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Research paper

Construction and performance analysis of diagonally shifted column structured RPM based regular quasi cyclic-LDPC codes with girth 10

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Abstract

In this manuscript, QC-LDPC (Quasi Cyclic Low-Density Parity Check Codes) are constructed by the diagonally Shifted Column structured Random Permutation Method (RPM) with girth 10. In this proposed method, Random Permutation based Parity Check Matrix, Base Matrix and Zero Matrices has been constructed with a column structured approach using Lower Upper decomposition technique. The row-column mapping Technique is applied to create the sparse behavior of Regular RPM based Quasi Cyclic LDPC Codes. The construction of code is obtainable by means of column weight three and row weight six. Those codes are decoded with Log Domain Sum Product Algorithm. The constructed Sparse Regular RPM-QC-LDPC Codes have low error performance and compared with other existing results.

Keywords: *Circulant and Diagonal Shifting; Frame Error Rate; Girth 10; Log Domain Sum Product Algorithm; QC-LDPC Codes; Random Permutation Matrix Semicolon.*

1. Introduction

Low Density Parity Check codes are represented through Linear Block codes, it consists sparse parity check matrix which has less amount of Non-zero elements. Quasi cyclic LDPC codes have been constructs using Cyclic or circulant shifting technique being applied. These codes are mostly used to apply error detection and correction method for the data channel coding in 5G mobile networks. The tanner bipartite graph technique is used for representation of LDPC codes and girth defined as the shortest cycle between variable nodes to check nodes in these graphs.

The Regular Quasi Cyclic-LDPC codes are constructed using Lower bound of circulant matrix with a perfect cyclic difference set from sidon sequence [1] and symmetrical structure of the parity check matrix based circulant permutation matrices is given in [2]. Recently, in [3] array cyclic and exponent matrix method have presented for the explicit construction of Type-I Quasi Cyclic-LDPC codes with girth 12 has length PJL^2. The QC-LDPC codes are constructed using the cyclic group ring technique with structured property and finite field method [4]. In [5], Circulant structured subsets based combinatorial designs of Quasi Cyclic-LDPC Codes with girth 6 and 8 have been presented. The 2-cycle and decoding failure in Gallager A algorithm is explained in [6] for decoding of irregular LDPC codes to improve error correction ability. In [7], Lower-upper decomposition, circular and counter shifting techniques based QC-LDPC codes with girth 6 have been constructed and decoded with Log Domain sum product Algorithm. The Quantum Quasi Cyclic-LDPC codes constructed using classical circular permutation matrices and orthogonal pairing of parity check matrices with cycle 4, performance of codes is also evaluated with an iterative belief propagation decoding Algorithm [8]. The approximation of box-plus operation with small value concepts used in Sum Product decoding of LDPC codes [9]. The spatially coupled Quasi Cyclic-LDPC codes constructed with structural, matrix-dispersion, Lower-Upper triangular decomposition and cyclic shifting techniques, those codes decoded with Min-Sum Algorithm [10]. In [11], the exponent matrix has been presented with a ring of integers, lower and upper bound technique and greatest common divisor Algorithm to give an explicit construction of Affine-Permutation matrices (APM)- LDPC codes with girths 6,8 and 10. The short cycle of constructing Quasi Cyclic-LDPC Codes has been removed by shifting of circulant permutation Matrix [12]. The scalable Quasi Cyclic-LDPC codes constructed in [13] using Block triple Diagonal Parity structure and a greedy algorithm for good cycle distribution to achieve better error performance with Belief Propagation decoding Algorithm. The adaptive and Variable node group shuffled decoding algorithm for Regular and irregular LDPC codes is presented to improve the performance of existing belief propagation decoders [14]. In [15], isomorphic classes of exponent matrices used for construction of cyclic structure based QC-LDPC codes with 6 and 8 girth. The bit-selection Algorithm in [16] proposed to reduce the problem of Trapping and stopping sets of Data shorting for systematic LDPC codes in lower error floor region. The piecewise systematic encoder has been presented with right and left feedback shift registers for encoding of circular structure based Quasi Cyclic-LDPC codes [17]. In [18], multiplicative and additive matrix dispersion of base matrices over the primitive element of Galois field GF (q) has been presented for the algebraic construction of Quasi Cyclic-LDPC codes. The conventional matrix unwrapping construction with masking a replicated version of spatially coupled Quasi Cyclic-LDPC codes are given in [19]. The unreliability based information bit flipping (UIBF) decoding Algorithm proposed in [20] with LLR vector and cyclic redundancy to reduce complexity of Sum-Product decoder. The transpose and mapping technique used to construction of an isomorphism algebraic system for circulant permutation matrix in [21], it also represented competent method for finding the girth of Quasi Cyclic-LDPC codes.

Hierarchical row-splitting technique used for construction of multiple rates QC-LDPC codes proposed in [22].

The QC-LDPC Codes with girth 8 are constructed using a circulant permutation matrix (CPM) based subtraction Method [23]. The error correction capability of column-weight-three regular LDPC codes under the Gallager A algorithm for iterative decoding Algorithm based on belief propagation has been analyzed in [24].

The partially stopped criterion proposed in [25] for the normalized probabilistic min-sum decoding algorithm for LDPC codes to reduce the power consumption of the check node units.

This paper presents Random permutation based Sparse Quasi Cyclic-LDPC codes denoted by H1 have length of M×N, which construct using lower-upper decomposition diagonally structured parity check matrix (PCM) of H_D. In fact, for each M×N, M× (N-M), M<N, base matrix is B, size of each basic sub-matrix is m and girth of code is g=10. RPM size m (prime) has been constructed by a column vector containing a random permutation of the integers from 1 to N is P_N. The column weight of H1 is denoted by W_{col}, taken 3 and Row weight is represented through W_{row}, set to be 6. This arrangement of code called Regular Quasi Cyclic-LDPC, otherwise irregular. The M×N has shifted matrix of S on P_N and Circulant matrix is represented by C on P_N.

The frame error performance of constructed Quasi Cyclic-LDPC codes is comparable with various code lengths are given in [3], [5], [9], [12] and [19].

The rest of the paper organized as follows: In section II construction of diagonally structured Parity Check Matrix. Section III represents a detail of column structured RPM-QC-LDPC Codes. In section IV, discuss simulation results and performance comparison of existing code-lengths. The conclusion of the script is given in section V.

2. Diagonally structured parity check matrix

The Lower-upper (LU) decomposition of an M×N parity check matrix is given by H_D, decomposition of H_D into a Lower M× (N-M) is diagonally structured matrix represented by L and an M× (N-M) upper diagonal matrix is given by U. Such that,

$$P_N \times H_D = L \times U \tag{1}$$

Where P_N is a Column Permutation Matrix

There is L_{ij} and $U_{ij}=0$ for i<j, here i and j are the permutation of binary polynomial numbers corresponding to row (1,2, 3....M) and Column (1,2, 3....N) are given in exchange of H_D.

The following permutation matrix P is given below:

$$P_{ij}=\begin{cases}1 & \text{if } j = Pi\\ 0 & \text{if } j \neq Pi\end{cases} \tag{2}$$

Left multiplication of matrices and permutation vectors with P (P_1, P_2, P_3....P_M) determined easily using the permutation matrix. $L_{i,j} = H_{Di,j}$ defines the lower row permutation $L = P_L \times H_D$ and upper row permutation $U=P_U \times H_D$ of a matrix H_D.

The factorization depends on the switching strategy. To obtain diagonally structured parity check vector, the steps performed are:

- Stumble on non-zero elements (1s) for the diagonally structured Matrix
- Find non-zero diagonal elements
- Find the first non-zero column of the matrix
- Find the minimum column weight for Parity check matrix
- Rearrange the columns of both Randomly Permuted H1and Diagonally Structured matrix H_D
- Fill the Lower and Upper matrices (LU) column by column
- Find the latter on rows of a matrix with non-zero elements given in column i
- Parity check vector originates by solving the sparse Lower Upper decomposition

3. Column structured RPM-QC-LDPC codes

In this paper, the sparse Regular (N, W_{col}, W_{row}) Quasi Cyclic-LDPC codes H1 constructs using the proposed method of diagonally shifted column structured random permutation sub-matrices of m. There is m= size of each sub-matrix and message length of code is $M=W_{col}\times m$, code-length is given by $N=W_{row}\times m$. The ensembles of matrices for H1= $(N\times W_{col})/W_{row} \times N$ and design rate R= $1-(W_{col}/W_{row})$, constructed Regular code for proposed method have (3, 6) weight with a 0.5 code rate. The H1 generated from column Circulant permutation sub matrices P_N and Zero matrices correspond to the N×N identity matrix with column cyclically shifted to right place of mod P_N.

The first sub-matrix constructed for the generation of basic sub-matrix is given by $H1_s$ by zero matrices of $(N/W_{row}) \times N$. The basic sub matrix B generated for i= (1, 2, 3, 4......N/W_{row}) and j= (i-1) $\times W_{row}+1$ to $i\times W_{row}$. The random column permutation of basic submatrix B constructed for t= (2, 3, 4....Wcol) is P_N, e.g. m=2 taken $W_{Col}=3$ and $W_{row}=6$. The constructed random column permutation matrix is given below in Fig. 1, it gets as an example of code-length (12, 6) at 0.5 code rate.

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1\\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0\\
1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1\\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}$$

Fig. 1: Random Column Permutation Matrix P_N for Code (12, 6).

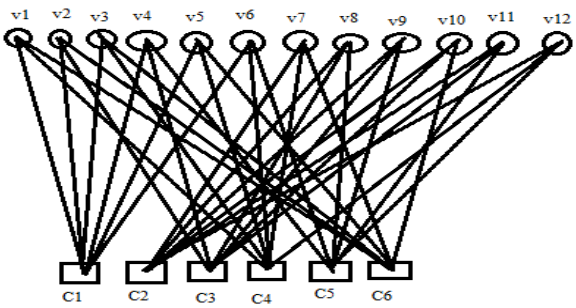

Fig. 2: Tanner Graph Representation of Code (12, 6) with Girth 10.

The diagonal matrix h_d is obtained for i= (1, 2, 3 ...W_{row}) and y= (1, 2, 3...W_{col}) at non-zero elements. Then, it diagonally shifted the non-zero elements in length of code N. For any vector x= (x_1, x_2, x_3, ...x_b), here a=(1,2,3,4.....W_{row}) and b= (2,3,4......W_{col}) are diagonally shifted to the left by $x_1(r)$. Let $\oplus$ denote modulo-2 addition respectively, x1= ($b\oplus a$) and 0<r<N-1. There is 10-cycle in the Tanner graph (V, C, e) given in Fig. 2 denoted by V= (V_1, V_2, V_3 ...V_N), C= (C_1, C_2, C_3 ...C_M) and e= edge is the shortest cycle in tanner graph. It exists a series of (V_1, C_1); (V_2, C_2); ....;(V_N, C_M) ;(V2, C_2) = (V_1, C_1), $V_d \neq V_d+1$, $C_e \neq C_e+1$, $0 \leq d \leq W_{col}$ and $0 \leq e \leq W_{row}$. The 10-cycle or girth in the tanner graph Fig. 2 expressed by the equation is given below:

$$(S_{i,r}-S_{d,r}\oplus i)+(S_{e,r}\oplus i-S_{i,r})+(S_{i,t}-S_{e,t}\oplus i)+(S_{d,t}\oplus i-S_{i,t})=0(\mathrm{mod}\, m,e) \quad (3)$$

10-cycle or girth in the Tanner graph related to base matrix B and column permutation matrix P_N, with g (B, P_N) ≥6. The condition to obtained 10-cycle in the graph are i<j, r<t and d<e.
The shift value of array matrix is denoted by H(x). It corresponds to contain binary non-zero elements of (b, a) and modulo-2 addition of a diagonally shifted array of b. The H(x) contain shifting value of the array, it's associated with base matrix B and column permutation matrix P_N from circulant shifting technique being applied. The constructed Quasi Cyclic-LDPC matrix H converted into sparse non-zero elements of H1. Then, randomly permuted the column of H1 and add two 1's with each row is not containing any 1. The lastly row-column pairing performed to get sparse regular diagonally shifted column structured RPM based Quasi Cyclic-LDPC Codes with girth 10. In this method, find the non-zero element's value from constructed code H1 is w and return the length of largest array dimension in H1 is given by c1. If more number of 1's are found in H1, then flip 1's in to 0's to make sparse property of Constructed Quasi Cyclic-LDPC Codes. Therefore, in this Proposed Constructions of RPM-QC-LDPC codes reduced the performance instead of circulant permutation matrix presented in [3].

4. Simulation results and discussion

The constructed RPM-Quasi Cyclic-LDPC codes with girth 10 simulated using MATLAB. In this section, constructed diagonally structured Quasi Cyclic-LDPC codes are comparable with any existing codes given in [3], [5], [9], [12] and [19]. The simulation uses Binary Phase Shift Keying modulation in excess of Additive White Gaussian Noise channels. The QC-LDPC Codes decoded with Simple Log Domain Sum Product Algorithm, where the largest amount of iteration set to be 10 and total number of frames is equal to 15. The Signal to Noise Ratio (E_b/N_0) defined between 0 to 5.0 dB. Here, frame error rate analyses from decoding of various code-words.
Five diagonally structured randomly constructed sparse regular Quasi Cyclic-LDPC codes for half code rate, it considered with code length (M, N) = Code A (816, 408), Code B (1008, 504), Code C (1032, 516), Code D (1764, 882) and Code E (1836, 918).
In Fig. 3, Frame error performance of Code A achieved nearly10^{-1} for 0 to 2.5 dB SNR region, which is 80% better performing that compared to LDPC code given in [9]. Another Code B is shown in Fig. 4, the FER performance of code improved from 10^{-3} to about 10^{-4} comparing with circulant permutation matrix based code represented in [12]. The resultant performance of code C is given in Fig. 5, it obtained ≈10^{-4} at 3.5 dB is comparable with Combinatorial Designs based Quasi Cyclic-LDPC code analyzed only for 3 decibels [5]. For comparison, regular (3, 6) type I PEG (Progressive Edge Growth) Quasi Cyclic-LDPC code with girth 12 is selected in [3]. It compared FER performance is 30% showing better results in Fig. 6, where SNR is given between 2 to 2.5 dB for Code D.
The FER performance of Code E shown in Fig. 7, it achieved ≈10^{-4} is better shown that compared to finite length algebraic construction based Quasi Cyclic-LDPC codes presented frame error rate performance ≈10^{-3} in [19].

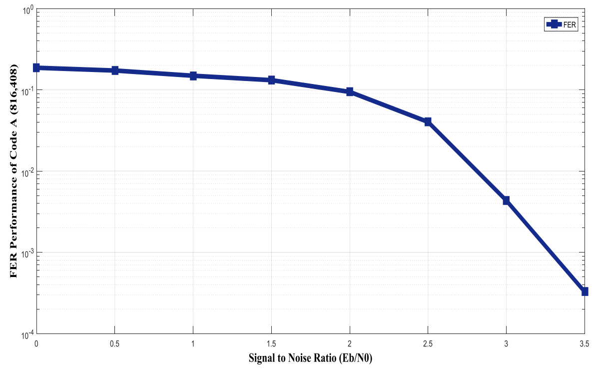

Fig. 3: Frame Error Rate Performance of Proposed RPM-QC-LDPC Code A (816, 408).

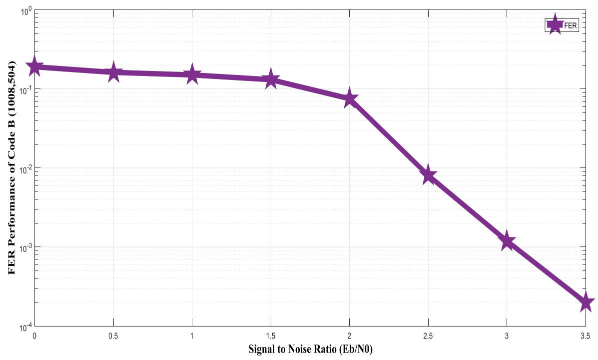

Fig. 4: Frame Error Rate Performance of Proposed RPM-QC-LDPC Code B (1008, 504).

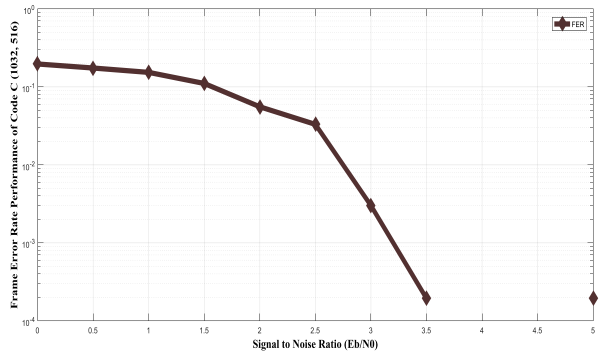

Fig. 5: Frame Error Rate Performance of Proposed RPM-QC-LDPC Code C (1032, 516).

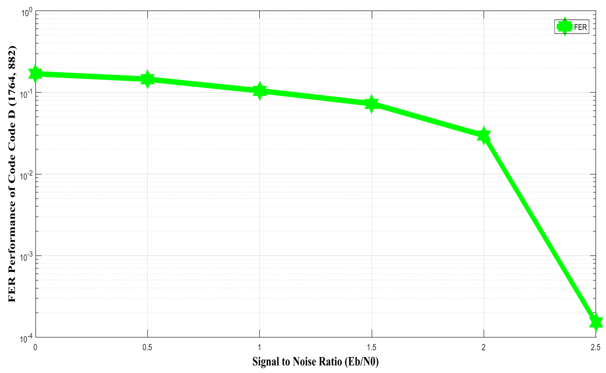

Fig. 6: Frame Error Rate Performance of Proposed RPM-QC-LDPC Code D (1764, 882).

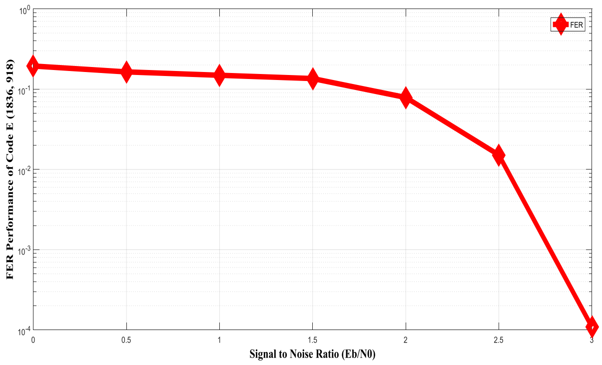

Fig. 7: Frame Error Rate Performance of Proposed RPM-QC-LDPC Code E (1836, 918).

5. Conclusion

In this paper, diagonal shifting column structured technique proposed for construction of RPM-QC-LDPC codes. There is random permutation matrix based construction presented to get better error performance instead of circulant permutation method. The row-column pairing technique is also presented for constructing sparse Quasi Cyclic-LDPC codes.
Simulation results demonstrate that the constructed RPM-Quasi Cyclic-LDPC codes do better than as compared with numerous Quasi Cyclic-LDPC Codes with Girth 6, 8, 10 and 12. The decoding complexity and construction of such codes with girth g is a further research problem for the future.

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