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Research paper

Inventory Model for Deteriorating Items with Life Time

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Abstract

In this paper, an inventory model of deteriorating items with life time has been discussed. The demand rate has been taken of cubic function of time which starts from zero in the beginning and ends to zero at the completion of the cycle. The deterioration rate is assumed to be of power pattern , i.e. $\alpha\beta t^{\beta-1}$. Shortages are allowed and fully backlogged.

Keywords: *Inventory; cubic function; life time; power pattern; shortage.*

1. Introduction:

Several researchers such as Silver & Meal (1973), Donaldson (1977) and Ritche (1984) discussed the inventory models involving time dependent demand pattern. Dave & Patel (1981) discussed an inventory model with time proportional demand. Goswami &Chaudhary (1981) discussed an EOQ model for deteriorating items with shortages and linear trend in demand. Mitra, et al. (1984) presented a simple procedure for EOQ model for the cases of increasing or decreasing linear trend in demand. Dutta & Pal (1992) derived an optimal inventory policy by considering demand rate in power pattern form.
Several researchers such as Shah & Jaiswal (1977), Covert & Phillip (1973) and Mishra (1975) discussed the inventory models for deteriorating items. Hari Kishan et al. (2010) discussed an inventory model of deteriorating product with life time and variational demand. Hari Kishan et al. (2014) developed an inventory model of deteriorating products with life time.
In this paper, an inventory model has been developed for deteriorating items with life time. The demand rate has taken of cubic function of time which starts with zero in the beginning and ends to zero at the completion of the cycle. The deterioration rate is taken of power pattern form, i.e. $\alpha\beta t^{\beta-1}$.

2. Assumptions and Notations:

2.1 Assumptions:

The following assumptions have been considered:
(i) Demand rate is taken of cubic form a $d(t) = -at^2(T-t)$
(ii) Both cases with shortages and without shortage are discussed
(iii)he deteriorating rate is deterministic and it is of power pattern form given by $\alpha\beta t^{\beta-1}$.

2.2 Notations:

The following notations have been used :
(i) A = the ordering cost per order.
(ii) C = the unit purchasing price.
(iii) h = unit holding cost per unit time.
(iv) c_d = unit deterioration cost.
(v) C_s =unit shortage cost.
(vi) T=the replenishment cycle time.
(vii) μ = the life time
(viii) $q(t)$ = stock level at any time *t*.
(ix) Q = maximum stock level.
(x) $TC(T)$= the annual total relevant cost, which is a function of T.
  a. (ix) T^* = the optimal cycle time of $TVC(T)$.
  b. (x) Q^* = the optimal order quantity.

3. Mathematical Model and Analysis:

3.1 Without Shortage:

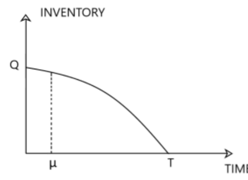

Figure 1

The governing differential equations of the stock status during the period $0 \le t \le T$ are given by

$$\frac{dq}{dt} = -at^2(T-t), \qquad 0 \le t \le \mu \qquad (1)$$

$$\frac{dq}{dt} + \alpha\beta t^{\beta-1}q = -at^2(T-t) \qquad \mu \le t \le T \qquad (2)$$

The boundary conditions are

$$q(0) = Q, \qquad \text{and} \qquad q(T) = 0 \qquad (3)$$

The solution of equation (1) is given by

$$q = -a\left[\frac{Tt^3}{3} - \frac{t^4}{4}\right] + c_1$$

Using the initial condition, we get $c_1 = Q$. Therefore

$$q = Q - a\left[\frac{Tt^3}{3} - \frac{t^4}{4}\right] \quad (4)$$

The solution of equation (2) is given by

$$qe^{\alpha t^\beta} = -a\left(\frac{Tt^3}{3} - \frac{t^4}{4} + \frac{\alpha T t^{\beta+3}}{\beta+3} + \frac{\alpha t^{\beta+4}}{\beta+4}\right) + c_2.$$

Using the boundary condition (3), we get

$c_2 = a\left(\frac{T^4}{12} + \frac{\alpha T^{\beta+1}}{(\beta+3)(\beta+4)}\right)$. Therefore

$$q = a\left[\left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right) - \left(\frac{Tt^3}{3} - \frac{t^4}{4} + \frac{\alpha T t^{\beta+3}}{\beta+3} + \frac{\alpha t^{\beta+4}}{\beta+4}\right)\right] e^{-\alpha t^\beta}. \quad (5)$$

From equations (4) and (5), we get

$$Q = a\left[\left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right) - \left(\frac{\alpha T\mu^{\beta+3}}{\beta+3} + \frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right] e^{-\alpha\mu^\beta} + a\alpha\mu^\beta\left(\frac{T\mu^3}{3} - \frac{\mu^4}{4}\right) \quad (6)$$

The holding cost is given by

$$HC = h\int_0^T q\,dt$$

$$= h\left[\int_0^\mu q\,dt + \int_\mu^T q\,dt\right]$$

$$= h\left[\int_0^\mu \left(Q - a\left(\frac{Tt^3}{3} - \frac{t^4}{4}\right)dt\right) + a\int_\mu^T \left[\left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right) - \left(\frac{Tt^3}{3} - \frac{t^4}{4} + \frac{\alpha T t^{\beta+3}}{\beta+3} + \frac{\alpha t^{\beta+4}}{\beta+4}\right)\right] e^{-\alpha t^\beta}\right]$$

$$= ah\Bigg[\mu\left[\left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right) - \left(\frac{\alpha T\mu^{\beta+3}}{\beta+3} + \frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right] e^{-\alpha\mu^\beta} + \alpha\mu^{\beta+1}\left(\frac{T\mu^3}{3} - \frac{\mu^4}{4}\right) - \left(\frac{T}{12} - \frac{\mu}{20}\right)\mu^4 + \left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right)(T-\mu)$$
$$- \left(\frac{T^5}{30} + \frac{2\alpha T^{\beta+5}}{(\beta+3)(\beta+5)}\right) + \left(\frac{T\mu^4}{12} - \frac{\mu^5}{20} + \frac{\alpha T\mu^{\beta+4}}{(\beta+3)(\beta+4)} + \frac{\alpha\mu^{\beta+5}}{(\beta+4)(\beta+5)}\right)$$
$$+ \alpha\left(\frac{T^{\beta+5}}{3(\beta+4)} - \frac{T^{\beta+5}}{(\beta+5)} + \frac{\alpha T^{2\beta+5}}{2(\beta+2)(\beta+3)} + \frac{\alpha T^{2\beta+5}}{(\beta+4)(2\beta+5)}\right)$$
$$- \alpha\left(\frac{T\mu^{\beta+4}}{3(\beta+4)} - \frac{\mu^{\beta+5}}{(\beta+5)} + \frac{\alpha T\mu^{2\beta+4}}{2(\beta+2)(\beta+3)} + \frac{\alpha\mu^{2\beta+5}}{(\beta+4)(2\beta+5)}\right)\Bigg] \quad (7)$$

The deterioration cost is given by

$$DC = c_d\left[Q - \int_0^T d(t)dt\right]$$

$$= c_d\left[a\left[\left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right) - \left(\frac{\alpha T\mu^{\beta+3}}{\beta+3} + \frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right] e^{-\alpha\mu^\beta} + a\alpha\mu^\beta\left(\frac{T\mu^3}{3} - \frac{\mu^4}{4}\right) - \frac{aT^4}{12}\right] \quad (8)$$

The average cost of the inventory system is given by

$$TC = \frac{1}{T}(HC + DC + A)$$

$$= \frac{ah}{T}\Bigg[\mu\left[\left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right) - \left(\frac{\alpha T\mu^{\beta+3}}{\beta+3} + \frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right] e^{-\alpha\mu^\beta} + \alpha\mu^{\beta+1}\left(\frac{T\mu^3}{3} - \frac{\mu^4}{4}\right) - \left(\frac{T}{12} - \frac{\mu}{20}\right)\mu^4 + \left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right)(T-\mu)$$
$$- \left(\frac{T^5}{30} + \frac{2\alpha T^{\beta+5}}{(\beta+3)(\beta+5)}\right) + \left(\frac{T\mu^4}{12} - \frac{\mu^5}{20} + \frac{\alpha T\mu^{\beta+4}}{(\beta+3)(\beta+4)} + \frac{\alpha\mu^{\beta+5}}{(\beta+4)(\beta+5)}\right)$$
$$+ \alpha\left(\frac{T^{\beta+5}}{3(\beta+4)} - \frac{T^{\beta+5}}{(\beta+5)} + \frac{\alpha T^{2\beta+5}}{2(\beta+2)(\beta+3)} + \frac{\alpha T^{2\beta+5}}{(\beta+4)(2\beta+5)}\right)$$
$$- \alpha\left(\frac{T\mu^{\beta+4}}{3(\beta+4)} - \frac{\mu^{\beta+5}}{(\beta+5)} + \frac{\alpha T\mu^{2\beta+4}}{2(\beta+2)(\beta+3)} + \frac{\alpha\mu^{2\beta+5}}{(\beta+4)(2\beta+5)}\right)\Bigg]$$
$$+ \frac{c_d}{T}\left[a\left[\left(\frac{T^4}{12} + \frac{\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}\right) - \left(\frac{\alpha T\mu^{\beta+3}}{\beta+3} + \frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right] e^{-\alpha\mu^\beta} + a\alpha\mu^\beta\left(\frac{T\mu^3}{3} - \frac{\mu^4}{4}\right) - \frac{aT^4}{12}\right] + \frac{A}{T} \quad (9)$$

For minimization of cost function *TC*, we have $\frac{dTC}{dT} = 0$ and $\frac{d^2TC}{dT^2} > 0$. Differentiating expression (9) with respect to *T* and putting it equal to zero, we may obtain the value of *T*. The value of *T* which satisfies the condition $\frac{d^2TC}{dT^2} > 0$ provided the optimal value of *T*. This may be represented by *T**. Putting the value of *T**, we may obtain the optimal value of cost.

3.2 With Shortage:

Let Q be the initial stock level after fulfilling the back orders in an inventry cycle. Let $q(t)$ be the inventory level at any time t. This inventory level is depleted due to demand in the life time period and due to demand and deterioration during deterioration period. Inventory becomes zero at time T_1. Then shortage starts and continues to the time *T*. Let *S* be the maximum shortage. This model has been shown in figure 2:

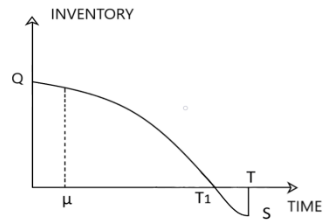

Figure 2

The governing differential equations of the stock status during the period $0 \le t \le T$ are given by

$$\frac{dq}{dt} = -at^2(T-t), \quad 0 \le t \le \mu \quad (10)$$

$$\frac{dq}{dt} + \alpha\beta t^{\beta-1}q = -at^2(T-t)$$

$$\mu \le t \le T_1 \quad (11)$$

$$\frac{dq}{dt} = -at^2(T-t)$$

$$T_1 \le t \le T \quad (12)$$

The boundary conditions are

$$q(0)=Q, \qquad q(T_1)=0 \text{ and } q(T)=-S \tag{13}$$

Solution of equation (10) is given by

$$q=Q-a\left[\frac{Tt^3}{3}-\frac{t^4}{4}\right] \qquad 0\le t\le\mu \tag{14}$$

Solution of equation (11) is given by

$$q=a\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)-\left(\frac{Tt^3}{3}-\frac{t^4}{4}+\frac{\alpha Tt^{\beta+3}}{\beta+3}+\frac{\alpha t^{\beta+4}}{\beta+4}\right)\right]e^{-\alpha t^\beta}$$

$$\mu\le t\le T_1 \tag{15}$$

Solution of equation (12) is given by

$$q=-S-a\left[\frac{Tt^3}{3}-\frac{t^4}{4}\right]+\frac{aT^4}{12}$$

$$T_1\le t\le T \tag{16}$$

From expressions (14) and (15), we have

$$Q=a\left[\frac{T\mu^3}{3}-\frac{\mu^4}{4}\right]+a\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)-\left(\frac{T\mu^3}{3}-\frac{\mu^4}{4}+\frac{\alpha T\mu^{\beta+3}}{\beta+3}+\frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right]e^{-\alpha\mu^\beta} \tag{17}$$

Using the boundary condition (13) in (16), we get

$$S=-a\left[\frac{TT_1^3}{3}-\frac{T_1^4}{4}\right]+\frac{aT^4}{12} \tag{18}$$

Substituting the values of Q and S in expressions (14) and (16), we get

$$q=a\left[\frac{T\mu^3}{3}-\frac{\mu^4}{4}\right]+a\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)-\left(\frac{T\mu^3}{3}-\frac{\mu^4}{4}+\frac{\alpha T\mu^{\beta+3}}{\beta+3}+\frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right]e^{-\alpha\mu^\beta}-a\left[\frac{Tt^3}{3}-\frac{t^4}{4}\right] \tag{19}$$
$$0\le t\le\mu$$

$$q=a\left[\frac{TT_1^3}{3}-\frac{T_1^4}{4}\right]-a\left[\frac{Tt^3}{3}-\frac{t^4}{4}\right] \tag{20}$$

$$T_1\le t\le T$$

The inventory holding cost for the entire cycle is given by

$$HC=h\int_0^T q\,dt$$
$$=h\left[\int_0^\mu q\,dt+\int_\mu^T q\,dt\right]$$
$$=h\Bigg[a\int_0^\mu\Bigg[\left[\frac{T\mu^3}{3}-\frac{\mu^4}{4}\right]+\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)-\left(\frac{T\mu^3}{3}-\frac{\mu^4}{4}+\frac{\alpha T\mu^{\beta+3}}{\beta+3}+\frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right]e^{-\alpha\mu^\beta}-\left[\frac{Tt^3}{3}-\frac{t^4}{4}\right]\Bigg]dt+a\int_\mu^{T_1}\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)-\left(\frac{Tt^3}{3}-\frac{t^4}{4}+\frac{\alpha Tt^{\beta+3}}{\beta+3}+\frac{\alpha t^{\beta+4}}{\beta+4}\right)\right]e^{-\alpha t^\beta}\Bigg]$$
$$=ha\Bigg[\left(\frac{T}{4}-\frac{\mu}{5}\right)\mu^4+\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)\mu-\left(\frac{T\mu^3}{3}-\frac{\mu^4}{4}+\frac{\alpha T\mu^{\beta+3}}{\beta+3}+\frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\mu\right]e^{-\alpha\mu^\beta}+\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)(T_1-\mu)-\left(\left(\frac{T}{12}-\frac{T_1}{20}\right)T_1^4+\frac{\alpha TT_1^{\beta+4}}{(\beta+3)(\beta+4)}+\frac{\alpha T_1^{\beta+5}}{(\beta+4)(\beta+5)}\right)$$
$$+\left(\left(\frac{T}{12}-\frac{T_1}{20}\right)\mu^4+\frac{\alpha T\mu^{\beta+4}}{(\beta+3)(\beta+4)}+\frac{\alpha\mu^{\beta+5}}{(\beta+4)(\beta+5)}\right)$$
$$+\alpha\left(\frac{TT_1^{\beta+4}}{3(\beta+4)}-\frac{T_1^{\beta+5}}{4(\beta+5)}+\frac{\alpha TT_1^{2\beta+4}}{2(\beta+3)(\beta+2)}+\frac{\alpha T_1^{2\beta+5}}{(2\beta+5)(\beta+4)}\right)$$
$$-\alpha\left(\frac{T\mu^{\beta+4}}{3(\beta+4)}-\frac{\mu^{\beta+5}}{4(\beta+5)}+\frac{\alpha T\mu^{2\beta+4}}{2(\beta+3)(\beta+2)}+\frac{\alpha\mu^{2\beta+5}}{(2\beta+5)(\beta+4)}\right)\Bigg] \tag{21}$$

The deterioration cost is given by

$$DC=c_d\left[Q-\int_0^{T_1}d(t)dt\right]$$
$$=c_d\left[a\left[\frac{T\mu^3}{3}-\frac{\mu^4}{4}\right]+a\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)-\left(\frac{T\mu^3}{3}-\frac{\mu^4}{4}+\frac{\alpha T\mu^{\beta+3}}{\beta+3}+\frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right]e^{-\alpha\mu^\beta}-a\int_0^{T_1}t^2(T-t)dt\right]$$
$$=ac_d\left[\left[\frac{T\mu^3}{3}-\frac{\mu^4}{4}\right]+\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)-\left(\frac{T\mu^3}{3}-\frac{\mu^4}{4}+\frac{\alpha T\mu^{\beta+3}}{\beta+3}+\frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right]e^{-\alpha\mu^\beta}-\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}\right)\right] \tag{22}$$

The shortage cost during the entire cycle is given by

$$SC=\int_{T_1}^T -q\,dt$$
$$=C_s a\int_{T_1}^T\left[\left[\frac{TT_1^3}{3}-\frac{T_1^4}{4}\right]-\left[\frac{Tt^3}{3}-\frac{t^4}{4}\right]\right]dt$$
$$=C_s a\left[\left[\frac{TT_1^3}{3}-\frac{T_1^4}{4}\right](T-T_1)-\frac{T^5}{30}+\left(\frac{T}{12}-\frac{T_1}{20}\right)T_1^4\right] \tag{23}$$

The average cost of the inventory system is given by

$$TC=\frac{1}{T}(HC+DC+SC+A)$$
$$=\frac{ha}{T}\Bigg[\left(\frac{T}{4}-\frac{\mu}{5}\right)\mu^4+\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)\mu-\left(\frac{T\mu^3}{3}-\frac{\mu^4}{4}+\frac{\alpha T\mu^{\beta+3}}{\beta+3}+\frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\mu\right]e^{-\alpha\mu^\beta}+\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)(T_1-\mu)$$
$$-\left(\left(\frac{T}{12}-\frac{T_1}{20}\right)T_1^4+\frac{\alpha TT_1^{\beta+4}}{(\beta+3)(\beta+4)}+\frac{\alpha T_1^{\beta+5}}{(\beta+4)(\beta+5)}\right)$$
$$+\left(\left(\frac{T}{12}-\frac{T_1}{20}\right)\mu^4+\frac{\alpha T\mu^{\beta+4}}{(\beta+3)(\beta+4)}+\frac{\alpha\mu^{\beta+5}}{(\beta+4)(\beta+5)}\right)$$
$$+\alpha\left(\frac{TT_1^{\beta+4}}{3(\beta+4)}-\frac{T_1^{\beta+5}}{4(\beta+5)}+\frac{\alpha TT_1^{2\beta+4}}{2(\beta+3)(\beta+2)}+\frac{\alpha T_1^{2\beta+5}}{(2\beta+5)(\beta+4)}\right)$$
$$-\alpha\left(\frac{T\mu^{\beta+4}}{3(\beta+4)}-\frac{\mu^{\beta+5}}{4(\beta+5)}+\frac{\alpha T\mu^{2\beta+4}}{2(\beta+3)(\beta+2)}+\frac{\alpha\mu^{2\beta+5}}{(2\beta+5)(\beta+4)}\right)\Bigg]$$
$$+\frac{ac_d}{T}\left[\left[\frac{T\mu^3}{3}-\frac{\mu^4}{4}\right]+\left[\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}+\frac{\alpha TT_1^{\beta+3}}{\beta+3}+\frac{\alpha T_1^{\beta+4}}{\beta+4}\right)-\left(\frac{T\mu^3}{3}-\frac{\mu^4}{4}+\frac{\alpha T\mu^{\beta+3}}{\beta+3}+\frac{\alpha\mu^{\beta+4}}{\beta+4}\right)\right]e^{-\alpha\mu^\beta}-\left(\frac{TT_1^3}{3}-\frac{T_1^4}{4}\right)\right]+\frac{C_s a}{T}\left[\left[\frac{TT_1^3}{3}-\frac{T_1^4}{4}\right](T-T_1)-\frac{T^5}{30}+\left(\frac{T}{12}-\frac{T_1}{20}\right)T_1^4\right] \tag{24}$$

For minimization of the cost function TC, $\frac{\partial TC}{\partial T_1}=0$, $\frac{\partial TC}{\partial T}=0$, $\frac{\partial^2 TC}{\partial T_1^2}>0$, $\frac{\partial^2 TC}{\partial T^2}>0$ and $\frac{\partial^2 TC}{\partial T_1^2}.\frac{\partial^2 TC}{\partial T^2}-\left(\frac{\partial^2 TC}{\partial T_1\partial T}\right)^2>0$. From above equations we may obtain the values of T_1 and T satisfying the above inequalities. These values are optimal values and may be denoted as T_1^* and T^*. Putting these values in the expression (24) the optimal value of cost function can be obtained.

4. Conclusions:

In this paper, an inventory model has been developed for deteriorating items with life time. The demand rate has taken of cubic function of time which starts with zero in the beginning and ends to zero at the completion of the cycle. The deterioration rate is taken of power pattern form, i.e. $\alpha\beta t^{\beta-1}$. Both cases without shortage and with shortage have been discussed. The cost minimization technique has been used to obtain the optimal values. The model can further be extended for other demand rate form and for finite time horizon.

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