



# Cellular range extension by optimized two piecewise companding technique for PAPR Reduction with M ary modulation in OFDM system

Himanshu Amritlal Patel <sup>1\*</sup>, Dr. D. J. Shah <sup>2</sup>

<sup>1</sup> EC Department, U V P C E, Ganpat University, Mehsana, Gujarat, India

<sup>2</sup> CEO & Director, Shruj LED Technologies Pvt. Ltd., Ahmedabad, Gujarat, India

\*Corresponding author E-mail: [hap03@ganpatuniversity.ac.in](mailto:hap03@ganpatuniversity.ac.in)

## Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is technique which is used in fifth generation wireless communication era. It is used in Digital Audio Broadcasting, Digital Video Broadcasting, Long Term Evolution etc. One of the drawbacks of OFDM is high Peak to Average Power Ratio (PAPR) in High Power Amplifier (HPA) at the transmitter end. Two Piecewise Companding (TPWC) technique is one of the method to overcome high PAPR problem in OFDM system. It is special technique in which the average power of signal before and after companding transform remains same. Two different functions are used in TPWC in which first function does the expansion of low amplitude signals while second function does the contraction of higher amplitude signals. The optimization parameters are found for Bit Error Rate (BER) of 10<sup>-5</sup> considering Eb/No of 19.5 dB with PAPR readings taken at Complementary Cumulative Distributive Function (CCDF) of 10<sup>-5</sup>. The companding method used gives the lower PAPR. Enhancement of power after PAPR reduction leads to cellular range extension.

**Keywords:** Bit Error Rate (BER); High Power Amplifier (HPA); Orthogonal Frequency Division Multiplexing (OFDM); Peak to Average Power Ratio (PAPR); Two Piecewise Companding (TPWC).

## 1. Introduction

Wireless communication becomes a part of our life. The start of mobile communication was for simple telephonic conversion. Now a day's with a large number of mobile subscribers having smart phones the requirement has gone for high speed internet connectivity. Under such a condition OFDM is scientific boon for us. An OFDM signal is super position of various sub channels signals which delivers a very high peak signal level. So there is large fluctuation of power in HPA in transmitter of OFDM system. Under such condition, a large linear range is required in HPA, otherwise the signal will go into the non linear range of HPA leads to in band radiation and out band radiation. There are various methods available to combat the high PAPR problems like Selective Mapping (SLM) Technique [1-3], Partial Transmit Sequence (PTS) method [4-5], Tone injection [6], Tone reservation [7], clipping [8-9] etc. Companding [10-14] is one of the methodologies based on mathematical foundation to reduce the high PAPR in HPA. TPWC method is very special method in which average power before and after applying companding transform remains same which gives true PAPR reduction. The peak power can be reduced by any PAPR reduction method which gives rise to the option of increase in power in HPA in such a way that peak power remains as it is in HPA without any companding. In other words the peak power in HPA is increased in such a way after companding it becomes equal to the peak power which was available without applying companding transform.

## 2. OFDM system & two piecewise companding (2PWC)

The block diagram of transmitter and receiver of OFDM system is given below for QPSK modulation. The binary stream is modulated and serial to parallel (S/P) conversion is being done before (Inverse Fast Fourier Transform) IFFT in the transmitter part of

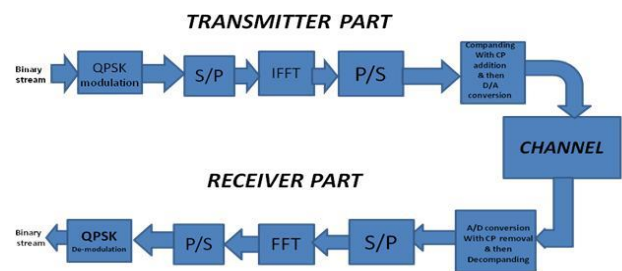


Fig. 1: Block Diagram of OFDM System.

OFDM system. The parallel to serial (P/S) is done before applying companding transform. Cyclic prefix is added and D/A is done. Companding, cyclic prefix and D/A conversion is shown in transmitter as a last block. After passing through channel, the reverse is done at the receiver end with FFT (Fast Fourier Transform) as shown in figure 1. The QPSK modulation can be replaced by any M ary QAM considered in this paper as shown in the figure 1. PAPR is usually defined mathematically as below;

$$PAPR = \frac{\text{Peak Power}}{\text{Average Power}}$$

$$PAPR = 10 \log_{10} \frac{\max[|x_n|^2]}{E[|x_n|^2]}$$

Here,  $\max[|x_n|^2]$  = maximum value of signal  $x_n$  and  $E[|x_n|^2]$  = expected value of signal  $x_n$ .  $x_n$  is an OFDM signal which is obtain after taking IFFT operation on input symbols  $X_k$ . [14]. The transfer characteristic of TPWC is shown below in with the relevant equations as in paper [13]: In transfer characteristic of TPWC case,  $u_1$  is first slope (greater than 1) while  $u_2$  (less than 1) is second slope.  $S$  is the intercept of  $u_2$  slope with Y axis and  $v = m \sigma$ .

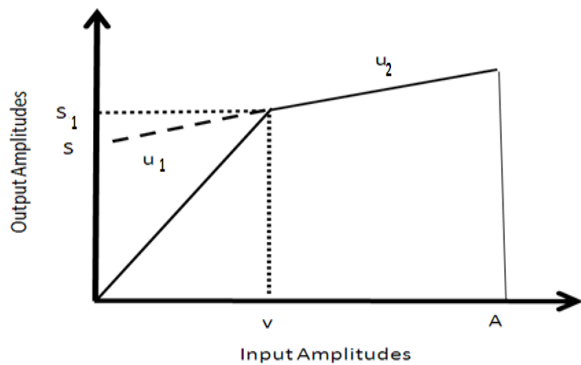


Fig. 2: Transfer Characteristic of TPWC [13].

$y_n$  is companding transform used at the transmitting end.

$$y_n = \begin{cases} (u_1 |x_n|) * \text{sgn}(x_n), & |x_n| \leq v \\ ((u_2 |x_n| + d_1) * \text{sgn}(x_n)), & |x_n| > v \end{cases} \quad [13] \quad (1)$$

If  $z_n$  is the output of decompanding transform at receiver end and  $r_n$  is a received signal after passing through channel at the decompanding end. then

$$z_n = \begin{cases} \left(\frac{1}{u_1}\right) |r_n| * \text{sgn}(r_n), & |r_n| \leq (u_1 * v) \\ \left(\frac{1}{u_2}\right) \{|r_n| - S\} * \text{sgn}(r_n), & |r_n| > (u_1 * v) \end{cases} \quad [13] \quad (2)$$

$$\begin{aligned} & \left[ \int_0^v (u_1 x)^2 f(x) dx + \int_v^\infty (u_2 x + S)^2 f(x) dx \right] \\ & = \int x^2 f(x) dx \end{aligned} \quad (3)$$

Where

$$f(x) = \frac{2x}{\sigma^2} e^{-x^2/\sigma^2}.$$

$$\int_0^\infty x^2 \frac{2x}{\sigma^2} e^{-x^2/\sigma^2} dx = \sigma^2$$

$$I = \int_0^v (u_1 x)^2 f(x) dx + \int_v^\infty (u_2 x + S)^2 f(x) dx$$

$$\text{Here } f(x) = \frac{2x}{\sigma^2} e^{-x^2/\sigma^2}$$

The expansion of I by putting the value of  $f(x)$  gives the value of I as below:

$$I = \left[ (u_1)^2 \int_0^v (x)^2 f(x) dx + (u_2)^2 \int_v^\infty (x)^2 f(x) dx + (S)^2 \int_v^\infty f(x) dx + 2 u_2 S \left( \int_v^\infty x f(x) dx \right) \right]$$

$$\text{But } S = (u_1 - u_2) v$$

$$S^2 = (u_1^2 + u_2^2 - 2 u_1 u_2) (m^2 \sigma^2)$$

$$I = \left[ (u_1)^2 \int_0^v (x)^2 f(x) dx + (u_2)^2 \int_v^\infty (x)^2 f(x) dx + (u_1^2 + u_2^2 - 2 u_1 u_2) m^2 \sigma^2 \int_v^\infty f(x) dx + 2 u_2 (u_1 - u_2) v \left( \int_v^\infty x f(x) dx \right) \right]$$

If I1 is given by:

$$I1 = (u_1)^2 \int_0^v (x)^2 f(x) dx$$

$$I1 = (u_1)^2 \sigma^2 [1 - (m^2 + 1)e^{-m^2}] \text{ is the solution.}$$

If I2 is given by:

$$I2 = (u_2)^2 \int_v^\infty (x)^2 f(x) dx$$

The solution of I2 is given as  $(u_2)^2 \sigma^2 [e^{-m^2} (m^2 + 1)]$ .

If I3 value is given by:

$$I3 = (u_1^2 + u_2^2 - 2 u_1 u_2) m^2 \sigma^2 \int_v^\infty f(x) dx$$

$$I3 = (u_1^2 + u_2^2 - 2 u_1 u_2) (m^2 \sigma^2) e^{-m^2}$$

Is the solution for the integration?

If I4 value is given by below equation:

$$I4 = 2 u_2 S \left( \int_v^\infty x f(x) dx \right)$$

$$I4 = \left\{ m^2 e^{-m^2} [2 u_1 u_2 m^2 e^{-m^2}] + [u_2 (u_1 - u_2) m \{(1 - \text{erf}(m)) \sqrt{\pi}\}] \right.$$

$$\left. - [2 u_2^2 m^2 e^{-m^2}] \right\} \sigma^2 \text{ is the solution.}$$

Now we can write  $I = I1 + I2 + I3 + I4$

But

$I =$

$\sigma^2$  as basic condition of equal average power for pre and post application of companding transform

So

$$\sigma^2 = \left\{ (u_1)^2 \sigma^2 [1 - (m^2 + 1)e^{-m^2}] \right\} + \left\{ (u_2)^2 \sigma^2 [e^{-m^2} (m^2 + 1)] \right\} +$$

$$\left\{ (u_1^2 + u_2^2 - 2 u_1 u_2) (m^2 \sigma^2) e^{-m^2} \right\} + \left\{ m^2 e^{-m^2} [2 u_1 u_2 m^2 e^{-m^2}] \right\} +$$

$$\left[ u_2 (u_1 - u_2) m \{(1 - \text{erf}(m)) \sqrt{\pi}\} \right] - [2 u_2^2 m^2 e^{-m^2}] \sigma^2$$

After expanding RHS a lots of entities gets cancelled. Thus remaining equation is as below:

$$(u_1)^2 - (u_1)^2 e^{-m^2} + (u_2)^2 e^{-m^2} + u_2 (u_1 - u_2) m \{(1 - \text{erf}(m)) \sqrt{\pi}\} - 1 = 0$$

Here;

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Let

$$A = 1 - e^{-m^2} \text{ \& } B = m \{(1 - \text{erf}(m)) \sqrt{\pi}\}$$

we will assume values of  $m$  and  $u_2$  and  $u_1$  will be calculated based on their value.

As square term is there the equation will be quadratic which can be solved by formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where x gives roots of equation

The paper [13] gave approximate solution with QPSK. The exact solution was given in the paper [14] with QPSK with CCDF of  $10^{-4}$ .

$$\text{If } A = 1 - e^{-m^2} \text{ and } B = m((1 - \text{erf}(m))\sqrt{\pi})$$

Then final equation for exact TPWC can be written as below.

$$((u_1^2 - u_2^2))A + (u_2)^2 + u_1u_2B - (u_2)^2B = 1 \quad [14]. \quad (4)$$

The optimization is done for cases of BPSK, QPSK, 16 QAM and 32 QAM using coding in MATLAB software based on the equation given above for exact TPWC. The CCDF value is  $10^{-5}$  instead of  $10^{-4}$ . The BER of  $10^{-5}$  at the  $E_b/N_0$  of 19.5 dB is considered for all cases in this paper.

### 3. Concept of range extension

PAPR reduction methods can reduce peaks in OFDM system. One way is to use this advantage as an increase the backup time of the mobile terminal. Another way is to increase the power in HPA of transmitter in such a way the reduced peaks after companding remains same with comparison to the peaks when no companding was applied.

Difference in transmitted power dB

$$= 10 \log_{10} (D_2 / D_1)^\alpha = G_p \text{ dB.}$$

$$\text{Hence we can write } (D_2 / D_1)^\alpha = 10^{(G_p/10)}$$

$$\text{In other word } [(D_2 / D_1) - 1] = \{ [10^{(G_p/10)}]^{(1/\alpha)} - 1 \}$$

$$= [(D_2 - D_1) / D_1]$$

But  $[(D_2 - D_1) / D_1] * 100\%$  is nothing but range extension percentage.

Hence,

$$\text{Range extension \%} = \{ ([10^{(G_p/10)}]^{(1/\alpha)} - 1) * 100\%$$

In other words assuming a path loss exponent ( $\alpha$ ) and considering that  $d_0$  is the original range and  $d_1$  is the range with a power gain of  $g_p$  dB, the incremental range extension (RE) by a power gain of  $g_p$  dB is given by

$$RE = [(d_1 - d_0) / d_0]$$

$$\text{Incremental range extension (RE)} = \left[ \alpha \sqrt{\left\{ (10)^{\left(\frac{g_p}{10}\right)} \right\}} - 1 \right] \quad (5) \quad [15].$$

Here,  $\alpha = \text{alpha}$ .

The alpha is nothing but free space path loss exponent in the wireless communication.

### 4. Results

The parameters which are considered during coding in MATLAB are as given in table considering CCDF of  $10^{-5}$  for PAPR reading. Bit Error Rate of  $10^{-5}$  at  $E_b/N_0$  of 19.5 dB is considered for all the cases of modulation.

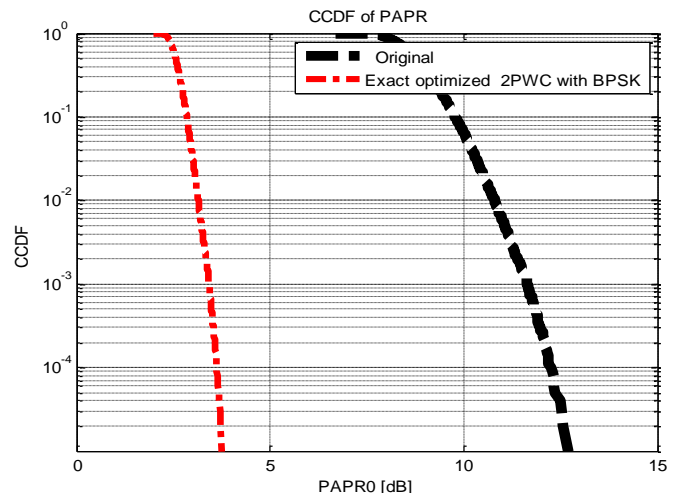
**Table 1:** General Parameters for MATLAB Coding

S.N.	Parameter	Parameter kind / Value
1	Modulation Scheme	BPSK, QPSK, 16 QAM & 32 QAM
2	No. of Carriers	1024
3	Cyclic Prefix used	256
4	Oversampling Factor	4
5	Channel used	AWGN
6	Optimization Parameters Considered	BER of $10^{-5}$ at $E_b/N_0$ of 19.5 dB

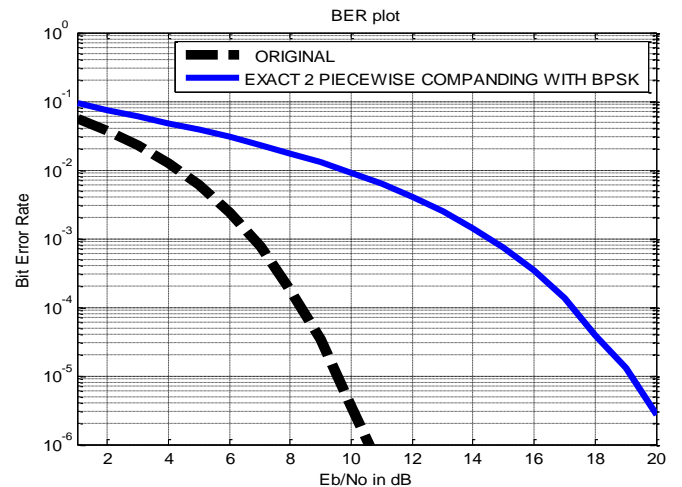
The BPSK has only two constellation points in the constellation diagram. The PAPR at CCDF of  $10^{-5}$  is considered in dB with  $u_1=1.5010$ ,  $u_2=0.1253$  and  $m=0.7262$  as an optimization parameters. These parameters are shown in table 2. Of course BER of  $10^{-5}$  is considered when  $E_b/N_0$  is 19.5 dB which is very clear from figure 4 of BER for BPSK.

**Table 2:** Optimization Parameters for BPSK

S.N.	Name of parameters	Parameters Value
1	$u_1$	1.5010
2	$u_2$	0.1253
3	$m$	0.7262
4	PAPR at CCDF of $10^{-5}$	3.85 dB



**Fig. 3:** PAPR Plot for BPSK.



**Fig. 4:** BER Plot for BPSK.

Quadrature Phase Shift Keying (QPSK) has four constellation points in the constellation diagram. The optimization parameters are considered as  $u_1=1.3700$ ,  $u_2=0.1253$  and  $m=0.8285$  in the case of QPSK for TPWC. The BER of  $10^{-5}$  is considered when  $E_b/N_0$  is 19.5 dB and PAPR at CCDF of  $10^{-5}$  is considered for this case.

**Table 3:** Optimization Parameters for QPSK

S. N.	Name of parameters	Parameters Value
1	$u_1$	1.3700
2	$u_2$	0.1253
3	$m$	0.8285
4	PAPR at CCDF of $10^{-5}$	4.04 dB

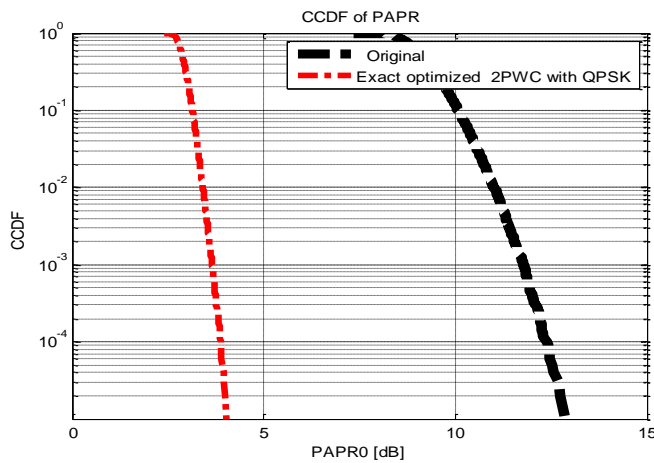


Fig. 5: PAPR Plot for QPSK.

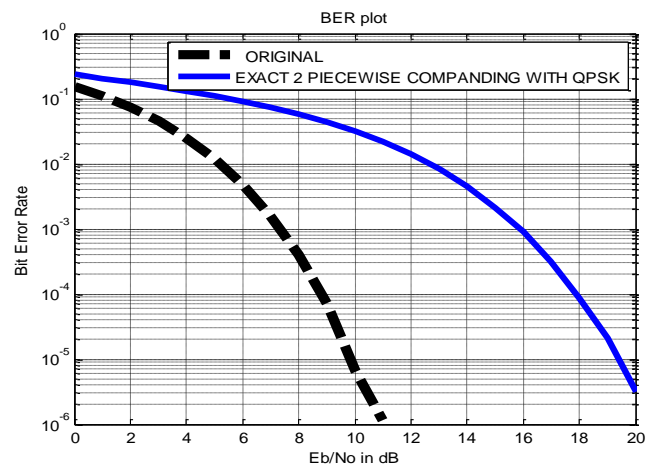


Fig. 6: BER Plot for QPSK.

There are sixteen constellation points in the constellation diagram of sixteen Quadrature Amplitude Modulation (16 QAM). The optimization parameters summary is given in table 5 with the standard condition mentioned earlier

Table 5: Optimization Parameters for 16 QAM

S.N.	Name of parameters	Parameters Value
1	u1	1.0820
2	u2	0.1541
3	m	1.34
4	PAPR at CCDF of $10^{-5}$	5.71 dB

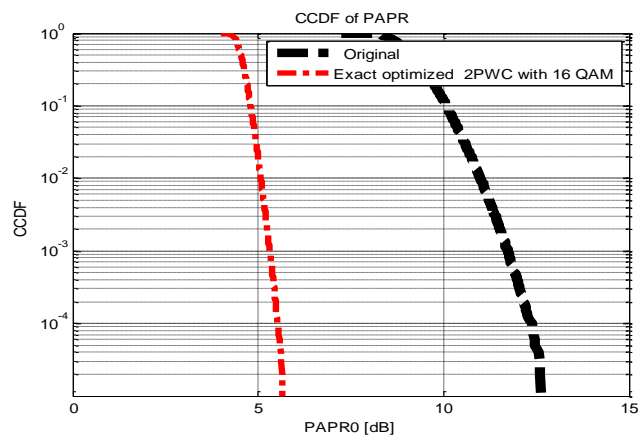


Fig. 7: PAPR Plot for 16 QAM.

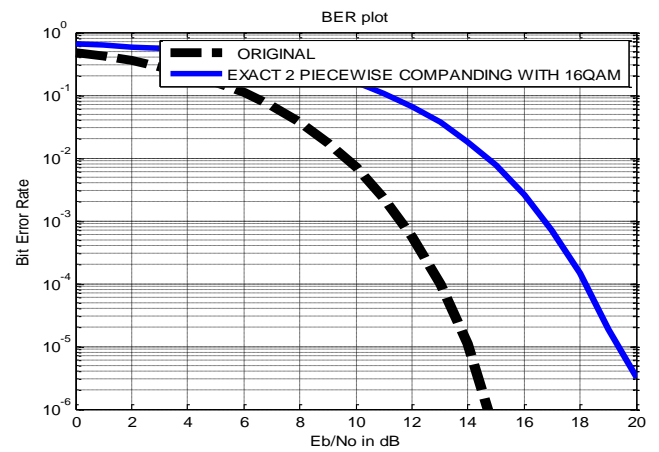


Fig. 8: BER Plot for 16 QAM.

Thirty two Quadrature Amplitude Modulation (32 QAM) has thirty two constellation points in the constellation diagram. It has highest constellation points in modulation scheme which are considered in this paper. So PAPR obtain should be highest among all the cases considered. The optimization parameters are considered in this case are  $u1=1.7492$ ,  $u2=0.1253$  and  $m=1.7492$  in the case of 32 QAM for TPWC. The BER of  $10^{-5}$  is considered when  $E_b/N_0$  is 19.5 dB and PAPR at CCDF of  $10^{-5}$  is considered for this case which is very much clear from figure 10.

Table 6: Optimization Parameters for 32 QAM

S.N.	Name of parameters	Parameters Value
1	u1	1.7492
2	u2	0.1253
3	m	1.7492
4	PAPR at CCDF of $10^{-5}$	6.55 dB

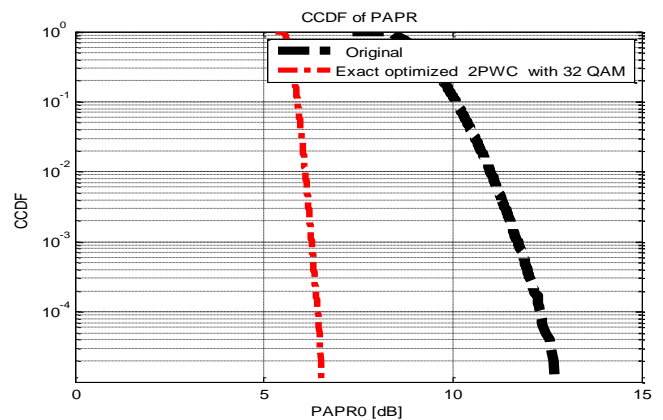


Fig. 9: PAPR Plot for 32 QAM.

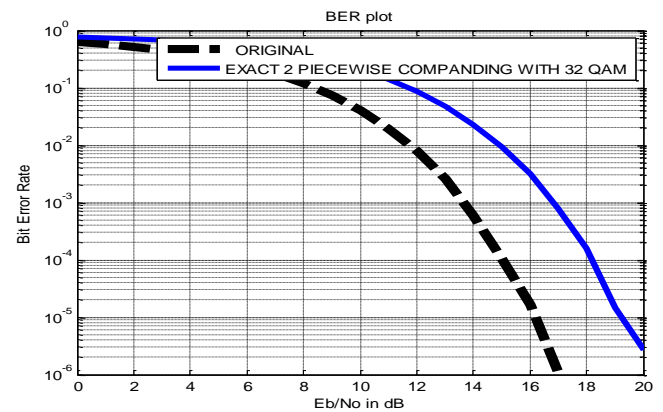


Fig. 10: BER Plot for 32 QAM.

Using equation (5) the value of range extension by reducing PAPR for different values of alpha is given below table 7.

**Table 7:** Comparison of PAPR for M Ary QAM & Percentage of Range Extension for Alpha= 2

S.N.	OFDM System Condition	PAPR at CCDF of $10^{-5}$	Difference in context of without 2PWC	Theoretical Percentage of Range Extension [Alpha= 2]
1	Without 2PWC	13 dB	0 dB	Zero percent
2	BPSK with 2PWC	3.85 dB	9.15 dB	186.74%
3	QPSK with 2PWC	4.04 dB	8.96 dB	180.54%
4	16 QAM with 2PWC	5.71 dB	7.29 dB	131.47%
5	32 QAM with 2PWC	6.55 dB	6.45 dB	110.13%

## 5. Conclusion

The table 7 is made based on the PAPR readings obtain in cases of BPSK, QPSK, 16 QAM and 32 QAM considering CCDF at  $10^{-5}$ .

**Table 8:** Comparison of PAPR for M Ary QAM

S.N.	OFDM System Condition	PAPR at CCDF of $10^{-5}$
1	Without 2PWC	13 dB
2	BPSK with 2PWC	3.85 dB
3	QPSK with 2PWC	4.04 dB
4	16 QAM with 2PWC	5.71 dB
5	32 QAM with 2PWC	6.55 dB

PAPR can be reduced by optimizing 2PWC in M ary QAM. The order of QAM decides the reduction of PAPR. If we use higher order QAM (like 32 QAM), PAPR reduces to 6.55 dB but not as much as lower order QAM like BPSK which gives PAPR of 3.85 dB at CCDF of  $10^{-5}$  or like QPSK which gives PAPR of 4.04 dB at CCDF of  $10^{-5}$ . The 16 QAM gives PAPR reduction of 5.71 dB. Different modulation schemes gives flexibility to reduce PAPR when channel is less or more noisy. In other words the concept of this paper can be used in adaptive modulation environment to reduce the PAPR. The concept of multiple input and multiple output (MIMO) can be used with the OFDM system which can further reduces the PAPR.

It can be seen that BPSK has highest percentage of range extension while 32 QAM has lowest percentage range extension. Thus we can say that if we do not want to increase the backup time of battery in wireless terminal then we can increase the range of the terminal. In both of the cases the performance of wireless link is enhanced whether by increasing back up time of the terminal equipment or the operating range of the wireless terminal.

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