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Research paper

Intuitionistic Fuzzy Soft Hypergraph

S.Thilagavathi

Assistant Professor
Department of Mathematics
Erode Arts &Science College, Erode
**Corresponding author E-mail: thilak.pri@gmail.com*

Abstract

In this paper the notions of intuitionistic fuzzy soft hypergraph and strong intuitionistic fuzzy soft hypergraph are introduced. Also studied about union of two intuitionistic fuzzy soft hypergraphs with suitable examples..

Keywords: *Intuitionistic fuzzy soft hypergraph, strong intuitionistic fuzzy soft hypergraph union of intuitionistic fuzzy soft hypergraphs*

1. Introduction

The concept of fuzzy soft set theory was developed to deal with vagueness in real life situation. A number of real life problems in engineering, medical sciences etc., implicate broad data and their solution bring in the application of principles based on uncertainty and imprecision. Such uncertainties can be dealt with fuzzy set theory initiated by Zadeh[11] and soft set theory presented by Molodstov[5]. Majietal [4] introduced the concept of intuitionistic fuzzy soft set.

The concept of fuzzy graph was first initiated by Rosenfeld [7]. The idea of intuitionistic fuzzy set as a generalization of fuzzy set first introduced by Atanassov [1] which had a wider application in many fields. Later the concept of intuitionistic fuzzy graph was persuaded by Atanssov [2]. A special case of Atanssovs intuitionistic fuzzy graph were introduced by Karunambigai and Parvathi [3]. Thumbakara and George [10] introduced soft graph. Mohinta and Samanta [8] introduced the concept of fuzzy soft graph in 2015. In 2009, Parvathi etal [6] introduce the concept of intuitionistic fuzzy graph and their certain operations. In this paper the basic notions of intuitionistic fuzzy soft hypergraph are introduced and discuss about its union.

2. Preliminaries

Definition 2.1.
A fuzzy set of a base set $V=\{v_1,v_2,\dots,v_n\}$ (non-empty set) is specified by its membership function μ; where $\mu: V\to[0,1]$ assinging to each $v_i\in V$, the degree or grade to which $V\in\mu$

Definition 2.2.
A pair (F,A) is called a soft set over U if and only if F is a mapping of A into the set of all subsets of the set U.

Definition 2.3.
A pair $(\bar{F},A)$ is called a fuzzy soft set over view where $\bar{F}$ is a mapping given by $\bar{F}: A\to I^U; I^U$ denotes the collection of all fuzzy subsets of $U; A\subseteq P$.

Definition 2.4.
A pair $(\breve{F},A)$ is called an intuitionistic fuzzy soft set over U where $\breve{F}$ is a mapping given by $\breve{F}: A\to IF^U; IF^U$ denotes the collection of all intuitionistic fuzzy subsets of $U; A\subseteq P$

Definition 2.5.
Let $V=\{v_1,v_2,\dots,v_n\}$ (non-empty set), P (parameter set) and $A\subseteq P$. Also
Let (i) $\mu_i: A\to I^U(V)$ ($I^U(V)$ denotes collection of all fuzzy subsets in V) $a\mapsto mu_i(a)=\mu_{ia}$ (Say), $a\in A$ and $\mu_{ia}: V\to[0,1], (A,\mu_i)$ fuzzy soft vertex. (ii) $\mu_{ij}: A\to I^U(V\times V)$ ($I^U(V\times V)$ denotes collection of all fuzzy subsets in $V\times V$) $a\mapsto \mu_{ij}(a)=\mu_{ija}$ (say), $a\in A$ and $\mu_{ija}: V\times V\to[0,1](A,\mu_i),(A,\mu_{ij})\mapsto \mu_{ija}(v_i,v_j),(A,\mu_{ij})$ fuzzy soft edge.
Then $\left((A,\mu_i),(A,\mu_{ij})\right)$ is called fuzzy soft graph if and only if $\mu_{ija}(v_i,v_j)\le\mu_{ia}(v_i)\Lambda\,\mu_i(v_j)$ for each $(v_i,v_j)\in V\times V$, for each $a\in A$ and $i,j=1,2,\dots,n$

Definition 2.6.
Let $G=(V,E)$ be a simple graph,
$V=\{v_1,v_2,\dots,v_n\}$ (non-empty set), $E\subseteq V\times V$, P (Parameter Set) and $A\subseteq P$. Also let

(1) μ_i is a membership function defined on V by $\mu_i: A\to IF^U(V)$ ($IF^U(V)$ denotes of all intuitionistic fuzzy subsets in V) $a\mapsto\mu_i(a)=\mu_{ia}$ (say), $a\in A$ and $\mu_{ia}: V\to[0,1]$, $v_i\mapsto\mu_{ia}(v_i)$, (A,μ_i) intuitionistic fuzzy soft vertex of membership function and γ_i is a non-membership function defined on V by $\gamma_i: A\to I^U(V)$ ($I^U(V)$ denotes collection of all intuitionistic fuzzy subsets in V) $a\mapsto\gamma_i(a)=\gamma_{ia}$ (say), $a\in A$ and $\gamma_{ia}: V\to[0,1]$, $v_i\mapsto\gamma_{ia}(v_i)$, (A,γ_i) intuitionistic fuzzy soft vertex non-membership function such that $0\le\mu_{ia(v_i)}+\gamma_{ia(v_i)}\le 1\forall v_i\in V$, $i=1,2,\dots,n$ and $a\in A$.

(2) μ_{ij} is a membership function defined on E by $\mu_{ij}: A\to IF^U(V\times V)$ ($IF^U(V\times V)$ denotes collection of all intuitionistic fuzzy subsets in E) $a\mapsto\mu_{ij}(a)=\mu_{ija}$ (say), $a\in A$ and $\mu_{ija}: V\times V\to[0,1]$, $(v_i,v_j)\mapsto\mu_{ija}(v_i,v_j).\gamma_{ij}$ is a non-

membership function defined on E by $\gamma_{ij}: A \to IF^U(V \times V)$ ($IF^U(V \times V)$ denotes collection of all intuitionistic fuzzy subsets in E) $a \mapsto \gamma_{ij}(a) = \gamma_{ija}$ (say), $a \in A$ and $\gamma_{ija}: V \times V \to [0,1]$, $(v_i, v_j) \mapsto v_{ija}(v_i, v_j)$ where (A, μ_{ij}), (A, γ_{ij}) are intuitionistic fuzzy soft edge membership and non-membership function satisfying

(i) $\mu_{ija}(v_i, v_j) \leq min(\mu_{ia}(v_i), \mu_{ia}(v_i))$

(ii) $\gamma_{ija}(v_i, v_j) \leq max(\gamma_{ia}(v_i), \gamma_{ia}(v_i))$

(iii) $0 \leq \mu_{ija}(v_i, v_j) + \gamma_{ija}(v_i, v_j) \leq 1$

$0 \leq \mu_{ija}(v_i, v_j)$, $\gamma_{ija}(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E, i, j = 1,2, \dots, n$ and $a \in A$ Then $G^* = \left(V, E, (A, \mu_i), (A, \gamma_i)(A, \mu_{ij}), (A, \gamma_{ij})\right)$ is said to be intuitionistic fuzzy soft graph (IFSG) and this IFSG is defined by $G^*_{A,V,E}$.

Definition 2.7.
An intuitionistic fuzzy hypergraph (IFHG) H is an ordered pair $H = (V, E)$ where

1. $V = \{v_1, v_2, \dots, v_n\}$, a finite set of intuitionistic fuzzy vertices,
2. $E = \{e_1, e_2, \dots, e_m\}$, a family of intuitionistic fuzzy subsets of V
3. $e_j = \left\{\left(v_i, \mu_j(v_i), \gamma_j(v_i)\right): \mu_j(v_i), \gamma_j(v_i) \geq o \text{ and } 0 \leq \mu_j(v_i) + \gamma_j(v_i) \leq 1\right\}$, $j = 1,2, \dots, m$,
4. $e_j \neq \emptyset$, $j = 1,2, \dots, m$
5. $U_j supp(e_j) = V, j = 1,2, \dots, m$

Here , the edges e_j are IFSs. $\mu_j(v_i)$ and $\gamma_j(v_i)$ denote the degrees of membership and non-membership of vertex v_i to edge e_j. Thus, the elements of the incidence matrix of IFHG are of the form $\left(a_{ij}\mu_j(v_i), \gamma_j(v_i)\right)$. The sets V and E are crisp sets.

Definition 2. 8.
An intuitionistic fuzzy soft hypergraph (IFSHG) H is an ordered pair $H = (V, E)$ where

1. $V = \{v_1, v_2, \dots, v_n\}$, a finite set of intuitionistic fuzzy vertices,
2. $E = \{e_1, e_2, \dots, e_m\}$, a family of intuitionistic fuzzy subsets of V
3. P (Parameter set) and $A \subseteq P$.
4. $e_j = \left\{\left(v_i, \mu_{ja}(v_i), \gamma_{ja}(v_i)\right): \mu_{ja}(v_i), \gamma_{ja}(v_i) \geq 0 \text{ and } 0 \leq \mu_{ja}(v_i) + \gamma_{ja}(v_i) \leq 1\right\}$, $j = 1,2, \dots, m$, $a \in A$
5. $e_j \neq \emptyset$, $j = 1,2, \dots, m$
6. $U_j supp(e_j) = V, j = 1,2, \dots, m$.

Here, the edges e_j are IFSs. (A, μ_{ja}) and (A, γ_{ja}) denote the degrees of membership and non-membership of vertex v_i to intuitionistic fuzzy soft edge e_j. Thus, the elements of the incidence matrix of IFHG are of the form $\left(v_i, \mu_{ja}(v_i), \gamma_{ja}(v_i)\right)$. The sets V and E are crisp sets.

Example 2.1.
Consider an IFSHG $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{(v_1, v_2, v_3), (v_2, v_4, v_5), (v_2, v_4), (v_4, v_6)\}$

Definition 2. 9 An intuitionistic fuzzy soft hypergraph is said to be strong intuitionistic fuzzy soft hypergraph if

$$\mu_{ija}(v_i, v_j) = \min(\mu_{ia}(v_i), \mu_{ia}(v_j)) \text{and}$$

$$\gamma_{ija}(v_i, v_j) = \max\left(\gamma_{ia}(v_i), \gamma_{ia}(v_j)\right) \forall (v_i, v_j) \epsilon E \text{and } a \in A$$

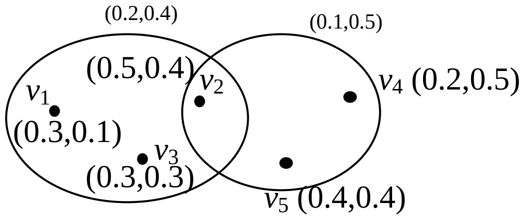

Fig.1: .*IFSHG* corresponding to the parameter a_1

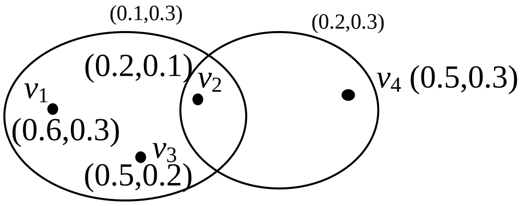

Fig.2:*IFSHG* corresponding to the parameter a_2

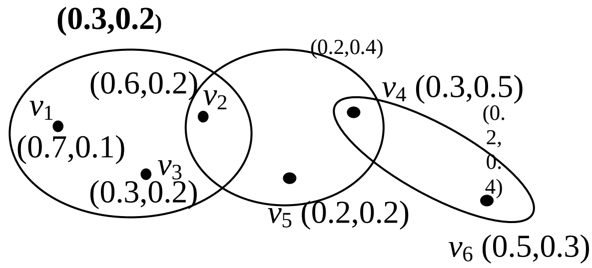

Fig.3:*IFSHG* corresponding to the parameter a_3

and its incidence matrix is as follows,

$$\langle\mu_{ia}, \gamma_{ia}\rangle \equiv \begin{pmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ a_1 & \langle 0.3,0.1\rangle & \langle 0.5,0.4\rangle & \langle 0.3,0.3\rangle & \langle 0.2,0.5\rangle & \langle 0.4,0.4\rangle & \langle 0,1\rangle \\ a_2 & \langle 0.6,0.3\rangle & \langle 0.2,0.1\rangle & \langle 0.5,0.2\rangle & \langle 0.5,0.3\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ a_3 & \langle 0.7,0.1\rangle & \langle 0.6,0.2\rangle & \langle 0.3,0.2\rangle & \langle 0.3,0.5\rangle & \langle 0.2,0.2\rangle & \langle 0.5,0.3\rangle \end{pmatrix}$$

$$\langle\mu_{ija}, \gamma_{ija}\rangle \equiv \begin{pmatrix} & (v_1, v_2, v_3) & (v_2, v_4, v_5) & (v_2, v_4) & (v_4, v_6) \\ a_1 & \langle 0.2,0.4\rangle & \langle 0.1,0.5\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ a_2 & \langle 0.1,0.3\rangle & \langle 0,1\rangle & \langle 0.2,0.3\rangle & \langle 0,1\rangle \\ a_3 & \langle 0.3,0.2\rangle & \langle 0.2,0.4\rangle & \langle 0,1\rangle & \langle 0.2,0.4\rangle \end{pmatrix}$$

Definition 2.10
Let $G = (V, E)$ be a simple graph, P (Parameter set). Also let $V_1, V_2 \subseteq V, E_1, E_2 \subseteq E, A, B \subseteq P$ and $G^*_{A,V_1,E_1} = (V_1, E_1, (A, \mu_i), (A, \gamma_i), (A, \mu_{ij}), (A, \gamma_{ij}))$ and $G^*_{B,V_2,E_2} = V_2, E_2, (B, \mu_i'), (B, \gamma_i'), (B, \mu_{ij}'), (B, \gamma_{ij}'))$ Be two IFSHGs with $V_1 \cap V_2 \neq \emptyset$. Then the union of IFSHGs $G^*_{C,V_3,E_3} = G^*_{A,V_1,E_1} \cup G^*_{B,V_2,E_2}$ with the condition that $\mu_{ij}(v_i, v_j) \geq \max(\mu_{ia}'(v_k))$ and $\gamma_{ij}(v_i, v_j) \geq \max\left(v_{ia}'(v_k)\right) \forall v_i, v_j, v_k \in V, a \in P$
And for any $i, j, k = 1,2, \dots, n$ is defined to be $G^*_{C,V_3,E_3} = (V_3, E_3, (C, \mu_i''), (C, \gamma_i''), (C, \mu_{ij}''), (C, \gamma_{ij}'')$ where $C = A \cup B, V_3 = V_1 \cup V_2$ and

$$\langle \mu_{ia}{''}, \gamma_{ia}{''} \rangle (v_i) = \begin{cases} \langle (\mu_{ia}, \gamma_{ia})(v_i) \rangle & \forall v_i \in V_1 | V_2 \text{ and } a \in A|B \\ \langle (0,1) \rangle & \forall v_i \in V_2 | V_1 \text{ and } a \in A|B \\ \langle (\mu_{ia}, \gamma_{ia})(v_i) \rangle & \forall v_i \in V_1 \cap V_2 \text{ and } a \in A|B \\ \langle \mu'_{ia}(v_i), \gamma'_{ia}(v_i) \rangle & \forall v_i \in V_2 | V_1 \text{ and } a \in B|A \\ \langle (0,1) \rangle & \forall v_i \in V_2 | V_1 \text{ and } a \in B|A \\ \langle \mu'_{ia}(v_i), \gamma'_{ia}(v_i) \rangle & \forall v_i \in V_1 \cap V_2 \text{ and } a \in B|A \\ \langle \mu_{ia}(v_i) \vee \mu'_{ia}(v_i) \rangle, \langle \gamma_{ia}(v_i) \wedge \gamma'_{ia}(v_i) \rangle & \forall v_i \in V_1 \cap V_2 \text{ and } a \in A \cap B \\ \langle (\mu_{ia}, \gamma_{ia})(v_i) \rangle & \forall v_i \in V_1 | V_2 \text{ and } a \in A \cap B \\ \langle \mu'_{ia}(v_i), \gamma'_{ia}(v_i) \rangle & \forall v_i \in V_2 | V_1 \text{ and } a \in A \cap B \end{cases}$$

$$\langle \mu_{ija}{''}, \gamma_{ija}{''} \rangle (v_i, v_j) = \begin{cases} \langle \mu_{ija}, \gamma_{ija}(v_i, v_j) \rangle & \forall (v_i, v_j) \in (V_1 \times V_1) | (V_2 \times V_2) \text{ and } a \in A|B \\ \langle (0,1) \rangle & \forall (v_i, v_j) \in (V_2 \times V_2) | (V_1 \times V_1) \text{ and } a \in A|B \\ \langle (\mu_{ija}, \gamma_{ija})(v_i, v_j) \rangle & \forall (v_i, v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2) \text{ and } a \in A|B \\ \langle (\mu'_{ija}, \gamma'_{ija})(v_i, v_j) \rangle & \forall (v_i, v_j) \in (V_2 \times V_2) | (V_1 \times V_1) \text{ and } a \in B|A \\ \langle (0,1) \rangle & \forall (v_i, v_j) \in (V_1 \times V_1) | (V_2 \times V_2) \text{ and } a \in B|A \\ \langle (\mu'_{ija}, \gamma'_{ija})(v_i, v_j) \rangle & \forall (v_i, v_j) \in (V_2 \times V_2) \cap (V_1 \times V_1) \text{ and } a \in B|A \\ \langle (\mu_{ija} \vee \mu'_{ija})(v_i, v_j) \rangle, \langle (\gamma_{ija} \wedge \gamma'_{ija})(v_i, v_j) \rangle & \forall (v_i, v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2) \text{ and } a \in A \cap B \\ \langle (\mu_{ija}, \gamma_{ija})(v_i, v_j) \rangle & \forall (v_i, v_j) \in (V_1 \times V_1) | (V_2 \times V_2) \text{ and } a \in A \cap B \\ \langle (\mu'_{ija}, \gamma'_{ija})(v_i, v_j) \rangle & \forall (v_i, v_j) \in (V_2 \times V_2) \cap (V_1 \times V_1) \text{ and } a \in A \cap B \end{cases}$$

Example 2.2.
The incidence matrix of G_1 and G_2 is given below

$$\langle \mu_{ia}, \gamma_{ia} \rangle \equiv \begin{pmatrix} G_1 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\ a_1 & \langle 0.3,0.3 \rangle & \langle 0.4,0.4 \rangle & \langle 0.6,0.3 \rangle & \langle 0.2,0.1 \rangle & \langle 0.5,0.4 \rangle & \langle 0.6,0.2 \rangle & \langle 0.7,0.1 \rangle \\ a_2 & \langle 0.5,0.2 \rangle & \langle 0.2,0.2 \rangle & \langle 0.7,0.2 \rangle & \langle 0.6,0.3 \rangle & \langle 0.3,0.2 \rangle & \langle 0.5,0.3 \rangle & \langle 0.1,0.8 \rangle \end{pmatrix}$$

$$\langle \mu_{ija}, \gamma_{ija} \rangle \equiv \begin{pmatrix} G_1 & (v_1, v_4, v_5) & (v_2, v_4, v_6) & (v_2, v_6, v_7) \\ a_1 & \langle 0.2,0.4 \rangle & \langle 0.1,0.1 \rangle & \langle 0.5,0.3 \rangle \\ a_2 & \langle 0.3,0.3 \rangle & \langle 0.1,0.2 \rangle & \langle 0.1,0.7 \rangle \end{pmatrix}$$

$$\langle \mu_{ia}, \gamma_{ia} \rangle \equiv \begin{pmatrix} G_2 & v_1 & v_2 & v_3 & v_4 & v_5 \\ a_2 & \langle 0.6,0.1 \rangle & \langle 0.7,0.1 \rangle & \langle 0.2,0.8 \rangle & \langle 0.5,0.4 \rangle & \langle 0.3,0.6 \rangle \\ a_3 & \langle 0.1,0.6 \rangle & \langle 0.5,0.2 \rangle & \langle 0.14,0.5 \rangle & \langle 0.62,0.1 \rangle & \langle 0.5,0.4 \rangle \end{pmatrix}$$

$$\langle \mu_{ija}, \gamma_{ija} \rangle \equiv \begin{pmatrix} G_2 & (v_1, v_2, v_3) & (v_2, v_4, v_5) \\ a_2 & \langle 0.2,0.7 \rangle & \langle 0.2,0.5 \rangle \\ a_3 & \langle 0.04,0.5 \rangle & \langle 0.02,0.3 \rangle \end{pmatrix}$$

Then the union is given by

$$\langle \mu_{ia}, \gamma_{ia} \rangle \equiv \begin{pmatrix} G_1 \cup G_2 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\ a_1 & \langle 0.3,0.3 \rangle & \langle 0.4,0.4 \rangle & \langle 0.6,0.3 \rangle & \langle 0.2,0.1 \rangle & \langle 0.5,0.4 \rangle & \langle 0.6,0.2 \rangle & \langle 0.7,0.1 \rangle \\ a_2 & \langle 0.6,0.1 \rangle & \langle 0.7,0.1 \rangle & \langle 0.7,0.2 \rangle & \langle 0.6,0.3 \rangle & \langle 0.3,0.2 \rangle & \langle 0.5,0.3 \rangle & \langle 0.1,0.8 \rangle \\ a_3 & \langle 0.1,0.6 \rangle & \langle 0.5,0.2 \rangle & \langle 0.14,0.5 \rangle & \langle 0.62,0.1 \rangle & \langle 0.5,0.4 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \end{pmatrix}$$

$$\langle \mu_{ija}, \gamma_{ija} \rangle \equiv \begin{pmatrix} G_1 \cup G_2 & (v_1, v_4, v_5) & (v_2, v_4, v_6) & (v_3, v_6, v_7) & (v_1, v_2, v_3) & (v_2, v_4, v_5) \\ a_1 & \langle 0.2,0.4 \rangle & \langle 0.1,0.1 \rangle & \langle 0.5,0.3 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ a_2 & \langle 0.3,0.3 \rangle & \langle 0.1,0.2 \rangle & \langle 0.1,0.7 \rangle & \langle 0.2,0.7 \rangle & \langle 0.2,0.5 \rangle \\ a_3 & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.04,0.5 \rangle & \langle 0.02,0.3 \rangle \end{pmatrix}$$

The graph of $G_1 \cup G_2$ is shown in the following figure.

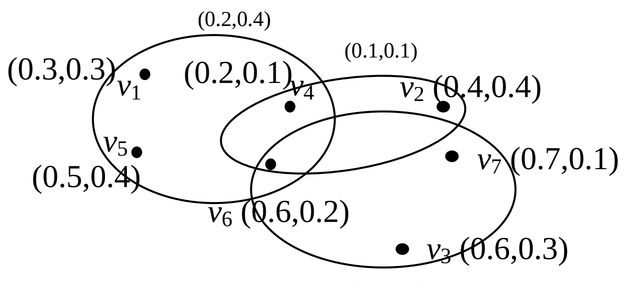

Fig.4: $G_1 \cup G_2$ corresponding to the parameter a_1

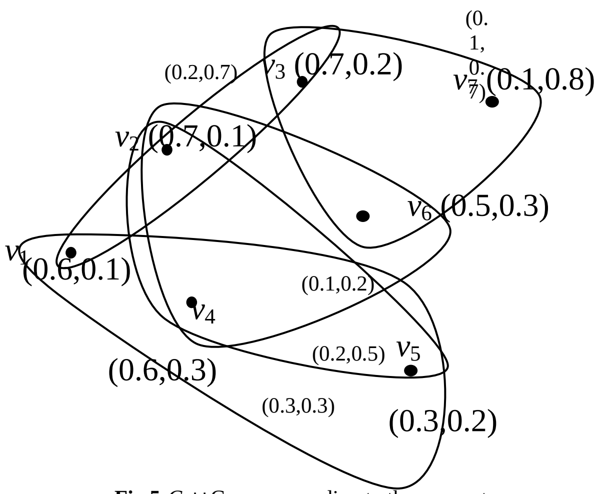

Fig.5: $G_1 \cup G_2$ corresponding to the parameter a_2

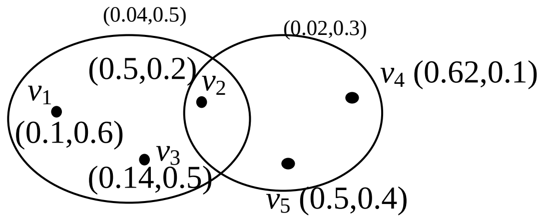

Fig.6: $G_1 \cup G_2$ corresponding to the parameter a_3

3. Conclusion

In the real world situations there are large number of issues that warrant rational, logical and scientific decisions that fit best for the deed of desired objective. The concept of intuitionistic fuzzy soft hypergraph and union of them has been explained with examples which has broader application in decision making models suitable for personal, social, technical, commercial and managerial problems.

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