotor serving for both axial and radial sections. The axial rotor has two discs with 8 pole each and radial rotor with 8 pole single outer structure. Slotted Torous Stator (STS)[7] is used in the proposed model due to the large number of turns per coil, otherwise Slotless Torous (NST) configuration would be preferred because of negligible cogging torque[21]. In NST geometry, for large number of turns, the airgap increases beyond the permissible limits in both section that reduces airgap flux density and effects performance adversely. The design parameters are given in the Table 1.

Table 1: Design Parameters of Proposed Model

Rated speed	1000 rpm
Rated frequency of supply	100hz
Peak Emf(1000 rpm)	125V
Number of poles	8
Air gap peak flux density	0.7T
Core inner diameter of axial rotor	140mm
Core Outer diameter of axial rotor	220mm
Core Axial length of stator	40mm
Number of coils	48
Number of turns per coil	8 X 4
Winding electrical loading	< 45 KA/m
Ratio of inner to outer radius	0.6-0.7
Remanence	1.1T
Demagnetizing field	900KA/m
Peak current(rated torque)	< 20A
PM Dimensions for axial rotor	40X5mm
PM material	NdFeB
Conductor size	0.709mm
Magnet thickness Radial rotor	4.5mm
Winding phases	3
Inner diameter of radial rotor	226mm
Outer diameter of radial rotor	245mm

The complete assembly of the proposed model with 8 pole radial section and 8 pole with each axial section and torous stator is shown in figs. 1. & 2. In the proposed model, external rotor design is simulated in which the electromagnetic torque generated will be preferably more than interior rotor design. The leakage of the magnetic flux and problem overfluxing of rotor parts is less. The magnetic flux remains more uniformly distributed and thereby electromagnetic stresses are not exceeding beyond the limiting value, further in the outer rotor geometry cooling of the magnets is better to interior rotor design and thereby demagnetization of rotor permanent magnet poles does not become more problematic. It is more advantageous for the direct wheel drive applications particularly in electric vehicles.

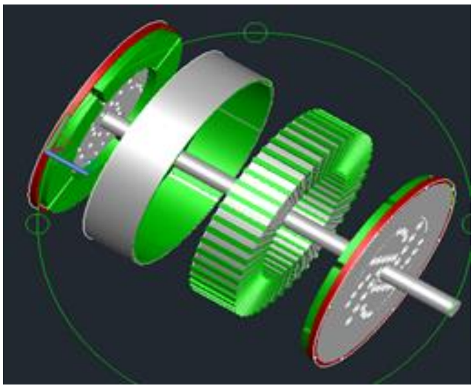

Fig. 1: Complete Assembly of CRAFPMSM.

The structural modifications of axial and radial sections in the proposed model have been investigated separately and the results then analytically combined to evaluate the cumulative effect of cogging torque in the CRAFPMSM.

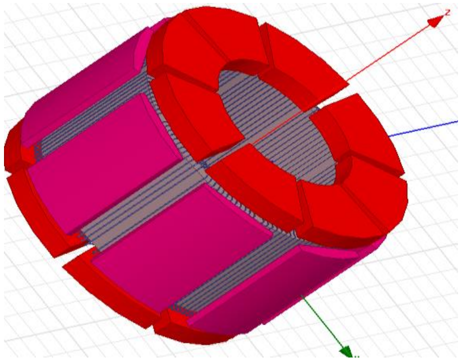

Fig. 2: 3D Radial and Axial Rotor and Inner TS Stator.

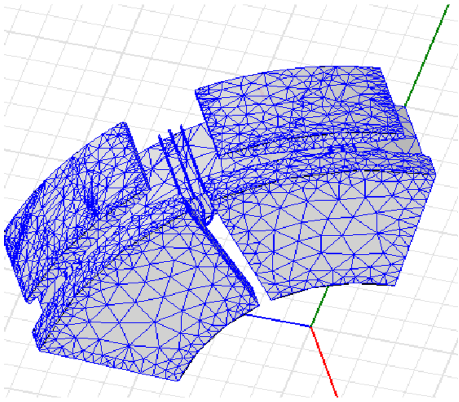

Fig. 3: 3D Section of Both Radial and Axial Rotor and Stator Meshed in FEM.

Fig. 3 presented the one fourth section of the proposed model in which meshing is done to locate flux tresses and high cogging torque zones in both the sections.

5. Cogging torque assessment in the conventional model

In the conventional model of CRAFPMSM presented by Singh G et. al [18], the electromagnetic torque is developed by two sections, one radial section and other axial section. The total torque has been evaluated analytically. The model consists of unskewed rotor poles, maximum possible pole arc ratio and conventionally opening stator slots as shown in fig. 4.

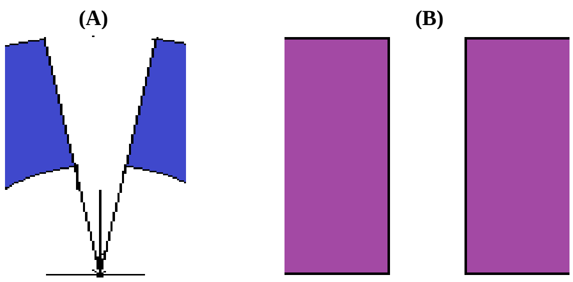

Fig. 4: Unskewed Rotor Poles (A) Axial Poles (B) Radial Poles

In the model the percentage of cogging torque in the torque profile is very high because of the attraction between stator slots and rotor poles occurred over whole portion simultaneously in both radial poles as well as in axial poles for two discs.

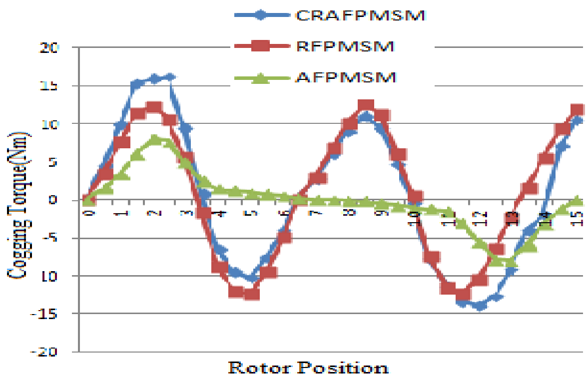

Fig. 5: Variation of Cogging Torque with Rotor Position for Axial and Radial Rotors and Combined Rotor.

Initially the cogging torque investigation was carried out for both the section radial and axial separately and the net cogging torque was evaluated by numerical methods. The rotor position is varied from 0^o to 15^o in steps of 1^o per step and assessment of cogging torque was made using the equations (1) and (4). The resultant cogging torque is simulated as shown in the fig. 5.

6. Design parameter modifications

Cogging torque minimization in the CRAFPMSM by structural modifications can be achieved in two ways: one is stator design modification and second is rotor design modifications.
Stator deign modification techniques are stator slot skewing, slot opening, fractional slots, and stator slot to rotor pole ratio. The stator design modifications to minimize the cogging torque component introduces structural complexity and consequently higher manufacturing cost. In general, practice of stator design modification to reduce cogging torque is avoided.
Rotor side methodology further classified into two categories: axial section modifications and radial section modifications.
3D simulation modelling was carried out using software Maxwell version 15 to investigate the influence of modifications of design parameters on the cogging torque.
There are various design parameter recorded in the literature [7] that influence the generation of cogging torque. In this paper following three key design parameters have been examined which predominately reduces cogging torque contribution in the main flux torque without causing much structural complexity.

1) Stator Slot opening.
2) Rotor pole skewing.
3) Pole arc ratio modification.

The multiobjective approach was made and the analysis has been carried out by the simultaneous employment of more than one modification. The results presented appropriate solution for cogging torque minimization.

6.1. The pole arc ratio

Pole arc ratio is the ratio of the pole arc to the pole pitch. The cogging torque is the function of the pole arc to pole pitch ratio. The pole arc ratio for minimum fundamental component of cogging torque without fringing effect is given by the relation [6]

$$\alpha_p = \frac{N_c - k(2P)}{N_c} \tag{5}$$

Here N_c is the smallest common multiple between the stator slot number and rotor pole pairs 2p, and k is constant. Its value varies from 1 to an integer less than N_c /2p.
When fringing effect is considered the optimum value of α_p will be increased slightly, typical increase ranges from 0.01 to 0.03[6].
Ajay kumar at el [11] explored FEM based leakage field analysis of the electromagnetic devices, similar technique is employed to

estimate fringing effect. For different slot to rotor pole combination the optimum value of α_p is given by the Table 2

Table 2: Pole Arc Ratio for Different Slot/Pole Combination

S.N.	No. stator slots	No. of rotor poles	Smallest common multiple(N_c)	α_p
1	12	8	12	0.66
2	24	8	24	0.83
3	48	8	48	0.91

Usually higher value of α_p is preferred to maximize the airgap flux and eventually higher average electromagnetic torque. It can be achieved by taking large number of stator slots for same number of pole pairs. The variation of peak cogging torque 12/8 slot to pole combination model for both radial section as well as for axial section with α_p has been obtained from 3D FEA simulation performed with unskewed rotor poles.

6.2. Rotor pole skewing

One of the most effective means of reducing the cogging torque is the skewing of rotor poles. This technique also improves the back EMF wave form and consequently reduces ripple torque component. Due to the simple flat geometry of the poles, the skewing in AFPM section is easier than in RFPM section having curved pole structures.
In the proposed model the skewing of both the sections of the machine have been analyzed separately by performing series of 3D FEA simulations

6.2.1. Skewing of AFPM section

In the AFPM section six types of skewing techniques have been explored. As shown in the fig. 4, Singh G et al.[18] carried out investigation with unskewed rotor poles keeping pole arc ratio same at the outer diameter and inner diameter of rotor disc. The cogging torque level is very in the torque profile. Since the cogging torque is generated due to the attraction between rotor pole and stator slot teeth, further the amplitude and nature of waveform is the function of pole arc ratio, the series 3D FEM simulations have been performed by varying pole arc ratio to obtain the maxima and minima of the cogging torque.
Conventional skewing is one of the effective solution and easy way of reducing the cogging torque in AFPM section. Fig. 6 (a). shows the conventional skewing in AFPM section. The inner pole edge is shifted from position XY to X1Y1 and the new shape of skewed pole is ABX1Y1. The both sides are revolved by equal mechanical degree w. r. t fan shaped pole edges. Series of simulation have been performed by varying the skew angle from zero to one slot pitch and the effect of skewing on the cogging torque is recorded as shown in fig. 8.
Triangular type of skewing results into triangular space between adjacent poles. The outer edges of the poles brought closure and the inner edges are moved away. The skewed magnet is shown in the fig. 6 (b). The pole arc ratio at the outer diameter of the rotor is increased and at the inner diameter decreased. The analysis is carried out by varying the skew angle through same mechanical degree as in conventional skewing to have better comparison
For small number of rotor poles, the parallel skewing technique is the highly significant in AFPM machine to reduce cogging torque. This technique has not introduced much structural complexity and cost. The edges of adjacent poles are made parallel to each other and the pole arc ratio at the outer diameter of rotor is more than at the inner diameter illustrated in fig. 6 (c) The simulations have been performed with different pole ratios. Good results are reflected as shown in the fig. 9.

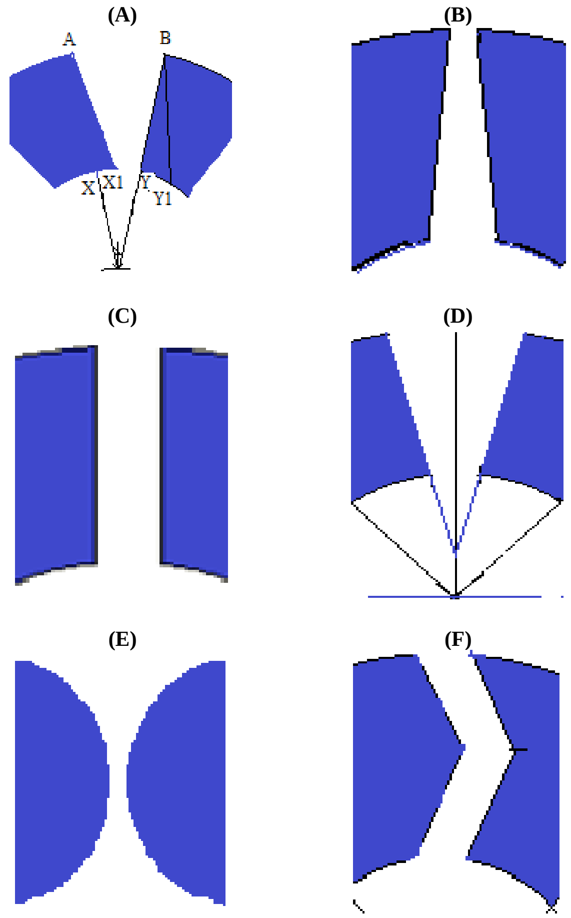

Fig. 6: Axial Rotor Magnet Skewing (A) Conventional Skewing (B) Triangle Shape Skewing (C) Parallel Skewing (D) Trapezoid Type Skewing (E) Round Shape Skewing (F) Dual Skewing.

Trapezoidal type of pole skewing in AFPM machine is realized by drawing pole edges closure at the inner diameter of rotor than at the outer diameter. The geometry of pole appeared trapezoidal as shown in fig. 6 (d) and pole arc ratio at the inner side increased as compared to outer side. The skew angle is increased to maximum square shaped magnets which is the extreme case of trapezoidal skewing. However the leakage flux at the inner edge is high that adversely effects the electromagnetic torque generated.
In order to reduce cogging torque, the round pole skewing technique is successfully employed in the AFPM section. The magnet edges facing the adjacent poles are made circular as shown in the fig. 6 (e). 3D FEA simulation is carried out with different diameters of round edges of the pole. The peak cogging torque becomes less and wave form turn out to be smoother. Further technique involved simple geometry.
Dual skewing is the most effective skewing technique and is realized by introducing magnet skewed at both the ends as shown in fig. 6 (f). The whole edge of the magnet is not influenced at same time by the stator slot teeth. The skewing angle at both the ends is kept identical to have symmetrical shape and therefore more sine waveform of back EMF. The space between poles is uniform and pole arc ratio remain unchanged along the edge for particular skew angle. The simulation work is performed for different skew angles maximum upto one slot pitch. The significant reduction cogging torque is achieved.

6.2.2. Skewing of RFPM section

The radial rotor poles have curved geometries, due to which the implementation of skewing causes lot of structural complications. In this paper some of the skewing techniques explored are parallel sided skewing, triangular skewing, round pole skewing and dual skewing. As shown in fig. 4 (b), the conventional magnet arrangement is investigated by Ozeki M et al [19]

The cogging torque component corrupted the torque profile of the machine to large extent due to the greater attraction between rotor pole and stator slot edges. The rotor pole faces of the adjacent poles are parallel to slot edges that yield huge amount of cogging torque.

As shown in fig.7 (a), the parallel sided skewing is the simplest form of skewing used in RFPM machines. The one side of the each pole is shifted along the circumference. Skewed magnet has acquired the shape ABEF obtained by the displacement of side CD of conventional pole to EF. The model is simulated with the different skew angles and the minima in the cogging torque waveform is located.

Triangular skewing technique is employed by bringing the pole edges of the adjacent poles near to each other at one side and displacing away on the other side. The pole geometry is illustrated in fig. 7 (b). The shape of inter-polar space becomes more triangular. The pole arc ratio gradually increases from the side with far away edges toward the end with closure pole edges. On the side of higher pole arc ratio the leakage flux is more and therefore it has become source of higher harmonics in main torque profile.

Round pole skewing in radial flux machine is common and readily used to minimize the cogging torque [7]. The edges of the adjacent poles facing each other have been made circular as shown in fig. 7 (c).

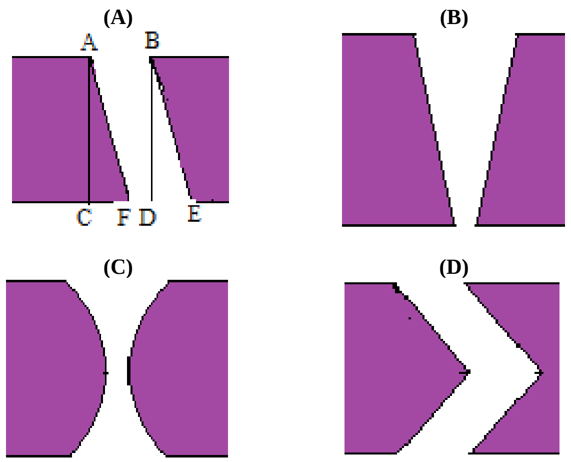

Fig. 7: Radial Rotor Magnet Skewing. A) Parallel Skewing B). Triangular Skewing C) Round Skewing and D) Dual Skewing.

The inter-polar separation is no longer uniform, it is minimum at the center and increases on either sides and in the same fashion the pole arc ratio varies. The machine is investigated with different radii and minimum cogging torque is achieved. The reduction of cogging torque with dual skewing of rotor pole effectively used in RFPM [22]. This technique readily illiminated the axial forces. However causes more complicated geometry and increases manufacturing cost. The pole shape is shown in fig. 7 (d). The skew angle is kept identical on both edges and the pole arc ratio is not disturbed through out the space between the poles.The cogging torque is evaluated with different skew angles. The method is very much succesful in reducing the cogging torque.

7. Results and discussions

In this section the influence of pole arc ratio and rotor pole skewing on the cogging torque is presented.

7.1. Influence of pole arc ratio

The peak cogging torque in radial section was recorded 2.6 N-m. Whereas in axial is 16.8 Nm. The minima in both the cases occurred at α_p = 0.68. The peak cogging torque in axial section is reduced to about 6.25% and in case of radial section to 25% at pole arc ratio 0.68. The variation of cogging torque with pole arc ratio in both the sections is shown in fig. 8. However the useful torque is very much sensitive to pole arc ratio variation. The cogging torque reduction is restricted to certain extent using the pole arc optimization technique. The actual value of α_p derived from 2D analytical model is slightly higher on account of fringing effect. Subject to the provisions of slot accommodation large value of N_c for same number of rotor poles is the best choice for achieving large electromagnetic torque and thereby reducing the ripple factor, though may the pole arc ratio sometime also depends upon machine topology.

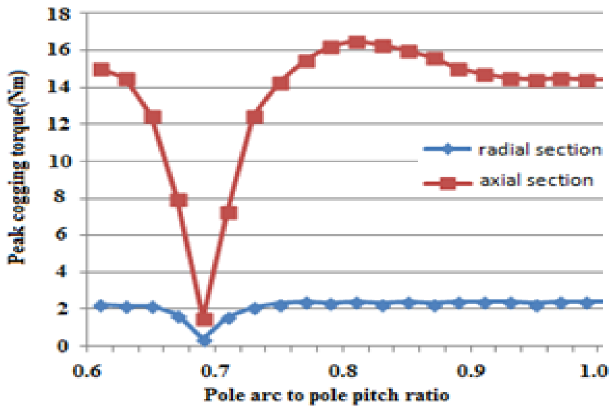

Fig. 8: Variation of Peak Cogging Torque with Pole Arc to Pole Pitch Ratio.

7.2. Influence of rotor pole skewing

The peak to peak cogging torque in the case of unskewed fan shaped poles in the proposed model was recorded 19.44 Nm. The conventional skewing results in the reduction of cogging torque. Observations revealed that with increase in the skew angle, the cogging torque decreased as shown in fig. 9. The investigations have been made by varying the skew angle upto one slot pitch. The peak to peak cogging torque beyond one slot pitch shows no noticeable reduction. The minimum cogging torque recorded was 2.16 Nm. obtained at skew angle equal to one slot pitch. Similar results are obtained in triangular skewing. The maximum reduction of cogging torque in this type of skewing method is 60%. The fig. 9 presented the comparison of the reduction of cogging torque of both conventional skewing and triangular skewing.

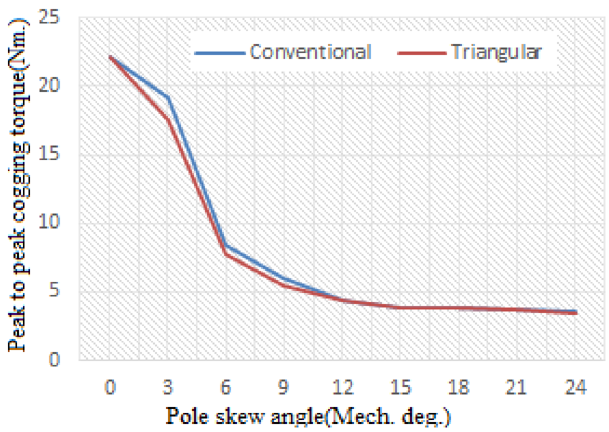

Fig. 9: Variation of Peak Cogging Torque vs. Skewing Angle for Conventional and Triangular Skewing.

The comparison in other type of skewing, parallel sided, trapezoidal, circular and dual type skewing is made by taking the same variation of skew angle in terms of magnet area to pole area or pole arc ratio. The results presented that parallel sided skewing offered better reduction in the cogging torque. In the pole arc ratio variation from 0.4 to 0.9, the minima of cogging torque 2.53 Nm is achieved at pole arc ratio 0.6 and the maxima corresponding to 0.7 and the maximum value of peak to peak cogging torque is recorded as 11 Nm. The reduction upto 40% have been achieved.

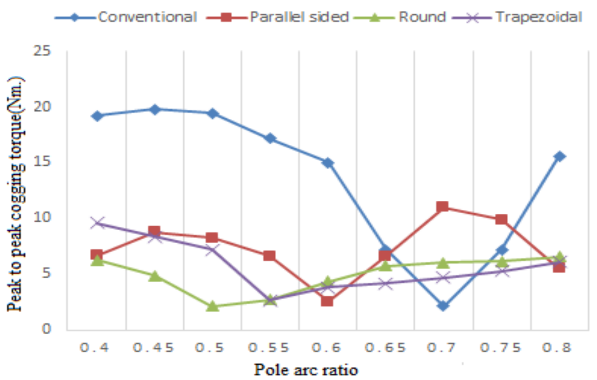

Fig. 10: Variation of Peak To Peak Cogging Torque with Pole Arc Ratio for Conventional, Parallel Sided, Round and Trapezoidal Skewing.

On the other hand in the trapezoidal skewing technique, the minimum cogging torque was 2.64 Nm. occurred at pole arc ratio 0.55 and maximum value was 9.6 Nm. The skewing was carried to squared shape pole geometry. Increase of skew angle beyond square shape geometry results into excessive flux leakage and thereby reduction in the useful torque.

Table 3: Comparison of Peak To Peak Cogging Torque in Various Skewing Techniques

Type of skewing	Minimum peak to peak cogging torque(Nm.)	Maximum peak to peak cogging torque(Nm.)
Conventional	3.56	19.2
Triangular	3.5	17.6
Parallel sided	2.53	11.0
Trapezoidal	2.64	9.6
Round	2.16	6.5
Dual	2.0	7.0

In the round shape pole skewing method the variation of peak cogging torque was measured as function of pole arc ratio that presented the pole cross sectional area. The maximum peak to peak cogging torque was recorded 6.5 Nm. corresponding to 75% of pole cross sectional area and minimum 2.16 Nm. corresponding to 55% of pole cross sectional area. The reduction upto 85% in the cogging torque is obtained with round skewing technique. The table 3 presented the maximum and minimum peak to peak cogging torque for various skewing techniques in axial section. The dual skewing technique results in more structural complexity. The skewing at both the edges is kept same the reduction in the cogging torque upto 45% is achieved.

In CRAFPMSM model the skew angle is changed by equal amount in both radial section and axial section. The cogging torque reduction upto 80% has been achieved skew angle was varied from 0 to 1 slot pitch. Fig. 8 shows that the peak cogging torque beyond 18^{o} (mech.) has not decreased to a noticeable value.

8. Conclusion

Cogging torque assessment results presented that major contribution occurred due to axial section of the machine. In totality 70% of the cogging torque is shared by AFPM section and rest comes from RFPM section. The main focus was to analyze the effect of pole arc ratio and magnet skew technique on cogging torque. Both the techniques simultaneously implemented and the examination of results obtained, concluded that in comparison to conventional model, there is large reduction in the deterioration of torque quality, however compromising with the average electromagnetic torque to small extent. The optimal pole arc ratio for all the skew technique is 0.75 at which maximum reduction in cogging torque is 55%. in round magnet skewing. 3D FE analysis of the prototype were also investigated for the minimization of the pulsating torque component in the resultant torque. The skew angle variation upto one slot pitch provided the fruitful results in reduction of cogging torque by 80%. Due to excessive leakage of flux the skewing is kept less than one slot pitch otherwise machine suffers from huge reduction of useful torque. For observing structural symmetry the skew angle changed by equal amount in both radial and axial section. Further improvement in torque quality by implementation of more techniques at time causes structural complexity.

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